Syllabus of Design of RC Structures (DRCS)

BTCVC 601 Design of RC Structures

Teaching Scheme: (3 Lectures + 1 Tutorial) hours/week

Course Contents

Module 1: Introduction (4 Lectures)

Basic Aspects of Structural Design, Introduction to Design Philosophies, Stress Strain behavior of Materials Working stress method, Ultimate load method and Limit state method, Comparison of Different Philosophies, Factor of Safety, Estimation of Loads.

Working Stress Method

Module 2: (8 Lectures)

Stress block parameters, permissible stresses, balanced, under reinforced and over reinforced section, analysis and design for flexure, shear, analysis and design of singly and doubly reinforced beams. Design of axial and uniaxial eccentric loaded columns, Isolated Column Footings, WSM design requirements as per Annexure B of IS 456:2000

Limit State Method

Module 3: Introduction to LSM (10 Lectures)

Introduction to limit state approach, types and classification of limit states, characteristics strength and characteristics load, load factor, partial safety factors, strain variation diagram, stress variation diagram, serviceability criteria

Limit State of Collapse in Shear and Bond

Design for shear: shear failure, types of shear reinforcement, minimum shear reinforcement, design of shear reinforcement

Design for bond: types, factors affecting, resistance, check for development length, detailing of reinforcement

Module 4: Limit State of Collapse in Flexure (16 Lectures)

Design of beams: Analysis and Design: Singly and Doubly Reinforced Beams, Flanged (L and T) sections.

Design of Slabs: One-Way and Two-Way Slab: Behavior of slabs, types, support conditions, analysis and design with various conditions Staircases, effective span and load distribution, design of dog- legged and open well stair case.

Module 5: Limit State of Collapse in Compression (10 Lectures)

Design of columns, and footings

Analysis and design of axially and eccentrically loaded short columns (Circular and Rectangular), construction of Interaction diagrams for uni-axial bending and its application in design, concept of design charts, concept of bi-axial bending, concept of interaction surface, Design of isolated column footing for axial load, and uni-axial bending.

Text Books

- **IS: 456-2000, IS: 456-1978, Bureau of Indian Standards, New Delhi**
- **Karve and Shah, "Limit State Theory & Design", Structures Publications, Pune**
- **Jain A.K., "Reinforced Concrete Design (Limit State)", Nemchand Brothers, Roorkee**
- **Sinha and Roy, "Fundamentals of Reinforced Concrete"**
- **Sinha S.N., "Reinforced Concrete Design, Vol. I, II", Tata Mc-Graw Hill**
- **Varghese P.C., "Limit State Design of Reinforced Concrete", Prentice Hall of India, New Delhi**
- **Mehra H. and V.N. Vazirani, "Limit State Design of Reinforced Concrete Structures", Khanna Publishers, N. Delhi, ISBN No: 978-81-7409-162-9**
- **Vazirani V.N. and Ratwani M.M., "Design of Reinforced Concrete Structures", Khanna Publishers, N. Delhi, ISBN No: 978-81-7409-232-8**
- **Pillai S Unnikrishna, and Menon Devdas., "Reinforced Concrete Design" Tata Mc-Graw Hill**

Reference Books

- **Punmia B.C., "Reinforced Concrete Design, Vol. I, II", Laxmi Publications**
- **Relevant Publications by Bureau of Indian Standards, New Delhi**

Course Outcomes: On completion of the course, the students will be able to comprehend the various design philosophies used in design of reinforced concrete. Analyze and design the reinforced concrete sections using working stress and limit state method.

Introduction

R.C.C. Structure design : a combination of concrete and steel reinforcement that are joined into one piece and work together in a structure. The term “reinforced concrete” is frequently used as a collective name for reinforced-concrete structural members and products. For a strong, ductile and durable construction the reinforcement needs to have the following properties at least:-

1.High relative strength

2. High toleration of tensile strain

3. Good bond to the concrete, irrespective of pH, moisture, and similar factors

4. Thermal compatibility, not causing unacceptable stresses (such as expansion or contraction) in response to changing temperatures.

5. Durability in the concrete environment, irrespective of corrosion or sustained stress for example

History

François Coignet was the first to use iron-reinforced concrete as a technique for constructing building structures. In 1853, Coignet built the first iron reinforced concrete structure, a four-story house at 72 rue Charles Michels in the suburbs of Paris. Coignet's descriptions of reinforcing concrete suggests that he did not do it for means of adding strength to the concrete but for keeping walls in monolithic construction from overturning. In 1854, English builder William B. Wilkinson reinforced the concrete roof and floors in the two-story house he was constructing. His positioning of the reinforcement demonstrated that, unlike his predecessors, he had knowledge of tensile stresses. Joseph Monier was a French gardener of the nineteenth century, a pioneer in the development of structural, prefabricated and reinforced concrete when dissatisfied with existing materials available for making durable flowerpots. He was granted a patent for reinforced flowerpots by means of mixing a wire mesh to a mortar shell In 1877. Before 1877 the use of concrete construction, though dating back to the Roman Empire, and having been reintroduced in the early 1800s, was not yet a proven scientific technology. Ernest L. Ransome was an English-born engineer and early innovator of the reinforced concrete techniques in the end of the 19th century. G. A. Wayss was a German civil engineer and a pioneer of the iron and steel concrete construction. In 1879. Wayss bought the German rights to Monier's patents and in 1884 One of the first skyscrapers made with reinforced concrete was the 16-story Ingalls Building in Cincinnati, constructed in 1904 The first reinforced concrete building in Southern California was the Laughlin Annex in Downtown Los Angeles, constructed in 1905 The National

Association of Cement Users (NACU) published in 1906 "Standard No. 1" in 1910 the "Standard Building Regulations for the Use of Reinforced Concrete".

Fig. 1. R.C.C Building

Necessity of Steel bar in concrete

1.R.C.C. having high strength in· compression but very weak in tension and for reinforcementvice-versa

2. The tensile strength of concrete is about 10-15% of its compressive strength, to overcome this difficulty it becomes necessary to reinforce the plain. cement concrete by placing steel bars in tensile zone of the concrete. (i.e. to increase tensile strength of tensile zone of the concrete section, concrete is to be reinforced).

In both the cases steel reinforcement is provided in tensile zone. only, such beam is known as singly reinforced beams. However, steel bars are also provided in compression zone is termed as anchor bars, to hold the stirrups in position. If the steel bars are provided on compression side which will assist the concrete in taking compression is known as doubly reinforced section.

Grade of Concrete According to IS. 456 : 2000,

The concrete mixes are designated as M10, M15, M20 -Ordinary concrete.

M25, M30, M35, M40, M45, M50, M55 - standard concrete.

M60, M65, M70, M75, M80-High strength concrete.

where, M – Concrete mix. Number - Ultimate compressive strength of 15cm cube at 28 days expressed in N/mm^2.

Function Of Reinforced Concrete

1. To resist direct or bending tension and compression.

2. To strengthen the concrete in compression also.

3. To resist diagonal tension due to shear.

4.To prevent buckling of main bars in column

5.To resist spiral cracking due to torsion

Advantages and Disadvantages of RCC

Strength : R.C.C. has very good strength in tension as well as compression.

Durability : .R.C.C. structures are durable if designed and laid properly. They can last up to 100 years. Mouldability : R.C.C. sections can be given any shape easily by properly designing the formwork. Thus, it is more suitable for architectural requirements.

Ductility : The steel reinforcement imparts ductility to the R.C.C. structures

Economy : R.C.C. is cheaper as compared to steel and prestressed concrete. There is an overall economy by using R.C.C. because its maintenance cost is low

Transportation: The raw material which are required for R.C.C. i.e. cement, sand aggregate, water and steel are easily available and can be a sported easily. Nowadays Ready Mix Concrete, is used for faster and better construction. (RMC is the concrete which is manufactured in the factory and transported to the site in green or plastic state).

Fire Resistance : R.C.C. structures are more fire resistant than other commonly used construction materials like steel and wood.

Permeability : R.C.C. is almost impermeable to moisture

Seismic Resistance : Properly designed R.C.C. structures are extremely resistant to earthquake

Disadvantages of RCC

1. R.C.C. structures are heavier than structures of other materials like steel, wood and glass etc.
2. R.C.C. needs lot of formwork, centering and shuttering to be fixed, thus require lot of space and skilled labour

3.Concrete takes time to attain its full strength. Thus, R.C.C structures can be used immediately after construction unlike steel structures

Assumption for design of member (WSM): (IS 456:2000, Page No. 80, Cl. No. B-1.3)

1. At any cross section, plane sections before bending remain plain after bending.

2. All tensile stresses are taken by reinforcement and none by concrete

3.The stress-strain relationship of steel and concrete under working loads, is a straight line.
4.There is perfect bond between steel arid concrete and no slip takes place between steel and concrete.

5. The modular ratio ‘m' has the value= $m = \frac{280}{3 \text{ x } 6_{cbc}}$

Modular Ratio is defined as the Ratio between Modulus of Elasticity of Steel and Modulus of Elasticity of Concrete. This is because, a Reinforced Concrete is made up of Both Steel and Concrete. In this case, Steel is a Tension member and Concrete is a Compression Member.

$$m = \frac{E_s}{E_c}$$

$$E_s = 2 \text{ x } 10^5 \text{ N/mm}^2$$

$$E_c = 5000\sqrt{F_{ck}}$$

$F_{ck} = Grade$ of Concrete in N/mm^2

Structural Properties

1.Compressive strength.

2.Tensile strength.

3.Modulus of elasticity and Poisson's ratio.,

4.Stress-strain relationship.

5.Shrinkage of Concrete

6.Creep of Concrete

1. Compressive Strength of Concrete

1. The compressive strength of hardened concrete is found by testing to failure 150 mm cubical (According to B.I.S) .) specimens after 28 days of standard curing. At least three specimens are made for test at specific age.

2. The compressive strength of concrete can also be found by testing to failure cylindrical specimens of diameter = 150 mm and height = 300 mm. Such test is carried out in United States. In India, the concrete grade is based on cube strength and if the cylinder is tested, the strength should be modified into equivalent cube strength. The ratio of the cylinder strength to the cube strength may be taken to be 0.8.

2.Tensile Strength of Concrete

Measurement of tensile strength by subjecting the specimen to direct tension is extremely difficult Therefore, indirect measurements for tensile strength are made. Tensile strength of concrete can be measured by two methods

1. Split-cylinder test (IS : 5816-1999)

2.Standard beam-test (modulus of rupture test) (IS :516)

3. Elastic Modulus of Concrete : The stress-strain curve for concrete is shown in Fig.2. It is obtained from a compression test on cylindrical specimens. Since, concrete is not an elastic material, therefore the obtained stress-strain curve for concrete is non-linear. The slope of the tangent drawn to the stress-strain curve is maximum at the origin and reduces to zero at the peak. To adopt this slope as the elastic modulus of concrete would be both erroneous· and inconvenient. However, the slope of the secant which is the line joining a point on the curve to the origin, does not vary too widely between the origin and the peak. As the lower portion of the stress-strain curve is relatively straight, an elastic modulus may be conveniently defined in that region. Thus, the elastic modulus of concrete is taken to be the slope of the secant drawn to the stress-strain curve at a point corresponding to 40% of the maximum stress. However, in the absence of test results, the modulus of elasticity is normally related to the compressive strength of concrete as

$E_c = 5000\sqrt{F_{ck}}$

$F_{ck} = Grade$ of Concrete in N/mm^2

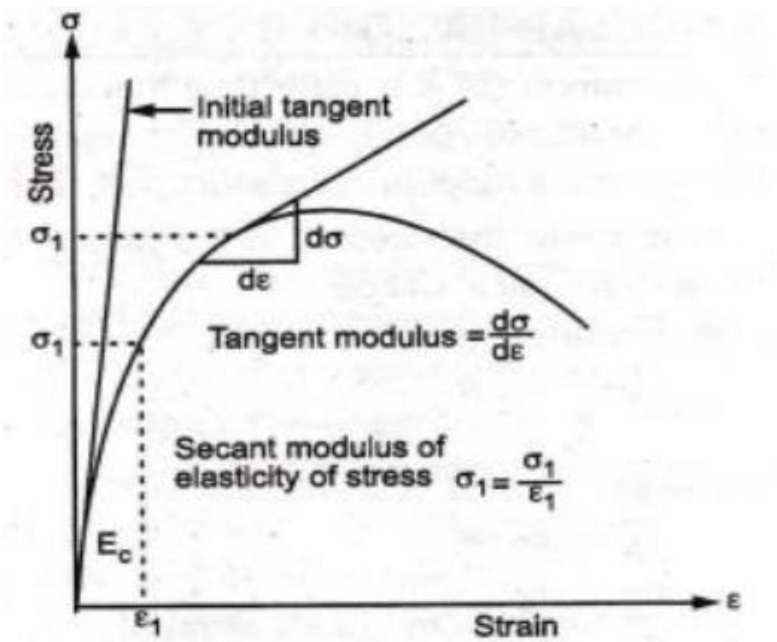
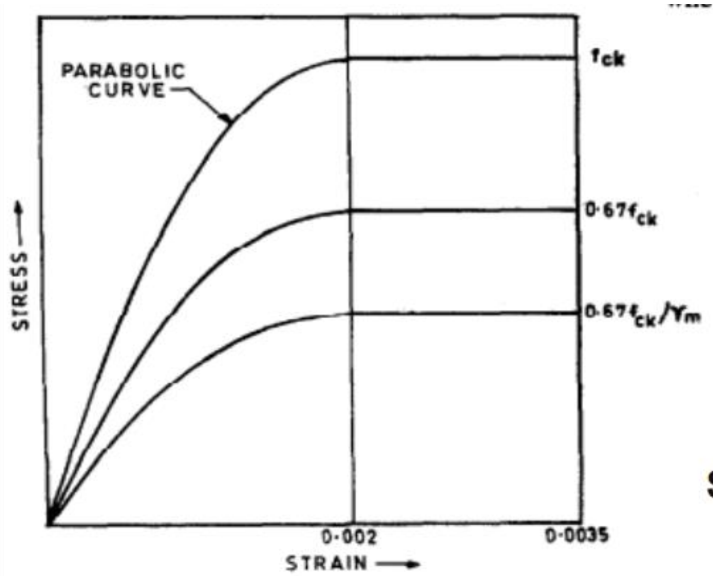
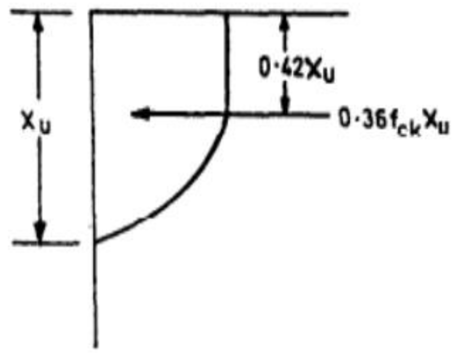


Fig. 2. Stress strain behavior of concrete

4. Stress- Strain Curve for concrete



Stress- Strain Curve for concrete



Stress block parameter

5. Shrinkage of Concrete :

The contraction of concrete per unit length during the process of hardening is known as Shrinkage. The total shrinkage of concrete depends upon the constituents of concrete, size of the members, environmental conditions and percentage of steel. The shrinkage is a long process and continues for many years particularly for mass concrete, The designer must provide the shrinkage steel to prevent shrinkage cracks. Greater the percentage of steel, lesser is the shrinkage because the reinforcement restrains the shrinkage. A curve showing shrinkage strain against time after commencement of drying is shown in Figure From Fig.3., it is clear that the rate of Shrinkage decreases with time. The total shrinkage strain may be in the range of 0.0002 to 0.0007 The IS : 456 recommends a value for shrinkage to be 0.0003 for the purpose of design.

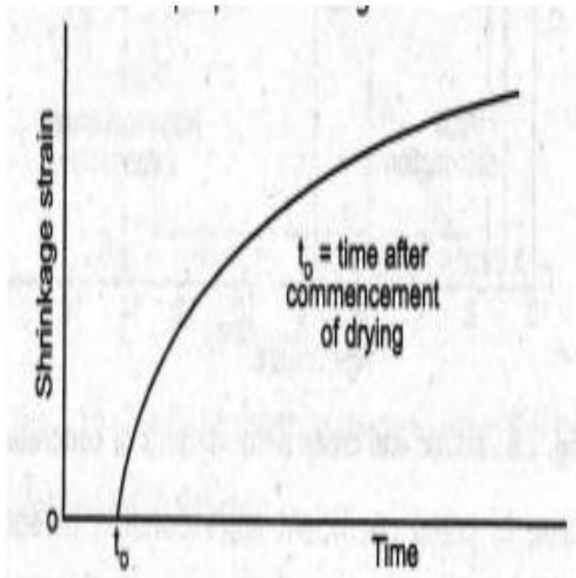


Fig.3. Shrinkage curve for concrete

6. **Creep:** A plastic deformation under sustain load

Methods of Design

The aim of design is to design, shape, size and connection details of the members so that the structural beam design will performed satisfactory during its right span. Following methods of design are used in concrete structures
1) Working Stress Method (WSM)
2) Ultimate Load Method (ULM)
3) Limit State Method (LSM)

1) Working Stress Method (WSM) :
Working Stress Method is the traditional method of design not only for Reinforced Concrete but also for structural steel and timber design. The conceptual basis of the WSM assumes that the structural material behaves in a linear elastic manner and that appropriate safety can be ensured by suitably limiting the stresses in the material due to the presumed working loads (service loads) on the structure. WSM also assumes that both the steel reinforcement and concrete act together and are perfectly elastic at all stages, and hence the modular ratio can be used to determine the stresses in steel and concrete. The stresses under the working loads are obtained by applying the methods of 'strength of materials' like the simple bending theory. The limitations due to non-linearity and buckling are neglected. The stresses caused by the 'characteristic' or service loads are checked against the permissible (allowable) stress, which is a fraction of the

ultimate or yield stress. The permissible stress may be defined in terms of a factor of safety, which takes care of the overload or other unknown factors.

Limitations of Working Stress Method

1.The main assumption of a linear elastic behavior and the implied assumption that the stresses under working loads can be kept within the 'permissible stresses' are found to be unrealistic. Many factors are responsible for this, such as the long-term effects of creep and shrinkage and other secondary effects.

2.The use of the imaginary concept of modular ratio results in larger percentage of compression steel and generally larger member sizes than the members designed using ultimate load or limit states design. However, as a result of the larger member sizes, they result in better performance during service.

2) Ultimate Load Method (ULM)

This is also known as load factor method or ultimate strength method. In this we make use of the nonlinear region of stress strain curves of steel and concrete. The safety is ensured by introducing load factor.

"Load factor is the ratio of ultimate strength to the service loads"

The ULM makes it possible to consider the effects of different loads acting simultaneously thus solving the shortcomings of WSM. As the ultimate strength of the material is considered we will get much slender sections for columns and beams compared to WSM method. But the serviceability criteria is not met because of large deflections and cracks in the sections. The fall-back in the method was that even though the nonlinear stress strain behaviour of was considered sections but the nonlinear analysis of the structural was not carried out for the load effects. Thus the stress distribution at ultimate load was just the magnification of service load by load factor following the linear elastic theory.

3) Limit State Method (LSM)

In limit state design method, the structure shall be designed withstand safety. All loads likely to act on it throughout its life span. It shall not suffer total collapse under accidental load such as from explosion or impact or human error to an extend beyond the local damages. The acceptable limit for safety, serviceability requirement before failure occurs it called limit state.

Steel structure are to be design and constructed to safety. The design requirement with regard to stability, strength, serviceability, brittle, fracture, fatigue, fire and durability such that they need the following points.

a) Remain free adequate re-ability be able to sustain all loads.

b) We have adequate durability under normal maintenance

c) Do not suffer overall damage as collapse

Sl. No.	Working Stress Method	Limit State Method
1.	This method is based on the elastic theory which assumes that concrete and steel are elastic, and the stress strain curve is linear for both.	This method is based on the actual stress-strain curves of steel and concrete. For concrete the stress-strain curve is non-linear.
2.	In this method the factor of safety is applied to the yield stresses to get permissible stresses.	In this method, partial safety factors are applied to get design values of stresses.
3.	No factor of safety is used for loads.	Design loads are obtained by multiplying partial safety factors of load to the working loads.
4.	Exact margin of safety is not known.	Exact margin of safety is known.
5.	This method gives thicker, sections, so less economical.	This method is more economical as it gives thinner sections.
6.	This method assumes that the actual loads, permissible stresses, and factors of safety are known. So it is called as deterministic method.	This method is based upon the probabilistic approach which depends upon the actual data or experience; hence it is called as non-deterministic method.
7.	Working stress method is also known as the plastic method	Limit State method is also known as the Elastic design
8.	In working stress method, the material follows Hooke’s law as stress is not allowed to cross the yield limit.	Limit state method, stress is allowed to cross the yield limit.
9.	This method gives more large sections, therefore less economical.	This method is more economical since it gives thinner sections.
10	This method assumes that the actual loads, permissible pressures, and factors of safety have been understood. So it's called a deterministic method.	This way is based upon the probabilistic approach that depends upon the real data or expertise; thus it's referred to as a non-deterministic method.

Types of loads on steels structure:

1) Dead Load:

Dead loads are permanent and stationary load which are transferred to the structure thought there their life span. Dead load is primaralily due to self weight of structural member. Permanent partition wall fixed permanent equipment and weight of different material.

Plain concrete- 25 KN/m^3

R.C.C concrete-25 KN/m^3

Soil-18 KN/m^3

Rolled steel-79 KN/m^3

IS Code used for dead load is IS 875-1987 Part-I

2) Live Load :

Live load are either moveable or moving load without any impact. These are assumed to be produced by the intended use or occupancy of the building including weight of movable portion. Live load is consider according to i.s.875 part-II.

Sr.no	Type of load	Maximum live load
1	Residential building	2 KN/m^2
2	Bank, office	3 KN/m^2
3	Classroom assembly hall	4 KN/m^2
4	Workshop, factory:- Light weight- Medium weight- Heavy weight-	 5 KN/m^2 7.5 KN/m^2 10 KN/m^2

3) Wind Load:

Wind load basically horizontal load causes by movement of air. Wind load is required to be consider in the design specially when the height of building exceeds the two times of dimensions transferred to expose surface. Wind depend upon intensity of wind pressure and shape of structure in case of truss design two type of wind type of wind pressure considered

1) Internal air pressure:-it depend on permeability of structure

2) External air pressure:-it depend on location of structure

Internal air pressure depends upon permeability of structure and external air pressure depends upon location.

IS Code used for wind load is 875 part-III.

4) Earthquake Load: If structure is situated in earthquake prone area, earthquake load may be considered due to earthquake shock structure vibration. Earthquake load are horizontal load caused by earthquake and shall be calculated in accordance with IS 1983 and revised IS 2017.

5) Snow Load: This depend upon latitude of placed. Design snow load depends upon shape of roof and this load act vertically and this load can be taken as 2.5 KN/m^2 per mm depends of snow.

6) Imposed Load: Imposed load caused by vibrator or impact or acceleration of person walking, produce live load but soldiers marching or frame supporting lifts produced impact load. Thus impact load is equal to imposed load incremental by some percentage depending on the intensity of impact.

7) Hydrostatic Pressure: Pressure of water is to be considered which are below the ground level. Hydrostatics pressure is calculated from established theories.

8) Temperature Effects: Due to change in temperature in structural member, extract or contract and produced the loading effects in member.

Load Combination:

The combination of the load are necessary to ensure the required safety and economic design. Load combination as per IS 875 Part IV

Dead Load (DL)

Live Load (LL)

Wind Load (WL)

Earthquake Load (EL)

Temporary Load (TL)

Combination

1) 1.5 (DL + LL)

2) 1.2 (DL+LL+EL)

3) 1.2 (DL+LL-EL)

4) 1.5 (DL + EL)

5) 1.5 (DL - EL)

6) 0.9 DL+ 1.5 EL

7) 0.9 DL-1.5 EL etc.

Design of Beam (WSM)

Singly and Doubly Reinforcement of beam

Beam can be defined as a structural member which carries all vertical loads and resists it from bending. There are various types of materials used for beam such as steel, wood, aluminum etc. But the most common material is reinforced cement concrete (RCC).

Depending upon different criteria RCC beam can be of different types such as –

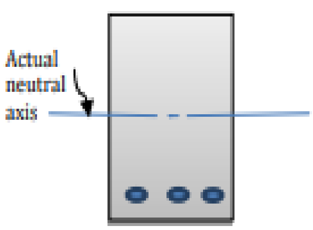
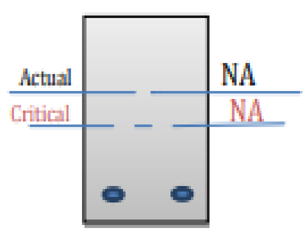
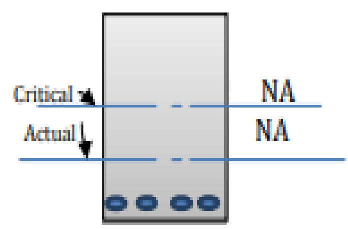
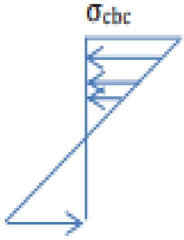
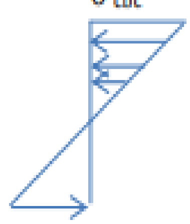
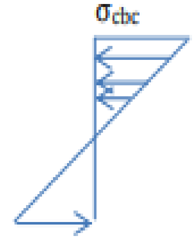
Depending upon shape beams can be T-beam, rectangular beam, etc.

Depending upon placement of reinforcement – singly reinforced beam, doubly reinforced beam and Flanged beams etc.

Singly reinforced beam: The beam that is **longitudinally reinforced** only in tension zone, it is known as a **singly reinforced beam**. In Such beams, the **ultimate bending moment and the tension** due to bending are carried by the reinforcement, while the **compression** is **carried by the concrete**. But it is not possible to provide reinforcement only in the tension zone, because we need to tie the stirrups. Therefore, two rebars are used in the compression zone to tie the stirrups, and the rebars act as false members only to hold the stirrups.

Doubly reinforced beam: The doubly reinforced beams have compression reinforcement in addition to the tension reinforcement, and this compression reinforcement can be on both sides of the beam (top or bottom face), depending on the type of beam, that is, simply supported or cantilever, respectively. The beam that is reinforced with steel in the tension and compression zone is known as the doubly reinforced beam. This type of beam is provided mainly when the depth of the beam is restricted. If a beam with limited depth is reinforced only on the tension side, it may not be strong enough to withstand the bending moment. The resistance moment cannot be increased by increasing the amount of steel in the stress zone. To increase, the beam is reinforced, but not more than 25%, on the tensioned side. Thus, a doubly reinforced beam is provided to increase the strength moment of a beam with limited dimensions. Steel reinforced beams in compression and tension zones are called doubly reinforced beams.

Flanged beams (T-Beams and L-Beams): The beams in which a portion of the slab acts together with the beam for resisting compression stress are called as flanged beams.

Balanced sections	Under reinforced sections	Over reinforced sections
Such type of section In which concrete and Steel attain its Permissible strength is termed as balanced section or critical section or economical section.	Such types of section in which steel attain its permissible stress but concrete attain stress lower than its permissible stress.	Such type of section in which concrete attain its permissible strength but steel remain below to its permissible strength.
This type sections occurs when amount of provided steel is neither less nor more than the steel required for a critical section.	This type of sections occurs when area of provided steel is less than the area of steel required for balanced section.	This type sections occurs when area of provided steel is more than the area of steel required for balanced section.
In this type of sections critical Neutral axis and Actual neutral axis are same line.  	Actual Neutral axis remains above than the critical neutral axis.  	Actual neutral axis remains below than critical neutral axis. 
Stress diagram σ_{cbc}  σ_{st}/m	Stress diagram σ'_{cbc}  σ_{st}/m $\sigma'_{cbc} < \sigma_{cbc}$	Stress diagram σ_{cbc}  σ'_{st}/m $\sigma'_{st} < \sigma_{st}$
Moment of resistence, $MR = bn_c\sigma_{cbc}\frac{(d - n_c/3)}{2}$ $= \sigma_{st} A_{st} (d - n/3)$ $= Qbd^2$	Moment of resistence, $MR = bn\sigma'_{cbc}\frac{(d - n/3)}{2}$ $= \sigma_{st} A_{st} (d - n/3)$	Moment of resistence, $MR = bn\sigma_{cbc}\frac{(d - n/3)}{2}$ $= \sigma'_{st} A_{st} (d - n/3)$

A singly reinforced beam section is shown in Fig. 2.3(a). To analyse this section, it is necessary to convert it into a transformed or equivalent section of concrete.

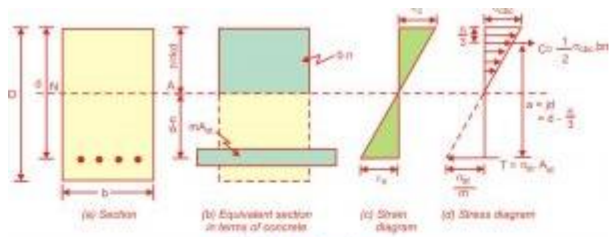


Fig. 2.3. Singly reinforced beam.

Equivalent or Transformed Section

As per the assumption (3), all the tensile stresses are taken by steel and none by concrete i.e., concrete in the tensile zone is cracked. So, the concrete area below the neutral axis is neglected and the effective area or the equivalent area of the section in terms of concrete is shown in Fig. 2.3(b). The equivalent area is equal to the area of concrete in the compression zone and an additional concrete area mA_{st} of concrete corresponding to steel area, A_{st}

Strain Diagram

As per the assumption (1) of elastic theory, the strain distribution is linear, with value zero at the neutral axis to maximum at the top and bottom fibre. The strain diagram for the given R.C.C. section is shown in Fig. 2.3(c).

Stress Diagram

As per the assumption (4) of the elastic theory the stress-strain relationship is linear for concrete. So, the stress diagram is also a straight line with value zero at neutral axis and varying linearly with the distance as shown in Fig. 2.3(d).

Maximum permissible stress at the top most fibre in concrete $=\sigma_{cbc}$

Maximum permissible stress in steel $=\sigma_{st}$

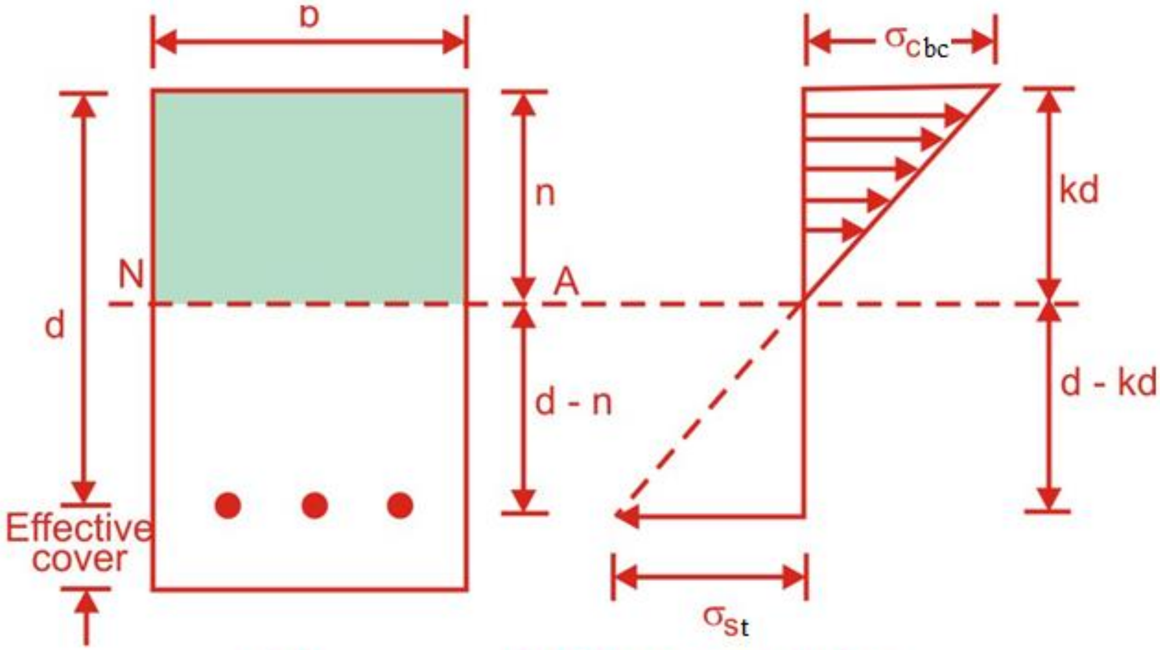
Maximum stress in equivalent concrete area at the level of steel$=\sigma st / m$

Note: 1. The suffix cbc in σ_{cbc} stands for permissible stress in *concrete in bending compression*.

2. The suffix *st* in σ_{st} stands for permissible stress in *steel in tension*.

Neutral Axis (n)

Neutral axis lies at the centre of gravity of the section. *It is defined as that axis at which the stresses are zero.* It divides the section into tension and compression zone. The position of the neutral axis depends upon the shape (dimensions) of the section and the amount of steel provided. The position of neutral axis of any rectangular section can be found by the following two methods :



(a) (b) Stress diagram for given loads

Let us consider the R.C.C. section shown in Fig. 2.4(a) the stress σ_{cbc} in concrete's top most fibre and σ_{st} in steel reinforcement are known.

From stress diagram:

$$\frac{6_{cbc}}{n} = \frac{6_{st} / m}{d - n}$$

From Similar triangles

$$\frac{m\ 6_{cbc}}{6_{st}} = \frac{n}{d - n}$$

If the stresses in concrete and steel are permissible then equation for n is written as

$$\frac{m\ 6_{cbc}}{6_{st}} = \frac{n}{d-n}$$

This neutral axis, corresponding to permissible values of stresses of concrete and steel is called as critical neutral axis n_c .

n_c =kd where k is the neutral axis depth factor.

$$\frac{m\ 6_{cbc}}{6_{st}} = \frac{kd}{d-kd}$$

On rearranging, we get

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}}$$

Putting $m = \frac{280}{3 \text{ x } 6_{\text{cbc}}}$

n the above equation for k, we can see that k does not depend upon grade of concrete. It depends upon grade of steel only.

$$k = \frac{(280/3)}{(280/3) + 6_{st}}$$

The moment of the tensile and compressive area should be equal at the neutral axis. The neutral axis obtained by this method is called as *actual neutral axis*.

Moment of compressive area = Area in compression × Distance between c.g. of compressive area and neutral axis

Moment of compressive area = $b.n.\frac{n}{2} = b.\frac{n^2}{2}$

Moment of tensile area = Equivalent tensile area × Distance of centroid of steel reinforcement from neutral axis

Moment of tensile area = m.Ast×(d−n)

Moment of compressive area = Moment of tensile area

$$b.\frac{n^2}{2} = \text{m.Ast}\times(\text{d}-\text{n})$$

It is a quadratic equation which will give two values of n. Out of these two values only one value (+ve) of n is possible.

Lever Arm

Lever arm is the distance between the resultant compressive force and the resultant tensile force. It is denoted as a in the stress diagram. As the compressive area is triangular, the resultant compressive force (C) will act at $\frac{n}{3}$

from the top compressive fibre. The resultant tensile force (T) will act the centroid of the steel reinforcement.

Lever arm = $z = d - \frac{n}{3}$

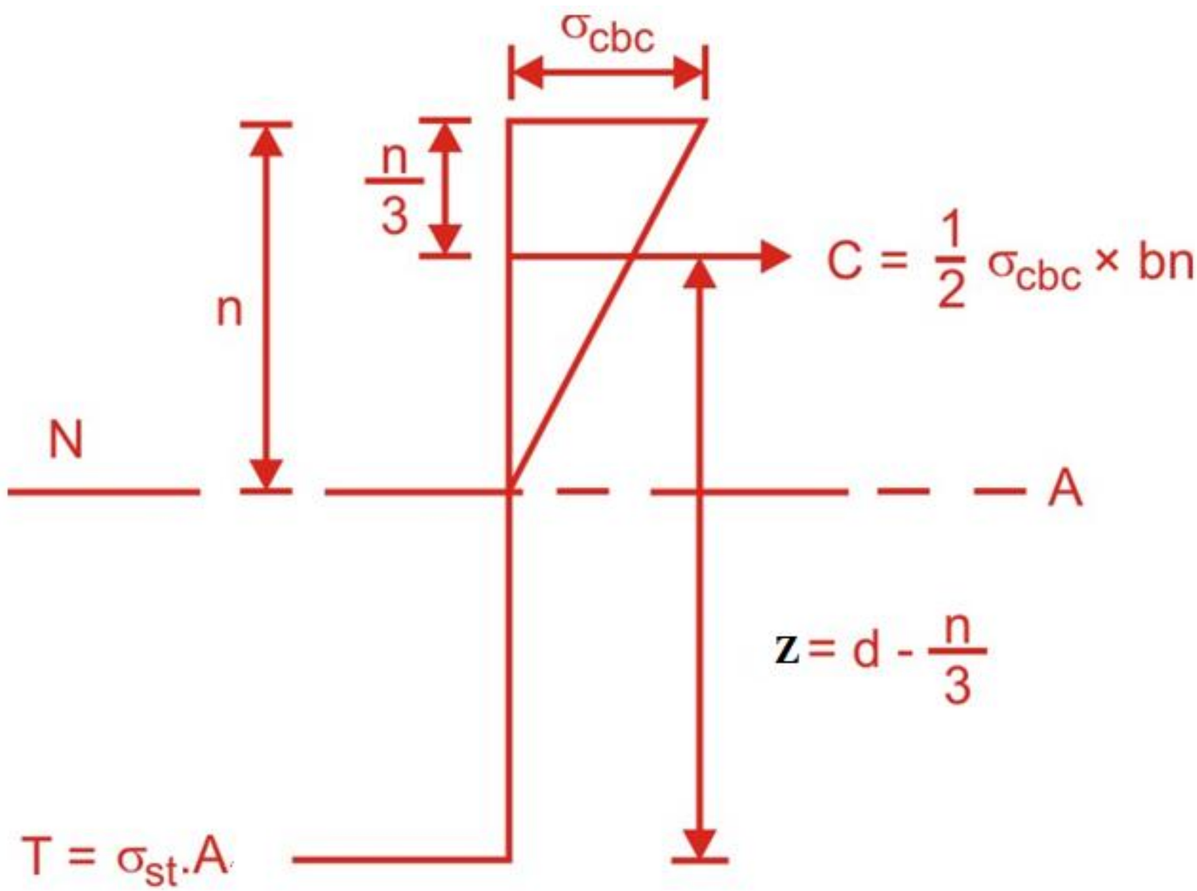
it is also expressed as z = jd where j is the lever arm depth factor.

$$jd = d - \frac{kd}{3}$$

$$j = 1 - \frac{k}{3}$$

Moment of Resistance (M_r)

Moment of resistance is the resistance offered by the beam against external loads. As there is no resultant force acting on the beam and the section is in equilibrium, the total compressive force is equal to the total tensile force. These two forces (equal and opposite separated by a distance) will form a couple (Fig. 2.5) and the moment of this couple is equal to the resisting moment or moment of resistance of the section.



Total compression = $C=\frac{1}{2}6_{cbc}$ b n acting at $\frac{n}{3}$

from top

Total tension = $T=6_{st}$ A_{st} acting at centroid of steel reinforcement.

Moment of resistance = C . z or T . z

$$Mr=\frac{1}{2}6_{cbc}\ \text{b n}\left[\text{d-}\frac{n}{3}\right]$$ for compression I

$$Mr=6_{st}\ A_{st}\left[\text{d-}\frac{n}{3}\right]$$ for tension II

Putting n = kd in the equation (I),

$$Mr = \frac{1}{2} 6_{cbc} \text{ b kd } \left[\text{d-} \frac{kd}{3} \right]$$

$$Mr = \frac{1}{2} 6_{cbc} \text{ k } \left[1\text{-} \frac{k}{3} \right] \text{ bd}^2$$

$$Mr = \frac{1}{2} 6_{cbc} \text{ k j bd}^2$$

$$Mr = \text{ Qbd}^2$$

where Q is called as resisting moment factor.

$$Q = \frac{1}{2} 6_{cbc} \text{ k j}$$

Percentage of Steel P_t

Equating compressive force C to tensile force T

$$bn \frac{6_{cbc}}{2} = 6_{st} \text{ A}_{\text{st}}$$

For balanced section

n=kd

$$\frac{bkd\,6_{cbc}}{2} = 6_{st} \text{ A}_{\text{st}}$$

$$\frac{\text{A}_{\text{st}}}{bd} = \frac{6_{cbc}\, k}{2 6_{st}}$$

Percentage steel

$$p_t = \frac{\text{A}_{\text{st}}}{bd} x100$$

$$\text{P}_{\text{t}} = \frac{6_{\text{cbc}} \text{ x k}}{2 \text{ x} 6_{\text{st}}} x100$$

The factor *k, j* and *Q* are constant for a given type of steel and concrete and do not depend upon the beam dimension. These are called as *design constants.* The value of *k, j, Q* and P_t are given in Table

Sr No	Grade of Concrete	Permissible stress in *concrete in bending compression* (6_{cbc})	Modular Ratio (m) $m = \frac{280}{3 \text{ x } 6_{cbc}}$	F_y= 250 N/mm^2 & 6_{st}=140 N/mm^2 (Up to & Including 20 mm Diameter) F_y= 250 N/mm^2 & 6_{st}=130 N/mm^2 (over 20 mm Diameter)			
				k $k = \frac{\text{m x } 6_{cbc}}{\text{m x } 6_{cbc} + 6_{st}}$	**j** $j = 1 - \frac{k}{3}$	**Q** $Q = \frac{6_{cbc} \text{ x k x j}}{2}$	**p_t** $p_t = \frac{6_{cbc} \text{ x k}}{2 \text{ x} 6_{st}} x100$
1	M_{15}	5.0	18.67	0.4	0.867	0.867	0.72
2	M_{20}	7.0	13.33	0.4	0.867	1.214	1.0
3	M_{25}	8.5	10.98	0.4	0.867	1.48	1.21
4	M_{30}	10.0	9.33	0.4	0.867	1.73	1.43

Sr No	Grade of Concrete	Permissible stress in *concrete in bending compression* (6_{cbc})	Modular Ratio (m) $m = \frac{280}{3 \text{ x } 6_{cbc}}$	F_y= 415 N/mm^2 & 6_{st}=230 N/mm^2			
				k $k = \frac{\text{m x } 6_{cbc}}{\text{m x } 6_{cbc} + 6_{st}}$	**j** $j = 1 - \frac{k}{3}$	**Q** $Q = \frac{6_{cbc} \text{ x k x j}}{2}$	**p_t** $p_t = \frac{6_{cbc} \text{ x k}}{2 \text{ x} 6_{st}} x100$
1	M_{15}	5.0	18.67	0.29	0.904	0.65	0.314
2	M_{20}	7.0	13.33	0.29	0.904	0.914	0.44
3	M_{25}	8.5	10.98	0.29	0.904	1.11	0.534
4	M_{30}	10.0	9.33	0.29	0.904	1.306	0.628

6_{st}=0.55 Fy -------------------------------- Fe 500

Sr No	**Grade of Concrete**	**Permissible stress in *concrete in bending compression* (6cbc)**	**Modular Ratio (m)** $m = \frac{280}{3 \text{ x } 6_{cbc}}$	**F_y= 500 N/mm^2 & 6_{st}=0.55 x500= 275 N/mm^2**			
				k $k = \frac{\text{m x } 6_{cbc}}{\text{m x } 6_{cbc} + 6_{st}}$	**j** $j = 1 - \frac{k}{3}$	**Q** $Q = \frac{6_{cbc} \text{ x k x j}}{2}$	**p_t** $p_t = \frac{6_{cbc} \text{ x k}}{2 \text{ x} 6_{st}} x100$
1	M_{15}	5.0	18.67	0.25	0.916	0.58	0.23
2	M_{20}	7.0	13.33	0.25	0.916	0.81	0.32
3	M_{25}	8.5	10.98	0.25	0.916	0.985	0.39
4	M_{30}	10.0	9.33	0.25	0.916	1.16	0.46

Formulae

1) Modular Ratio (m) (IS 456:2000, P .No:80, C. No: B-1.3)

$$m = \frac{280}{3 \text{ x } 6_{cbc}}$$

2) Neutral Axis depth factor (k)

$$k = \frac{m \ 6_{cbc}}{m \ 6_{cbc} + 6_{st}}$$

3) Lever arm factor (j)

$$j = 1 - \frac{k}{3}$$

4) Moment resisting factor (Q)

$$\text{Q} = \frac{1}{2} 6_{cbc} \text{jk}$$

5) Critical moment resistance (Mr)

$$\text{Mr} = Qbd^2$$

$$\text{Q} = \frac{1}{2} 6_{cbc} \text{jk}$$

6) Tensile Force (T)

$T = A_{st}\ 6_{st}$

OR

$T = m.A_{st} \times (d - n)$

7) Compressive force (C)

$C = b.\frac{n^2}{2}$

OR

$C = \frac{1}{2} 6_{cbc} \text{ x b x n}$

8) Depth of neutral axis (n)

C = T

$b.\frac{n^2}{2} = m.A_{st} \times (d - n)$

n= ?

9) **Depth of critical neutral axis (n_c)**

$n_c = kd$

10) Lever Arm (z)

Lever arm =z= $\left(\text{d-}\ \frac{n}{3} \right)$

11) Moment of resistance (Mr)

Mr = T z

$Mr = A_{st}\ 6_{st} \left(\text{d-}\ \frac{n}{3} \right)$

12) Moment of resistance (Mr)

Mr = C z

$Mr = \frac{1}{2} 6_{cbc} \text{ x b x n} \left(\text{d-}\ \frac{n}{3} \right)$

13) Area of steel

$A_{st} = \frac{M}{6_{st}\ j\ d}$

14) Percentage of steel

$p_t = \frac{A_{st}}{bd} x100$

$P_t = \frac{6_{cbc} \text{ x k}}{2 \text{ x} 6_{st}} x100$

15) **If** $n \prec n_c$ then section is under reinforced

16) **If** $n = n_c$ then section is balance section

17) **If** $n > n_c$ then section is over reinforced, if section is over reinforced then consider it as balance section.

IN SINGLY REINFORCED BEAM THERE ARE THREE TYPES OF PROBLEMS

1) To find moment of resistance of section

2) To find the maximum tensile stress (6_{st}) in steel and compression stress (6_{cbc}) in concrete

3) To find the area of tensile Steel (A_{st})

Type I

To find moment of resistance of section

Stepwise Procedure

To find :- The moment of resistance of section

Given Data:- b,d,,Ast, Fy, Fck

d = D - Effective cover

d = D - d'

d'=Effective cover = Clear cover + $\frac{\phi}{2}$

6_{cbc} = ? N/mm^2 (IS 456:2000, Table No: 21, P No:81)

6_{st} = ? N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$m = \frac{280}{3 \text{ x } 6_{cbc}}$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of neutral axis (n)

C = T

$$b.\frac{n^2}{2} = m.A_{st}\times(d-n)$$

n= ?

STEP 2: To find depth of critical neutral axis (n_c)

$n_c = kd$

$k = \dfrac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}}$

STEP 3: To compare n and n_c

a) If $n \prec n_c$ then section is under reinforced

b) **If** $n = n_c$ then section is balance section

c) **If** $n > n_c$ then section is over reinforced, if section is over reinforced then consider it as balance section.

STEP 4: To find moment of resistance

Mr = T z

$Mr = A_{st}\ 6_{st} \left(d- \dfrac{n}{3} \right)$

STEP 5: To find superimposed load

Superimposed load = Total working load **-** self weight of beam

Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25

Density of Concrete = 25 KN/m^3

Examples

1) A reinforced concrete beam 250 mm X 300mm overall depth is reinforced with 3 bars of 12 mm diameter at the bottom. The clear cover of 25 mm, Calculate ultimate moment of resistance of section also find superimposed UDL over a simply supported span of 3 m. Used Fe 250 and M_{15}. Using WSM.

Solution:- **To find :- The moment of resistance of section**

Given Data:- b = 250 mm

D = 300 mm

ϕ = 12 mm

No of bar = 3

Clear cover = 25 mm

d'= Effective cover = Clear cover + $\frac{\phi}{2}$

d'=Effective cover = 25 + $\frac{12}{2}$ = 31 mm

Effective depth =d = D - d' = 300 -31 = 269 mm

Ast = 3 X $\frac{\pi}{4}$ X ϕ^2 = 3 X $\frac{\pi}{4}$ X 12^2 = 339.29 mm^2

L = 3 m

M_{15}, 6_{cbc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 250 , 6_{st} = 140 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 5} = 18.666$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of neutral axis (n)

C = T

$$b.\frac{n^2}{2} = \text{m.A}_{st} \times (\text{d}-\text{n})$$

$$250x\frac{n^2}{2} = 18.666\ x339.29\ x(269 - n)$$

$$125n^2 = 6333.18\ (269 - n)$$

$$125n^2 = 1.703x10^6 - 6333.18n$$

$$125n^2 + 6333.18n - 1.703x10^6 = 0$$

$$n = 94.006\ mm$$

STEP 2: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.666x\ 5}{(18.666x\ 5) + 140} = 0.3999$$

n_c = 0.3999 x 269=107.57 mm

STEP 3: To compare n and n_c

$\mathbf{n} \prec n_c$

$94.006 < 107.57$

then section is under reinforced

STEP 4: To find moment of resistance

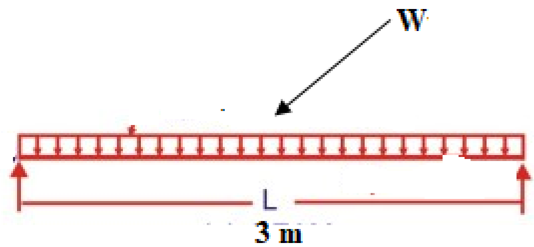
Mr = T z

$Mr = A_{st}\ 6_{st} \left(d- \frac{n}{3} \right)$

$Mr = 339.29 \text{ x } 140 \text{ x } \left(269- \frac{94.006}{3} \right)$

$Mr = 11.2892x10^6$ Nmm = 11.2892 KNm (1)

STEP 5: To find superimposed load



Maximum bending moment = $\frac{Wl^2}{8} = \frac{Wx3^2}{8}$ =1.125 W..(2)

Equating (1) and (2)

11.2892 = 1.125 W

W = 10.03 KN/m

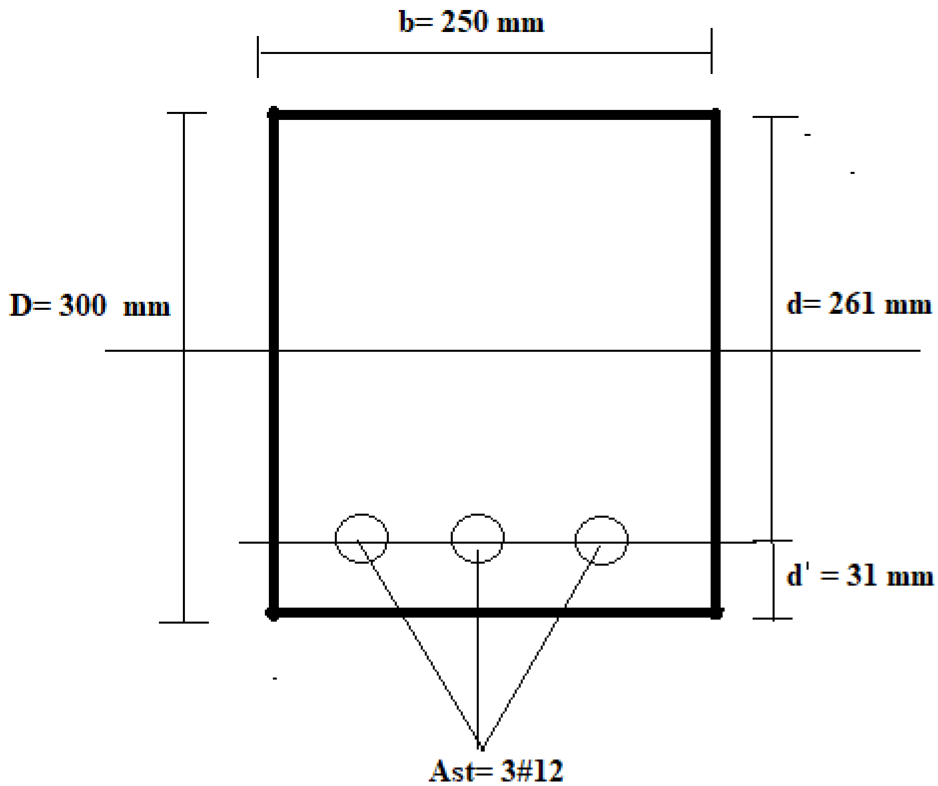
Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25= (0.25 X 0.3) X 25

Self weight of beam = 1.875 KN/m

Superimposed load = Total working load - self weight of beam

Superimposed load = 10.03 - 1.875 = 8.159 KN/m



2) A reinforced concrete beam 250 mm X 400mm overall depth is reinforced with 4 bars of 12 mm diameter at the bottom. The clear cover of 25 mm, Calculate ultimate moment of resistance of section also find superimposed UDL over a simply supported span of 4 m. Used Fe 250 and M_{15}. Using WSM.

Solution:- **To find :- The moment of resistance of section**

Given Data:- b = 250 mm

D = 400 mm

ϕ = 12 mm

No of bar = 4

Clear cover = 25 mm

d'= Effective cover = Clear cover + $\frac{\phi}{2}$

d'=Effective cover = 25 + $\frac{12}{2}$ = 31 mm

Effective depth =d = D - d' = 400 -31 = 369 mm

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 12^2 = 452.39 mm^2

L = 4 m

M_{15}, 6_{cbc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 250 , 6_{st} = 140 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 5} = 18.666$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of neutral axis (n)

C = T

$$b.\frac{n^2}{2} = \text{m.A}_{\text{st}} \times (\text{d} - \text{n})$$

$$250x\frac{n^2}{2} = 18.666\ x452.39\ x(369 - n)$$

$$125n^2 = 8444.31\ (369 - n)$$

$$125n^2 = 3.115x10^6 - 8444.31n$$

$$125n^2 + 8444.31n - 3.115x10^6 = 0$$

$$n = 127.65\ mm$$

STEP 2: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.666x\ 5}{(18.666x\ 5) + 140} = 0.3999$$

n_c = 0.3999 x 369=147.563 mm

STEP 3: To compare n and n_c

$\mathbf{n} \prec \text{n}_c$

$127.65 < 147.563$

then section is under reinforced

STEP 4: To find moment of resistance

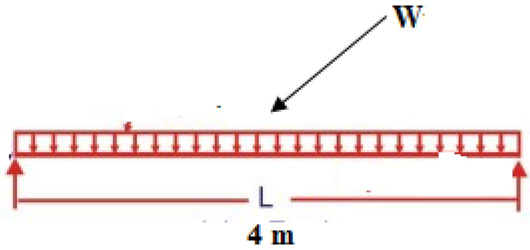
$Mr = T\ z$

$Mr = A_{st}\ 6_{st}\left(d- \frac{n}{3}\right)$

$Mr = 452.39 \text{ x } 140 \text{ x }\left(369- \frac{127.67}{3}\right)$

$Mr = 20.6751x10^{6}$ Nmm = 20.6751 KNm (1)

STEP 5: To find superimposed load



Maximum bending moment = $\frac{Wl^2}{8} = \frac{Wx4^2}{8}$ =2 W..(2)

Equating (1) and (2)

20.675 = 2 W

W = 10.3375KN/m

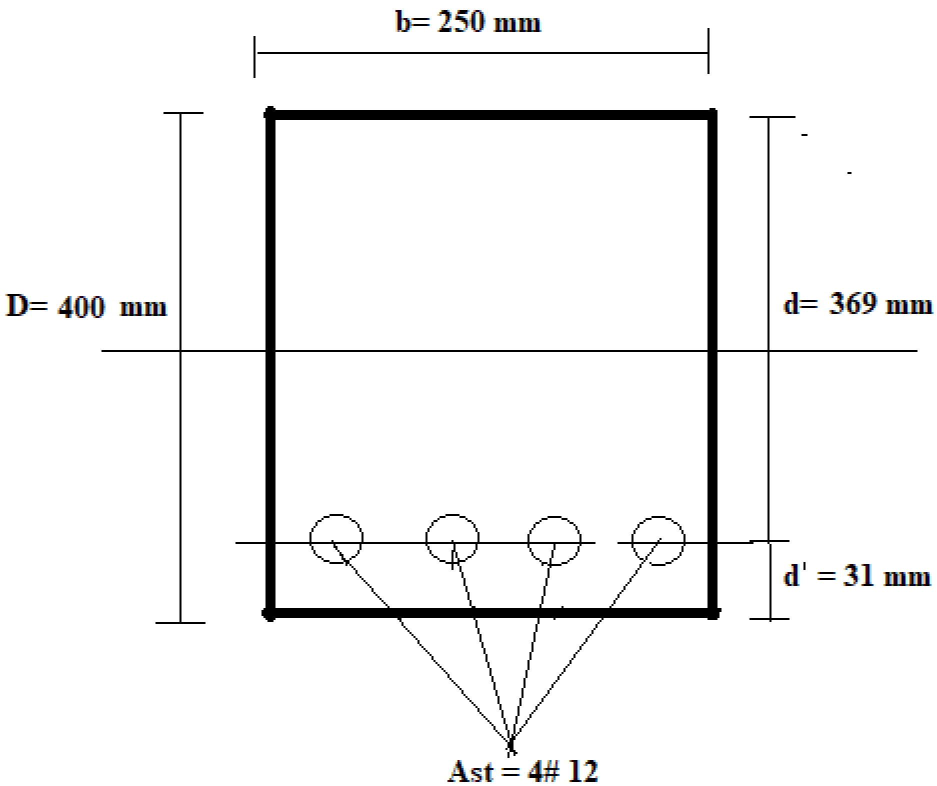
Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25= (0.25 X 0.4) X 25

Self weight of beam = 2.5 KN/m

Superimposed load = Total working load - self weight of beam

Superimposed load = 10.3375 – 2.5 = 7.8375KN/m



3) A reinforced concrete beam 300 mm X 500mm overall depth is reinforced with 4 bars of 16 mm diameter on tension side with 40 mm effective cover. Calculate ultimate moment of resistance of section also find superimposed UDL over a simply supported span of 5 m. Used Fe 250 and M_{15}. Using WSM.

Solution:- **To find :- The moment of resistance of section**

Given Data:- b = 300 mm

D = 500 mm

ϕ = 16 mm

No of bar = 4

d'=Effective cover = 40 mm

Effective depth =d = D - d' = 500 -40 = 460 mm

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 16^2 = 804.25 mm^2

L = 5 m

M_{15}, 6_{cbc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 250 , 6_{st} = 140 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 5} = 18.666$$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of neutral axis (n)

C = T

$$b.\frac{n^2}{2} = \text{m.A}_{st} \times (\text{d}-\text{n})$$

$$300x\frac{n^2}{2} = 18.666\ x804.25\ x(460-n)$$

$$150n^2 = 15012.13\ (460-n)$$

$$150n^2 = 6.9055x10^6 - 15012.13n$$

$$150n^2 + 15012.13n - 6.9055x10^6 = 0$$

$$n = 170.279\ mm$$

STEP 2: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.666x\ 5}{(18.666x\ 5) + 140} = 0.3999$$

$$n_c = 0.3999 \text{ x } 460 = 183.954 \text{ mm}$$

STEP 3: To compare n and n_c

$$\mathbf{n} \prec \text{n}_c$$

$$170.279 < 183.954$$

then section is under reinforced

STEP 4: To find moment of resistance

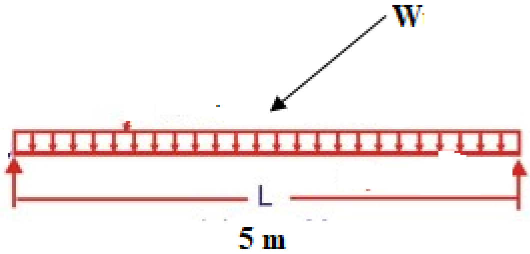
Mr = T z

$$\text{Mr} = \text{A}_{st}\ 6_{st} \left(\text{d- } \frac{n}{3} \right)$$

$$\text{Mr} = 804.25 \text{ x } 140 \text{ x } \left(460\text{- } \frac{170.279}{3} \right)$$

$$\text{Mr} = 45.400x10^6 \text{ Nmm} = 45.400 \text{ KNm} \qquad (1)$$

STEP 5: To find superimposed load



Maximum bending moment = $\frac{Wl^2}{8} = \frac{Wx5^2}{8}$ = 3.125 W...(2)

Equating (1) and (2)

45.40 = 3.125 W

Wu = 14.528 KN/m

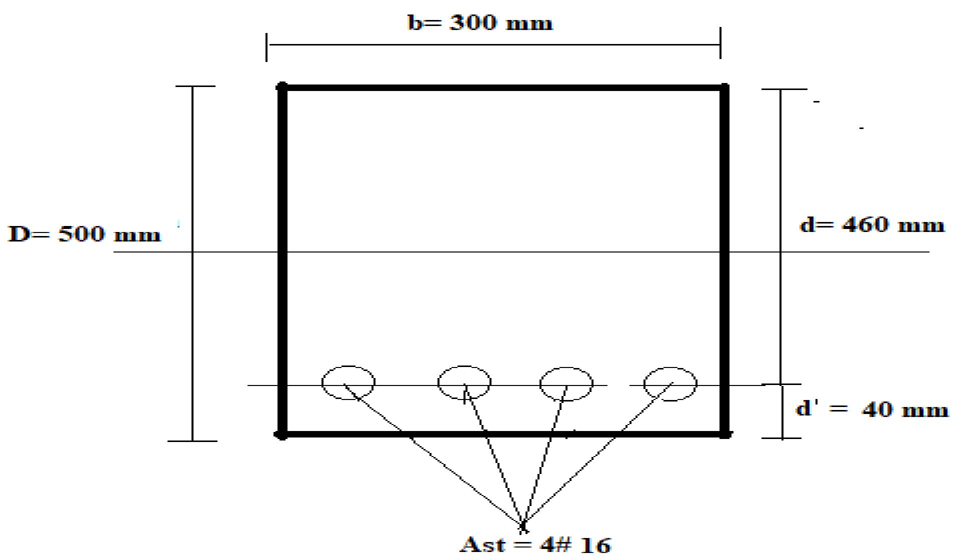
Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25= (0.3 X 0.5) X 25

Self weight of beam = 3.75 KN/m

Superimposed load = Total working load - self weight of beam

Superimposed load = 14.528 - 3.75 = 10.778 KN/m



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Type II :- To find the maximum tensile stress (6_{st}) in steel and compression stress (6_{cbc}) in concrete

Given Data:

STEP 1: To find depth of neutral axis (n)

$$C = T$$

$$b.\frac{n^2}{2} = m.A_{st}\times(d-n)$$

n=?

STEP 2: To find maximum stress in steel (6_{st})

$$Mr = T\ z$$

$$Mr = A_{st}\ 6_{st}\left(d-\frac{n}{3}\right)$$

$$6_{st} = ?$$

STEP 3: To find maximum stress in concrete (6_{cbc})

$$Mr = C\ z$$

$$Mr = \frac{1}{2}6_{cbc}\, x\ b\ x\ n\left(d-\frac{n}{3}\right)$$

$$6_{cbc} = ?$$

1) **Calculate the maximum compression stress in concrete and tensile stress in reinforcing steel for a R.C beam of 3.6 m effective span having cross section of 300 mm X 600 mm overall depth with 4 bars of 20 mm diameter and clear cover of 25 mm. The beam is loaded with a superimposed udl of 80 KN/m. Use m=18.66 using WSM.**
   Solution :

Given Data:- b = 300 mm

D = 600 mm

ϕ = 20 mm

No of bar = 4

Clear Cover= 25 mm

d'=Effective cover = clear cover + $\frac{\phi}{2}$ =25+(20/2)=35 mm

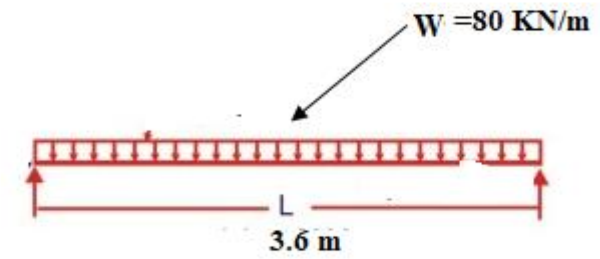
Effective depth =d = D - d' = 600 -35 = 565 mm

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 20^2 = 1256.63 mm^2

L = 3.6 m

m=18.66

udl = 80 KN/m



Maximum bending moment = $\frac{Wl^2}{8} = \frac{80x3.6^2}{8} = 129.6$ KNm

STEP 1: To find depth of neutral axis (n)

C = T

$b.\frac{n^2}{2}$ = m.A_{st}×(d−n)

$$300x\frac{n^2}{2} = 18.66\ x1256.63\ x(565-n)$$
$$150n^2 = 23448.84\ (565-n)$$
$$150n^2 = 13.2485x10^6 - 23448.843n$$
$$150n^2 + 23448.843n - 13.2485x10^6 = 0$$
$$n = 229.136\ mm$$

STEP 2: To find maximum stress in steel (6_{st})

Mr = T z

Mr = A_{st} 6_{st} $\left(\text{d-}\ \frac{n}{3}\right)$

$129.6 \text{ x } 10^6 = 1256.63 \text{ x } 6_{st} \text{ x } \left(565-\ \frac{229.136}{3}\right)$

6_{st} =211.06 N/mm^2

STEP 3: To find maximum stress in concrete (6_{cbc})

$$Mr = C\ z$$

$$Mr = \frac{1}{2} 6_{cbc} \text{x b x n} \left(\text{d-}\ \frac{n}{3} \right)$$

$$129.6 \text{ x } 10^6 = \frac{1}{2} 6_{cbc} \text{x } 300 \text{ x } 229.136 \text{ x } \left(565\text{-}\ \frac{229.136}{3} \right)$$

$$6_{cbc} = 7.7169 \text{ N/mm}^2$$

2) **Calculate the maximum compression stress in concrete and tensile stress in reinforcing steel for a R.C beam of 6 m effective span having cross section of 300 mm X 500 mm effective depth with 3 bars of 16 mm diameter. The beam is loaded with a superimposed udl of 10 KN/m. Use M_{20} using WSM.**
Solution :

Given Data:- b = 300 mm

Effective depth =d = 500 mm

ϕ = 16 mm

No of bar = 3

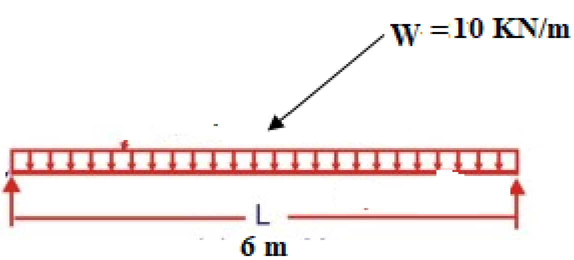
Ast = 3X $\frac{\pi}{4}$ X ϕ^2 = 3 X $\frac{\pi}{4}$ X 16^2 = 603.185 mm^2

L = 6 m

M_{20}, 6_{cbc} = 7 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

$$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 7} = 13.33$$

udl = 10 KN/m



Maximum bending moment = $\frac{Wl^2}{8} = \frac{10x6^2}{8} = 45$ KNm

STEP 1: To find depth of neutral axis (n)

C = T

$$b.\frac{n^2}{2} = m.A_{st} \times (d-n)$$

$$300x\frac{n^2}{2} = 13.33\ x603.185\ x(500-n)$$

$$150n^2 = 8040.4563\ (500-n)$$

$$150n^2 = 4.020x10^6 - 8040.4563n$$

$$150n^2 + 8040.4563n - 4.020x10^6 = 0$$

$$n = 139.084\ mm$$

STEP 2: To find maximum stress in steel (6_{st})

$$Mr = T\ z$$

$$Mr = A_{st}\ 6_{st}\left(d- \frac{n}{3}\right)$$

$$45 \text{ x } 10^6 = 603.185 \text{ x } 6_{st} \text{ x } \left(500- \frac{139.084}{3}\right)$$

$$6_{st} = 164.45 \text{ N/mm}^2$$

STEP 3: To find maximum stress in concrete (6_{cbc})

$$Mr = C\ z$$

$$Mr = \frac{1}{2} 6_{cbc} \text{ x b x n } \left(d- \frac{n}{3}\right)$$

$$45 \text{ x } 10^6 = \frac{1}{2} 6_{cbc} \text{ x } 300 \text{ x } 139.084 \text{ x } \left(500- \frac{139.084}{3}\right)$$

$$6_{cbc} = 4.7548 \text{ N/mm}^2$$

3) **Calculate the maximum compression stress in concrete and tensile stress in reinforcing steel for a R.C beam of 3.6 m effective span having cross section of 300 mm X 700 mm overall depth with 4 bars of 25 mm diameter and effective cover of 30 mm. The beam is having BM of 130 KNm. Use m=18.66 using WSM.**

Given Data:- b = 300 mm

D=Overall depth =700 mm

d'=Effective cover = 30 mm

Effective depth =d = D - d' = 700 -30 = 670 mm

ϕ = 25 mm

No of bar = 4

Ast = 4X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 25^2 = 1963.495 mm^2

B.M = M=**130 KNm=130 x 10^6 Nmm**

$m = 18.66$

STEP 1: To find depth of neutral axis (n)

C = T

$$b.\frac{n^2}{2} = m.A_{st} \times (d-n)$$

$$300x\frac{n^2}{2} = 18.66\ x1963.495\ x(670-n)$$
$$150n^2 = 36638.81\ (670-n)$$
$$150n^2 = 24.5480x10^6 - 36638.81n$$
$$150n^2 + 36638.81n - 24.5480x10^6 = 0$$
$$n = 300.444\ mm$$

STEP 2: To find maximum stress in steel (6_{st})

Mr = T z

$$Mr = A_{st}\ 6_{st} \left(d- \frac{n}{3} \right)$$

$$130 \text{ x } 10^6 = 1963.495 \text{ x } 6_{st} \text{ x } \left(670- \frac{300.444}{3} \right)$$

6_{st} =116.185 N/mm^2

STEP 3: To find maximum stress in concrete (6_{cbc})

$$Mr = C\ z$$

$$Mr = \frac{1}{2} 6_{cbc} x\ b\ x\ n \left(d- \frac{n}{3} \right)$$

$$130\ x\ 10^{6} = \frac{1}{2} 6_{cbc} x\ 300\ x\ 300.444\ x \left(670- \frac{300.444}{3} \right)$$

$$6_{cbc} = 5.0620\ N/mm^{2}$$

Type III :- To find the area of tensile Steel (Ast)

Given Data;

STEP 1: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}}$$

STEP 2: To find moment of resistance

$$Mr = C\ z$$

$$Mr = \frac{1}{2} 6_{cbc} x\ b\ x\ n_c \left(d- \frac{n_c}{3} \right)$$

STEP 3: To equate BM (M) and Ultimate moment (Mr)

STEP 4: To find area of steel (A_{st})

$$A_{st} = \frac{M}{6_{st}\ j\ d}$$

$$j = 1 - \frac{k}{3}$$

$$A_{st} = ?$$

Assume diameter of bar = Φ

Number of bars = $\frac{Ast}{(\pi/4)\ x\ \phi^{2}}$ =

1) **The cross section of a rectangular beam has to resist a bending moment of 100 KNm. If the beam is 250 mm wide. Find effective depth and tensile reinforcement required. Use M_{15} and Fe 250. Use WSM**

Solution:

BM= M= 100 KNm= 100 x 10^6 Nmm

b=250 mm

M_{15}, 6_{cbc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 250 , 6_{st} = 140 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 5} = 18.66$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of critical neutral axis (n_c)

$$n_c = kd$$
$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.66 x\ 5}{(18.66 x\ 5) + 140} = 0.3999$$
$$n_c = 0.3999 \text{ d}$$

STEP 2: To find moment of resistance

$$\text{Mr} = \text{C z}$$
$$\text{Mr} = \frac{1}{2} 6_{cbc} \text{x b x } n_c \left(\text{d-} \frac{n_c}{3} \right)$$
$$\text{Mr} = \frac{1}{2} \text{x5x 250 x 0.3999 d} \left(\text{d-} \frac{0.3999d}{3} \right)$$
$$\text{Mr} = 216.62 d^2$$

STEP 3: To equate BM (M) and Ultimate moment (Mr)

100 x 10^6 =216.62 d^2

d= 679.43 mm $\cong$ 680 mm

STEP 4: To find area of steel (A_{st})

$$A_{st} = \frac{M}{6_{st}\ j\ d}$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.3999}{3} = 0.8667$$

$$A_{st} = \frac{100 \text{ x } 10^6}{140x0.8667x680} = 1211.97mm^2$$

Assume diameter of bar = Φ = 20 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2}$ **=** $\frac{1211.97}{(\pi/4) \text{ x } 20^2} = 3.857 \cong 4$

2) **Design a reinforced concrete beam subjected to a bending moment of 30 KNm. Use M_{20} and Fe 415. Keep width of beam equal to half the effective depth. Use WSM**

Solution:

Given Data:-

BM= M= 30 KNm= 30 x 10^6 Nmm

M_{20}, 6_{cbc} = 7 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6_{st} = 230 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$$m = \frac{280}{3 \text{ x } 6_{\text{cbc}}} = \frac{280}{3 \text{ x } 7} = 13.33$$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

$$b = \frac{d}{2}$$

STEP 1: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{13.33x\ 7}{(13.33x\ 7) + 230} = 0.289$$

$$n_c = 0.289 \text{ d}$$

STEP 2: To find moment of resistance

$$Mr = C\ z$$

$$Mr = \frac{1}{2} 6_{cbc} x\ b\ x\ n_c \left(d- \frac{n_c}{3} \right)$$

$$Mr = \frac{1}{2} x7x\ \frac{d}{2}\ x\ 0.289\ d \left(d- \frac{0.289d}{3} \right)$$

$$Mr = 0.4570 d^3$$

STEP 3: To equate BM (M) and Ultimate moment (Mr)

$30\ x\ 10^6 = 0.457\ d^3$

d= 399.64 mm $\cong$ 400 mm

$$b = \frac{d}{2} = \frac{400}{2} = 200\ mm$$

STEP 4: To find area of steel (A_{st})

$$A_{st} = \frac{M}{6_{st}\ j\ d}$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.289}{3} = 0.9036$$

$$A_{st} = \frac{30\ x\ 10^6}{230x0.9036x400} = 360.87 mm^2$$

Assume diameter of bar = Φ = 12 mm

Number of bars = $\frac{Ast}{(\pi/4)\ x\ \phi^2}$ = $\frac{360.87}{(\pi/4)\ x\ 12^2} = 3.19 \cong 4$

3) **A cantilever beam of 3.0 m span consist of udl of 25 KN/m inclusive of its seft weight. Find the steel area for balanced section if it is reinforced in tension only. The width of beam is half the effective depth. Use M_{15} and Fe 415. Use WSM**

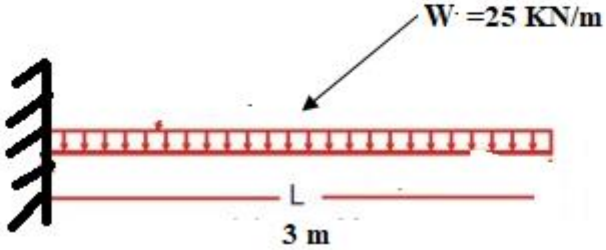
Solution:

Given Data:-

Cantilever Beam

L= 3.0 m

Udl =25 KN/m



Maximum bending moment = $\frac{Wl^2}{2} = \frac{25x3^2}{2} = 112.5 \text{ KN m}$

M_{15}, 6_{cbc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6_{st} = 230 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 5} = 18.666$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

$b = \frac{d}{2}$

STEP 1: To find depth of critical neutral axis (n_c)

$$n_c = kd$$
$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.666x\ 5}{(18.666x\ 5) + 230} = 0.289$$
$$n_c = 0.289 \text{ d}$$

STEP 2: To find moment of resistance

$$\text{Mr = C z}$$
$$\text{Mr} = \frac{1}{2} 6_{cbc} \text{x b x } n_c \left(\text{d-} \frac{n_c}{3}\right)$$
$$\text{Mr} = \frac{1}{2} \text{x5x} \frac{d}{2} \text{ x 0.289 d} \left(\text{d-} \frac{0.289d}{3}\right)$$
$$\text{Mr} = 0.3264 d^3$$

STEP 3: To equate BM (M) and Ultimate moment (Mr)

112.5 x 10^6 =0.3264 d^3

d= 701.09 mm $\cong$ 710 mm

$$b = \frac{d}{2} = \frac{710}{2} = 355\ mm$$

STEP 4: To find area of steel (A_{st})

$$A_{st} = \frac{M}{6_{st}\ j\ d}$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.289}{3} = 0.9036$$

$$A_{st} = \frac{112.5 \text{ x } 10^6}{230x0.9036x710} = 762.41\ mm^2$$

Assume diameter of bar = Φ = 20 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2}$ = $\frac{762.41}{(\pi/4) \text{ x } 20^2}$ $= 2.426 \cong 3$

4) Determine moment of resistance and area of tensile steel required for a section of R.C. beam of 300 mm X 550 mm effective depth. Use M_{15} and Fe 415. Use WSM

Solution: b=300 mm

d= 550 mm

M_{15}, $6_{cbc} = 5$ N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , $6_{st} = 230$ N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 5} = 18.666$$

STEP 1: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.666x\ 5}{(18.666x\ 5) + 230} = 0.289$$

$$n_c = 0.289 \text{ x } 550 = 158.95 \text{ mm}$$

STEP 2: To find moment of resistance

$$Mr = C\ z$$

$$Mr = \frac{1}{2} \sigma_{cbc} \text{ x b x } n_c \left(d- \frac{n_c}{3} \right)$$

$$Mr = \frac{1}{2} \text{ x } 5\text{x}300 \text{ x}158.95 \left(550- \frac{158.95}{3} \right)$$

$$Mr = 59.25 \text{ x } 10^6 \text{ Nmm}$$

STEP 3: To find area of steel (A_{st})

$$A_{st} = \frac{M}{\sigma_{st}\ j\ d}$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.289}{3} = 0.9036$$

$$A_{st} = \frac{59.25 \text{ x } 10^6}{230x0.9036x550} = 518.35\ mm^2$$

Assume diameter of bar = Φ = 16 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2}$ = $\frac{518.35}{(\pi/4) \text{ x } 16^2} = 2.578 \cong 3$

DOUBLY REINFORCED BEAM

Introduction

The balanced section of a beam is the most economical section from the requirement of steel point of view. If the area of steel reinforcement is doubled, the moment of resistance of a balanced section is increased only by about 22%.

For a design moment *M*, if the size of the section is restricted due to head room constraint or architectural constraints and the moment of resistance of the singly reinforced section is less than M, there are two methods to design such sections.

1) Increase the concrete mix to increase the capacity of the section.
2) Reinforcement are provided in compression zone to give additional strength to the concrete compression. Such sections are called *doubly reinforced* sections.

The reinforcement in compression zone has following advantages:

1) It permits smaller size beams which look aesthetic.
2) It reduces the long term deflections and increase ductility of the beam.
3) It can be used as anchor bars for positioning the stirrups
4) They are provided, even when not required for strengths in the seismic zone to withstand repeated reversals produced.

According to IS 456:2000,the compressive stress in compression on steel should be calculated by multiplying the stress in surrounding concrete by 1.5 m . The stress in compression steel so found should not exceed the permissible values.

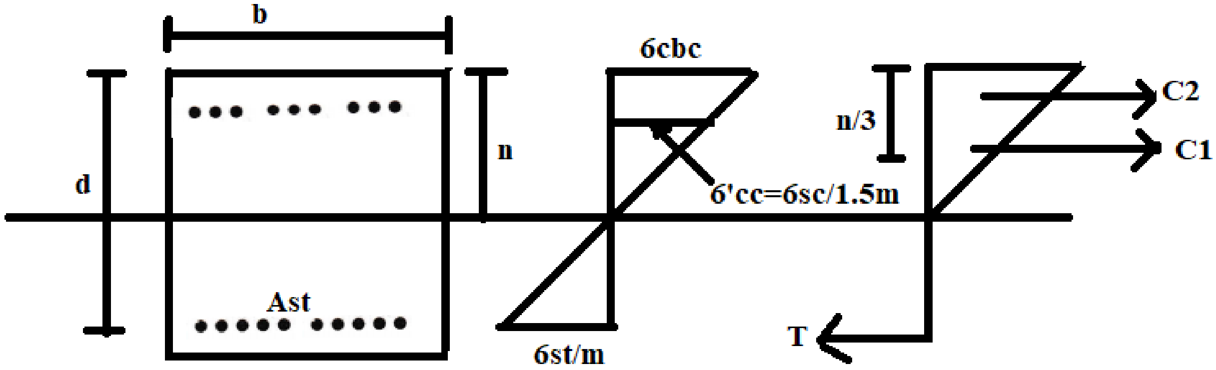
Location of Neutral Axis

Figure shows a doubly reinforced section. The neutral axis of a doubly reinforced section can be found by finding the centre of gravity of the combined section consisting of concrete in compression only and steel in compression and tension both.

Let, *b*=breadth of beam

d = effective depth of beam

d'= depth of centre of compression steel = *ed*
e = compressive steel depth factor =dc/d
6_{cbc}= maximum stress in concrete



6st= Maximum stress in tension steel

6’cc= Stress in concrete surrounding compression steel

6sc= Stress in compression steel

Ast= Area of tensile stress

Asc= Area of compression steel

n= depth of neutral axis

From stress diagram

$$\frac{6_{cc}}{\frac{6_{st}}{m}} = \frac{n}{d-n} = \frac{kd}{d-kd}$$

$$k = \frac{m6_{cc}}{m6_{cc} + 6_{st}}$$

Neglecting the concrete in tensile zone and equating the moment of compressive area about NA to the moment of tensile area about NA.

$$\frac{bn^2}{2} + 1.5mA_{sc}(n-d') - A_{sc}(n-d') = mA_{st}(d-n)$$

$$\frac{bn^2}{2} + (1.5m-1)A_{sc}(n-d') = mA_{st}(d-n)$$

IN DOUBLY REINFORCED BEAM THERE ARE THREE TYPES OF PROBLEMS

1) To find moment of resistance of section

2) To find the maximum tensile stress (6_{st}) in steel and compression stress (6_{cbc}) in concrete

3) To find the area of tensile Steel (A_{st}) and compressive steel (A_{sc})

Type I : To find moment of resistance of section

Design Procedure

Given Data

STEP 1: To find depth of neutral axis (n)

$$C = T$$
$$C_1 + C_2 = T$$
$$\frac{bn^2}{2} + (1.5m - 1)A_{sc}(n - d') = mA_{st}(d - n)$$
$$n = ?$$

STEP 2: To find depth of critical neutral axis (n_c)

$$n_c = kd$$
$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}}$$
$$n_c = ?$$

STEP 3: To compare n and n_c

n < n_c,then the section is under reinforced
n = n_c,then the section is balanced Section
n > n_c,then the section is over reinforced

$$6'_{cbc} = \frac{n - d'}{n} 6_{cbc}$$
$$6'_{cbc} = ?$$

STEP 4: To find moment of resistance

$$M = CZ$$
$$M = C_1 Z_1 + C_2 Z_2$$
$$M = \frac{1}{2} 6_{cbc} bn\left(d - \frac{n}{3}\right) + (1.5m - 1) A_{sc} 6'_{cbc} (d - d')$$

To find moment of resistance of section

1) A beam section , 280 mm wide and 540 mm overall depth reinforced with 5 bars of 20 mm diameter in the tension side and 4 bars of 20 mm diameter in compression side. The cover to the centre of both reinforcement is 30 mm. Determine the moment of resistance of the section, if M_{15} grade of concrete and bars of Fe 250 grade are used. Use WSM
Solution:

Given Data

Width of rectangular section = b = 280 mm

Overall Depth = D= 540 mm

Effective cover = d' = 30 mm

Effective Depth of rectangular section = d = D-d'=540-30=510 mm

Number of bar on tension side = 5

Area of steel on tension side = Ast =5 X $\frac{\pi}{4} X \ \phi^2$ = 5X $\frac{\pi}{4} X \ 20^2$ = 1570.79 mm^2

Number of bar on compressive side = 4

Area of steel on compressive side = Asc =4 X $\frac{\pi}{4} X \ \phi^2$ = 4 X $\frac{\pi}{4} X \ 20^2$ =1256.63 mm^2

M_{15}, 6_{cbc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 250 , 6_{st} = 140 N/mm^2 <u>Diameter is 20 mm</u> (IS 456:2000, Table No: 22, P No:82)

$$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 5} = 18.66$$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of neutral axis (n)

$$C = T$$

$$C_1 + C_2 = T$$

$$\frac{bn^2}{2} + (1.5m - 1)A_{sc}(n - d') = mA_{st}(d - n)$$

$$\frac{280n^2}{2} + (1.5x18.66 - 1)x1256.63(n - 30) = 18.66x1570.79(510 - n)$$

$$140n^2 + 33916.44n - 1.0174x10^6 = 14.948x10^6 - 29310.94n$$

$$140n^2 + 63227.38n - 15.9654x10^6 = 0$$

$$n = 180.426\ mm$$

STEP 2: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.66 \text{ x } 5}{(18.66 \text{ x } 5) + 140} = 0.3999$$

$$n_c = 0.3999x510 = 203.94\ mm$$

STEP 3: To compare n and n_c

n $< n_c$

$180.426 < 203.949$

then the section is under reinforced

$$6'_{cbc} = \frac{n - d'}{n} 6_{cbc}$$

$$6'_{cbc} = \frac{180.426 - 30}{180.426} x5$$

$$6'_{cbc} = 4.168 \text{N/mm}^2$$

STEP 4: To find moment of resistance

$$M = CZ$$

$$M = C_1 Z_1 + C_2 Z_2$$

$$M = \frac{1}{2} 6_{cbc} bn\left(d - \frac{n}{3}\right) + (1.5m - 1)A_{sc} 6'_{cbc}(d - d')$$

$$M = \frac{1}{2} x5x280x180.426\left(510 - \frac{180.426}{3}\right) + (1.5x18.66 - 1)x1256.63x4.168x(510 - 30)$$

$$M = 124.67 \text{x } 10^6 \text{ Nmm}$$

$$M = 124.67 \text{ KNm}$$

2) A beam section , 300 mm wide and 600 mm overall depth reinforced with 4 bars of 25 mm diameter in the tension side and 4 bars of 12 mm diameter in compression side. The cover to the centre of both reinforcement is 30 mm. Determine the moment of resistance of the section, if M_{20} grade of concrete and HYSD bars of Fe 415 grade are used. Use WSM
Solution:

Given Data

Width of rectangular section = b = 300 mm

Overall Depth = D= 600 mm

Effective cover = d' = 30 mm

Effective Depth of rectangular section = d = D-d'=600-30=570 mm

Number of bar on tension side = 4

Area of steel on tension side = Ast = 4 X $\frac{\pi}{4} X \quad \phi^2 = 4X \ \frac{\pi}{4} X \ 25^2 =$ 1963.5 mm^2

Number of bar on compressive side = 4

Area of steel on compressive side = Asc =4 X $\frac{\pi}{4} X \ \phi^2 = 4 \text{ X } \frac{\pi}{4} X \ 12^2 =$452.4 mm^2

M_{20}, 6_{cbc} = 7 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6_{st} = 230 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 7} = 13.33$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of neutral axis (n)

$$C = T$$
$$C_1 + C_2 = T$$
$$\frac{bn^2}{2} + (1.5m-1)A_{sc}(n-d') = mA_{st}(d-n)$$
$$\frac{300n^2}{2} + (1.5x13.33-1)x452.4(n-30) = 13.33x1963.5\ (570-n)$$
$$150n^2 + 34766.8n - 15176669 = 0$$
$$n = 222.65\ mm$$

STEP 2: To find depth of critical neutral axis (n_c)

$$n_c = kd$$
$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{13.33\ \text{x}\ 7}{(13.33\ \text{x}\ 7) + 230} = 0.288$$
$$n_c = 0.288x570 = 164.60\ mm$$

STEP 3: To compare n and n_c

$\mathbf{n} > \mathrm{n_c}$

$222.65 > 164.60$

then the section is over reinforced

$\mathbf{n} > \mathrm{n_c}$, the stress in concrete will reach its maximum permissible value first. Hence, stress in concrete surrounding the compression steel

$$6'_{cbc} = \frac{n-d'}{n} 6_{cbc}$$
$$6'_{cbc} = \frac{222.65-30}{222.65} x7$$
$$6'_{cbc} = 6.057\ \text{N/mm}^2$$

STEP 4: To find moment of resistance

$$M = CZ$$
$$M = C_1Z_1 + C_2Z_2$$
$$M = \frac{1}{2} 6_{cbc} bn\left(d - \frac{n}{3}\right) + (1.5m-1)A_{sc} 6'_{cbc}(d-d')$$
$$M = \frac{1}{2} x7x300x222.65\left(570 - \frac{222.65}{3}\right) + (1.5x13.33-1)x452.4x6.057x(570-30)$$
$$M = 144.01\ \text{x}\ 10^6\ \text{Nmm}$$
$$M = 144.01\ \text{KNm}$$

3) A beam section , 250 mm wide and 415 mm effective depth reinforced with 4 bars of 20 mm diameter in the tension side and 2 bars of 16 mm diameter in compression side. The cover to the centre of both reinforcement is 35 mm. Determine the moment of resistance of the section, if M_{20} grade of concrete and HYSD bars of Fe 415 grade are used. Use WSM
Solution:

Given Data

Width of rectangular section = b = 250 mm

Effective Depth of rectangular section = d =415 mm

Overall Depth = D= d+d'=415+35=440 mm

Effective cover = d' = 35 mm

Number of bar on tension side = 4

Area of steel on tension side = Ast = 4 X $\frac{\pi}{4} X \ \phi^2 = 4X \ \frac{\pi}{4} X \ 20^2 = 1256.6 \text{ mm}^2$

Number of bar on compressive side = 2

Area of steel on compressive side = Asc =2 X $\frac{\pi}{4} X \ \phi^2 = 2X \ \frac{\pi}{4} X \ 16^2 = 402.1 \text{ mm}^2$

M_{20}, 6_{cbc} = 7 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6_{st} = 230 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 7} = 13.33$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of neutral axis (n)

$C = T$

$C_1 + C_2 = T$

$$\frac{bn^2}{2} + (1.5m - 1)A_{sc}(n - d') = mA_{st}(d - n)$$

$$\frac{250n^2}{2} + (1.5x13.33 - 1)x402.1(n - 35) = 13.33x1256.6\ (415 - n)$$

$$125n^2 + 7637.88n - 267.326x10^3 = 6.9514x10^6 - 16750.47n$$

$$125n^2 + 24388.35 - 7.218x10^6 = 0$$

$$n = 161.793mm$$

STEP 2: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{13.33 \text{ x } 7}{(13.33 \text{ x } 7) + 230} = 0.288$$

$$n_c = 0.288x415 = 119.52\ mm$$

STEP 3: To compare n and n_c

n > n_c

161.793 > 119.52

then the section is over reinforced

n > n_c, the stress in concrete will reach its maximum permissible value first. Hence, stress in concrete surrounding the compression steel

$$6'_{cbc} = \frac{n - d'}{n} 6_{cbc}$$

$$6'_{cbc} = \frac{161.793 - 35}{161.793} x7$$

$$6'_{cbc} = 5.485 \text{ N/mm}^2$$

STEP 4: To find moment of resistance

$$M = CZ$$
$$M = C_1 Z_1 + C_2 Z_2$$
$$M = \frac{1}{2} 6_{cbc} bn \left(d - \frac{n}{3} \right) + (1.5m - 1) A_{sc} 6'_{cbc} (d - d')$$
$$M = \frac{1}{2} x7x250x161.793 \left(415 - \frac{161.793}{3} \right) + (1.5x13.33 - 1)x402.1x5.485x(415 - 35)$$
$$M = 67.035 \text{ x } 10^6 \text{ Nmm}$$
$$M = 67.035 \text{ KNm}$$

Type II :- To find the maximum tensile stress (6_{st}) in steel and compression stress (6_{cbc}) in concrete

Given Data

STEP 1: To find depth of neutral axis (n)

$$C = T$$
$$C_1 + C_2 = T$$
$$\frac{bn^2}{2} + (1.5m - 1) A_{sc} (n - d') = mA_{st} (d - n)$$
$$n = ?$$

STEP 2: To find maximum stress in concrete (6_{cbc})

$$M = CZ$$
$$M = C_1 Z_1 + C_2 Z_2$$
$$M = \frac{1}{2} 6_{cbc} bn \left(d - \frac{n}{3} \right) + (1.5m - 1) A_{sc} 6'_{cbc} (d - d') \quad \text{------------------------(1)}$$
$$6'_{cbc} = 6_{cbc} \left(\frac{n - d'}{n} \right) \text{--(2)}$$

Subt equation (2) in eqution (1)

$$6_{cbc} = ?$$

STEP 3: To find maximum stress in steel (6_{st})

$$6_{st} = 1.5 \text{ m } 6'_{cbc}$$

1) Find maximum compressive stress in concrete and tensile stress in steel in a doubly reinforced section for the following data
   b=300 mm , d=450 mm , d'= 45 mm , M= 110 KNm

Ast= 5 bars of 25 mm diameter
Asc= 2 bars of 25 mm diameter
m=13 **Use WSM**

Solution:

Given Data

b=300 mm , d=450 mm , d'= 45 mm , M= 110 KNm

Number of bar on tension side = 5

Area of steel on tension side = Ast = 5 X $\frac{\pi}{4} X \phi^2$ = 5X $\frac{\pi}{4} X\ 25^2$ = 2454.37 mm^2

Number of bar on compressive side = 2

Area of steel on compressive side = Asc =2 X $\frac{\pi}{4} X\ \phi^2$ = 2X $\frac{\pi}{4} X\ 25^2$ =981.75 mm^2

m=13

STEP 1: To find depth of neutral axis (n)

$$C = T$$
$$C_1 + C_2 = T$$
$$\frac{bn^2}{2} + (1.5m-1)A_{sc}(n-d') = mA_{st}(d-n)$$
$$\frac{300n^2}{2} + (1.5x13-1)x981.75(n-45) = 13x2454.37(450-n)$$
$$150n^2 + 18162.375n - 817.30x10^3 = 14.358x10^6 - 31906.81n$$
$$150n^2 + 50069.18n - 15.1753x10^6 = 0$$
$$n = 192.3\ mm$$

STEP 2: To find maximum stress in concrete (6$_{cbc}$)

$M = CZ$

$M = C_1 Z_1 + C_2 Z_2$

$$M = \frac{1}{2} 6_{cbc} bn\left(d - \frac{n}{3}\right) + (1.5m - 1)A_{sc} 6'_{cbc} (d - d') \qquad \text{------------------------(1)}$$

$$6'_{cbc} = 6_{cbc} \left(\frac{n - d'}{n}\right)$$

$$6'_{cbc} = 6_{cbc} \left(\frac{192.3 - 45}{192.3}\right)$$

$$6'_{cbc} = 0.766 \; 6_{cbc} \qquad \text{---(2)}$$

Subt equation (2) in eqution (1)

$$M = \frac{1}{2} 6_{cbc} bn\left(d - \frac{n}{3}\right) + (1.5m - 1)A_{sc} 6'_{cbc} (d - d')$$

$$110x10^6 = \frac{1}{2} x6_{cbc} x300x192.3\left(450 - \frac{192.3}{3}\right) + (1.5x13 - 1)984.75x0.766x6_{cbc}(450 - 45)$$

$$6_{cbc} = 6.56 \text{ N/mm}^2$$

STEP 3: To find maximum stress in steel (6_{st})

6_{st} =1.5 m $6'_{cbc}$

$6'_{cbc} = 0.766x6_{cbc}$

$6'_{cbc} = 0.766$ x6.56=5.024 N/mm^2

6_{st} =1.5 x13 x 5.024

6_{st} = 97.986 N/mm^2

2) Find maximum compressive stress in concrete and tensile stress in steel in a doubly reinforced section for the following data
   b=200 mm , d=450 mm , d'= 30 mm , M= 100 KNm
   Ast= 4 bars of 25 mm diameter
   Asc= 3 bars of 22 mm diameter
   m=18.66 **Use WSM**

Solution:

Given Data

b=200 mm , d=450 mm , d'= 30 mm , M= 100 KNm

Number of bar on tension side = 4

Area of steel on tension side = Ast = 4 X $\frac{\pi}{4} X \ \phi^2$ = 4X $\frac{\pi}{4} X \ 25^2$ = 1963.49 mm^2

Number of bar on compressive side = 3

Area of steel on compressive side = Asc =3 X $\frac{\pi}{4} X \ \phi^2$ = 3X $\frac{\pi}{4} X \ 22^2$ =1140.39 mm^2

m=18.66

STEP 1: To find depth of neutral axis (n)

$$C = T$$
$$C_1 + C_2 = T$$
$$\frac{bn^2}{2} + (1.5m - 1)A_{sc}(n - d') = mA_{st}(d - n)$$
$$\frac{200n^2}{2} + (1.5x18.66 - 1)x1140.39(n - 30) = 18.66x1963.49(450 - n)$$
$$100n^2 + 30779.12n - 923.37x10^3 = 16.487x10^6 - 36638.72n$$
$$100n^2 + 67417.84n - 17.41x10^6 = 0$$
$$n = 199.31\ mm$$

STEP 2: To find maximum stress in concrete (6_{cbc})

$$M = CZ$$
$$M = C_1Z_1 + C_2Z_2$$
$$M = \frac{1}{2}6_{cbc}bn\left(d - \frac{n}{3}\right) + (1.5m - 1)A_{sc}6'_{cbc}(d - d') \qquad \text{------------------------(1)}$$
$$6'_{cbc} = 6_{cbc}\left(\frac{n - d'}{n}\right)$$
$$6'_{cbc} = 6_{cbc}\left(\frac{199.31 - 30}{199.31}\right)$$
$$6'_{cbc} = 0.8494\ 6_{cbc} \qquad \text{---(2)}$$

Subt equation (2) in eqution (1)

$$M = \frac{1}{2}6_{cbc}bn\left(d - \frac{n}{3}\right) + (1.5m - 1)A_{sc}6'_{cbc}(d - d')$$
$$100x10^6 = \frac{1}{2}x6_{cbc}x200x199.31\left(450 - \frac{199.31}{3}\right) + (1.5x18.66 - 1)x1140.39x0.8494x6_{cbc}(450 - 30)$$
$$6_{cbc} = 5.369\ \text{N/mm}^2$$

STEP 3: To find maximum stress in steel (6_{st})

6_{st} =1.5 m $6'_{cbc}$

$6'_{cbc} = 0.8494 x 6_{cbc}$

$6'_{cbc} = 0.8494 \text{ x} 5.369 = 4.560 \text{ N/mm}^2$

6_{st} =1.5 x18.66 x 4.560

6_{st} = 127.63 N/mm^2

3) Find maximum compressive stress in concrete and tensile stress in steel in a doubly reinforced section for the following data
   b=300 mm , d=500 mm , d'= 25 mm , M= 120 KNm
   Ast= 4 bars of 20 mm diameter
   Asc= 4 bars of 20 mm diameter
   m=13.33 **Use WSM**

Solution:

Given Data

b=300 mm , d=500 mm , d'= 25 mm , M= 120 KNm

Number of bar on tension side = 4

Area of steel on tension side = Ast = 4 X $\frac{\pi}{4} X \ \phi^2 = 4X \ \frac{\pi}{4} X \ 20^2 =$ 1256.63 mm^2

Number of bar on compressive side = 4

Area of steel on compressive side = Asc =4 X $\frac{\pi}{4} X \ \phi^2 = 4X \ \frac{\pi}{4} X \ 20^2$ =1256.63 mm^2

m=13.33

STEP 1: To find depth of neutral axis (n)

$$C = T$$
$$C_1 + C_2 = T$$
$$\frac{bn^2}{2} + (1.5m - 1)A_{sc}(n - d') = mA_{st}(d - n)$$
$$\frac{300n^2}{2} + (1.5x13.33 - 1)x1256.63(n - 25) = 13.33x1256.63x(500 - n)$$
$$150n^2 + 23869.68n - 596.742x10^3 = 8.375x10^6 - 16750.87n$$
$$150n^2 + 40620.55n - 8.9721x10^6 = 0$$
$$n = 144.12\ mm$$

STEP 2: To find maximum stress in concrete (6_{cbc})

$$M = CZ$$
$$M = C_1Z_1 + C_2Z_2$$
$$M = \frac{1}{2}6_{cbc}bn\left(d - \frac{n}{3}\right) + (1.5m - 1)A_{sc}6'_{cbc}(d - d') \qquad \text{------------------------(1)}$$
$$6'_{cbc} = 6_{cbc}\left(\frac{n - d'}{n}\right)$$
$$6'_{cbc} = 6_{cbc}\left(\frac{144.12 - 25}{144.12}\right)$$
$$6'_{cbc} = 0.8265\ 6_{cbc} \qquad \text{--(2)}$$

Subt equation (2) in eqution (1)

$$M = \frac{1}{2}6_{cbc}bn\left(d - \frac{n}{3}\right) + (1.5m - 1)A_{sc}6'_{cbc}(d - d')$$
$$120x10^6 = \frac{1}{2}x6_{cbc}x300x144.12\left(500 - \frac{144.12}{3}\right) + (1.5x13.33 - 1)x1256.63x0.8265x6_{cbc}(500 - 25)$$
$$6_{cbc} = 6.269\ \text{N/mm}^2$$

STEP 3: To find maximum stress in steel (6_{st})

6_{st} =1.5 m $6'_{cbc}$

$6'_{cbc} = 0.8265x6_{cbc}$

$6'_{cbc}$ = 0.8265 x6.269=5.181 N/mm^2

6_{st} =1.5 x13.33 x 5.181

6_{st} = 103.594 N/mm^2

Type III :- To find the area of tensile Steel (A_{st}) and compressive steel (Asc)

Given Data;

STEP 1: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}}$$

STEP 2: To find moment of resistance

$$Mr = C\ z$$

$$Mr = \frac{1}{2} 6_{cbc} \text{x b x } n_c \left(d - \frac{n_c}{3} \right)$$

$$Mr = ?\text{---}(1)$$

M>Mr

The section is doubly reinforced section

STEP 3: Additional moment

M_1 =M-Mr

STEP 4: To find area of steel in tension (A_{st})

To find area of steel (A_{st1})

$$A_{st_1} = \frac{M_r}{6_{st}\ j\ d}$$

$$j = 1 - \frac{k}{3}$$

$$A_{st_1} = ?$$

To find area of steel (A_{st2})

$$A_{st_2} = \frac{M_1}{6_{st}\ (d - d')}$$

$$A_{st_2} = ?$$

Total area of steel in tension

Ast= A_{st1}+A_{st2}

Assume diameter of bar = Φ

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2}$

STEP 5: To find area of steel in compression (A_{sc})

$$\text{m } A_{st_2}(d - n_c) = (1.5m - 1)A_{sc}(n_c - d')$$
$$A_{sc} = ?$$

Assume diameter of bar = Φ

Number of bars = $\frac{A_{sc}}{(\pi/4) \text{ x } \phi^2}$

1) **Determine reinforcement to be provided for a R.C beam of 6 m effective span and cross sectional area is 300 mm X 600 mm overall depth with effective cover of 40 mm . The bending moment is 132.75 KNm. Use M_{15} and Fe 415.Use WSM**

Solution:

BM= M= 132.75 KNm= 132.75 x 10^6 Nmm

Width of beam =b=300 mm

Over all depth of beam =D=600 mm

Effective cover =d'=40 mm

Effective depth =d= D- d'=600-40=560 mm

M_{15}, 6_{cbc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6_{st} = 230 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$m = \frac{280}{3 \text{ x } 6_{\text{cbc}}} = \frac{280}{3 \text{ x } 5} = 18.66$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of critical neutral axis (n_c)

$$n_c = kd$$
$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.66x\ 5}{(18.66x\ 5) + 230} = 0.2885$$
$$n_c = 0.2885 \text{x d}$$
$$n_c = 0.2885 \text{x } 560 = 161.56 \text{ mm}$$

STEP 2: To find moment of resistance

Mr = C z

$$Mr = \frac{1}{2} 6_{cbc} x\ b\ x\ n_c \left(d- \frac{n_c}{3} \right)$$

$$Mr = \frac{1}{2} x5x\ 300\ x\ 161.56 \left(560- \frac{161.56}{3} \right)$$

$Mr = 61.329x10^6$ Nmm

Mr =61.329 kNm---(1)

M>Mr

$132.75 > 91.329$

The section is doubly reinforced section

STEP 3: Additional moment

M_1 =M-Mr

M_1 =132.75-61.329=71.424 KNm

STEP 4: To find area of steel in tension (A_{st})

To find area of steel (A_{st1})

$$A_{st_1} = \frac{M_r}{6_{st}\ j\ d}$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.2885}{3} = 0.9038$$

$$A_{st_1} = \frac{61.329\ x\ 10^6}{230x0.9038x560} = 526.83mm^2$$

To find area of steel (A_{st2})

$$A_{st_2} = \frac{M_1}{6_{st}\ (d-d')}$$

$$A_{st_2} = \frac{71.424x10^6}{230x\ (560-40)}$$

$$A_{st_2} = 597.19mm^2$$

Total area of steel in tension

Ast= A_{st1}+A_{st2}

Ast= 526.83+597.19=1121.020 mm^2

Assume diameter of bar = Φ = 20 mm

Number of bars = $\dfrac{Ast}{(\pi/4) \text{ x } \phi^2} = \dfrac{1124.02}{(\pi/4) \text{ x } 20^2} = 3.577 \cong 4$

STEP 5: To find area of steel in compression (A_{sc})

$$\text{m } A_{st_2}(d-n_c) = (1.5m-1)A_{sc}(n_c - d')$$

$$18.66 \, x 597.19 \, x \, (560\text{-}161.56) = (1.5 \text{ x} 18.66\text{-}1) \, A_{sc}(161.56-40)$$

$$A_{sc} = 1353.29 \, mm^2$$

Assume diameter of bar = Φ = 20 mm

Number of bars = $\dfrac{Asc}{(\pi/4) \text{ x } \phi^2} = \dfrac{1353.29}{(\pi/4) \text{ x } 20^2} = 4.30 \cong 5$

2) **Determine reinforcement to be provided for a R.C beam having cross section is 250mm X 530 mm overall depth with effective cover of 30 mm . The bending moment is 100 KNm. Use M_{15} and Fe 415. Use WSM**

Solution:

BM= M= 100 KNm= 100 x 10^6 Nmm

Width of beam =b=250mm

Over all depth of beam =D=530 mm

Effective cover =d'=30 mm

Effective depth =d= D- d'=530-30=500 mm

M_{15}, 6_{cbc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6_{st} = 230 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 5} = 18.66$$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.66x\ 5}{(18.66x\ 5) + 230} = 0.2885$$

$$n_c = 0.2885\text{x d}$$

$$n_c = 0.2885\text{x } 500 = 144.25 \text{ mm}$$

STEP 2: To find moment of resistance

Mr = C z

$$\text{Mr} = \frac{1}{2} 6_{\text{cbc}} \text{x b x n}_{\text{c}} \left(\text{d-} \frac{n_c}{3} \right)$$

$$\text{Mr} = \frac{1}{2} \text{x5x 250 x 144.25} \left(500\text{-} \frac{144.25}{3} \right)$$

$$\text{Mr} = 40.743x10^6 \text{ Nmm}$$

Mr =40.743 kNm---(1)

M>Mr

$100 > 40.743$

The section is doubly reinforced section

STEP 3: Additional moment

M_1 =M-Mr

M_1 =100-40.743=59.257 KNm

STEP 4: To find area of steel in tension (A_{st})

To find area of steel (A_{st1})

$$A_{st_1} = \frac{M_r}{6_{st}\ j\ d}$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.2885}{3} = 0.9038$$

$$A_{st_1} = \frac{40.743 \text{ x } 10^6}{230x0.9038x500} = 391.99mm^2$$

To find area of steel (A_{st2})

$$A_{st_2} = \frac{M_1}{6_{st}\ (d - d')}$$

$$A_{st_2} = \frac{59.257 x 10^6}{230 x\ (500 - 30)}$$

$$A_{st_2} = 548.16 mm^2$$

Total area of steel in tension

Ast= A_{st1}+A_{st2}

Ast= 391.99+548.16=940.158 mm^2

Assume diameter of bar = Φ = 20 mm

Number of bars = $\frac{Ast}{(\pi/4)\ \text{x}\ \phi^2}$ = $\frac{940.158}{(\pi/4)\ \text{x}\ 20^2}$ $=2.992 \cong 3$

STEP 5: To find area of steel in compression (A_{sc})

$$\text{m A}_{st_2}(d - n_c) = (1.5m - 1)A_{sc}(n_c - d')$$

$$18.66\ x 548.16\ x\ (500\text{-}144.25) = (1.5\ \text{x}18.66\text{-}1)\ A_{sc}(144.25 - 30)$$

$$A_{sc} = 1180.06\ mm^2$$

Assume diameter of bar = Φ = 16 mm

Number of bars = $\frac{Asc}{(\pi/4)\ \text{x}\ \phi^2}$ = $\frac{1180.06}{(\pi/4)\ \text{x}\ 16^2}$ $=5.869 \cong 6$

3) **Determine reinforcement to be provided for a R.C beam having cross section is 250mm X 600 mm overall depth with effective cover of 50 mm . The bending moment is 95 KNm. Use M_{15} and Fe 250. Use WSM**

Solution:

BM= M= 95 KNm= 95 x 10^6 Nmm

Width of beam =b=250mm

Over all depth of beam =D=600 mm

Effective cover =d'=50 mm

Effective depth =d= D- d'=600-50=550 mm

M_{15}, 6_{cbc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 250, 6_{st} = 140 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

$m = \dfrac{280}{3 \text{ x } 6_{cbc}} = \dfrac{280}{3 \text{ x } 5} = 18.66$ **(IS 456:2000, P .No:80, C. No: B-1.3)**

STEP 1: To find depth of critical neutral axis (n_c)

$$n_c = kd$$

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{18.66x\ 5}{(18.66x\ 5) + 140} = 0.3999$$

$$n_c = 0.3999 \text{x d}$$

$$n_c = 0.3999 \text{x } 550 = 219.945 \text{ mm}$$

STEP 2: To find moment of resistance

Mr = C z

$$\text{Mr} = \frac{1}{2} 6_{cbc} \text{x b x } n_c \left(\text{d-} \frac{n_c}{3} \right)$$

$$\text{Mr} = \frac{1}{2} \text{x5x } 250 \text{ x } 219.945 \left(550\text{-} \frac{219.945}{3} \right)$$

$$\text{Mr} = 65.52x10^6 \text{ Nmm}$$

Mr =65.52 kNm---(1)

M>Mr

$95 > 65.52$

The section is doubly reinforced section

STEP 3: Additional moment

M_1 =M-Mr

M_1 =95-65.52=29.48KNm

STEP 4: To find area of steel in tension (A_{st})

To find area of steel (A_{st1})

$$A_{st_1} = \frac{M_r}{6_{st}\ j\ d}$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.3999}{3} = 0.8667$$

$$A_{st_1} = \frac{65.52 \text{ x } 10^6}{140x0.8667x550} = 981.78mm^2$$

To find area of steel (A_{st2})

$$A_{st_2} = \frac{M_1}{6_{st}\ (d - d')}$$

$$A_{st_2} = \frac{29.48x10^6}{140x\ (550 - 50)}$$

$$A_{st_2} = 421.14mm^2$$

Total area of steel in tension

Ast= A_{st1}+A_{st2}

Ast= 981.78+421.014=1402.92 mm^2

Assume diameter of bar = Φ = 20 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2}$ = $\frac{1402.92}{(\pi/4) \text{ x } 20^2}$ =4.46 $\cong$ 5

STEP 5: To find area of steel in compression (A_{sc})

$$\text{m A}_{\text{st}_2}(d - n_c) = (1.5m - 1)A_{sc}(n_c - d')$$

18.66 x421.014 x (550-219.945) =(1.5 x18.66-1) $A_{sc}(219.945 - 50)$

$$A_{sc} = 565.30\ mm^2$$

Assume diameter of bar = Φ = 16 mm

Number of bars = $\frac{Asc}{(\pi/4) \text{ x } \phi^2}$ = $\frac{1180.06}{(\pi/4) \text{ x } 16^2}$ =5.869 $\cong$ 6

Column

Introduction:

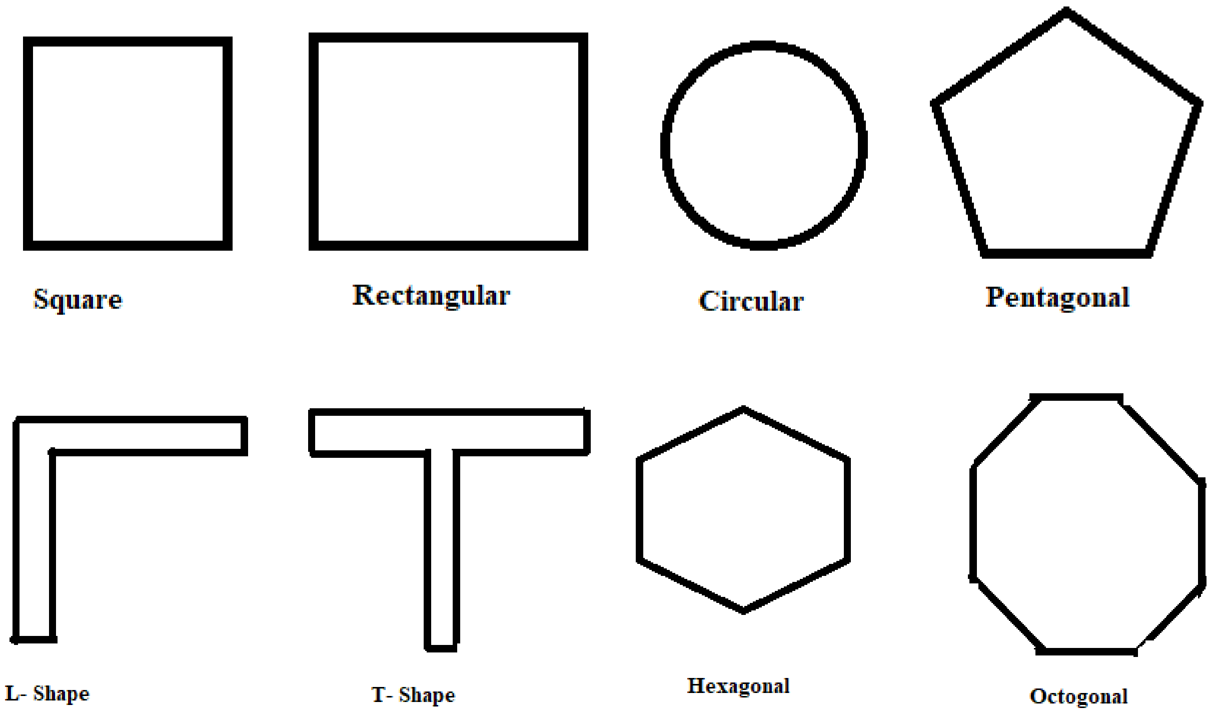
In reinforrced concrete construction, a compression member having its effective length greater than 3 times its least lateral dimension is called as column or Strut. A vertical compression member coming under above defination is usually called a column, while that in any other directions, as in case of frames and trusses, is called strut. A column with an effective length less than three times the least lateral dimension is called a pedestal. Column is an important element of every reinforced concrete structures. These are used to transfer the load of super structure to the foundation safely. Mainly column, struts and pedestals are used as compression members in buildings, bridges, supporting system of tanks, factories and many more such structures.

Types of Column:

Column are classified based on different criteria such as

1) Shapes of cross section
2) Material of constuction
3) Types of loading
4) Slenderness ratio
5) Types of lateral reinforcement

1) Shapes of cross section:
   On the basis of shape of cross section of the column, the column may be classified as following
   a) Square
   b) Rectangular
   c) Circular
   d) Pentagonal
   e) Hexagonal
   f) Octogonal
   g) T- shape or L- Shape etc



2) Material of constuction

   Column may be classified as following, as per material used for constuction

a) Timber Columns: Timber column are generally used for light loads. They are used in small trusses and wooden houses. They are called posts
b) Masonary Columns: These are used for light loads.
c) R.C.C. Columns: R.C.C. Column are used for mostly all types of buildings and other R.C.C structures link thanks, bridges etc
d) Steel columns: Steel columns are used for heavy loads.
e) Composite columns: Composite columns are used for heavy loads. They consist of steel sections like joists embedded in R.C.C. Section.

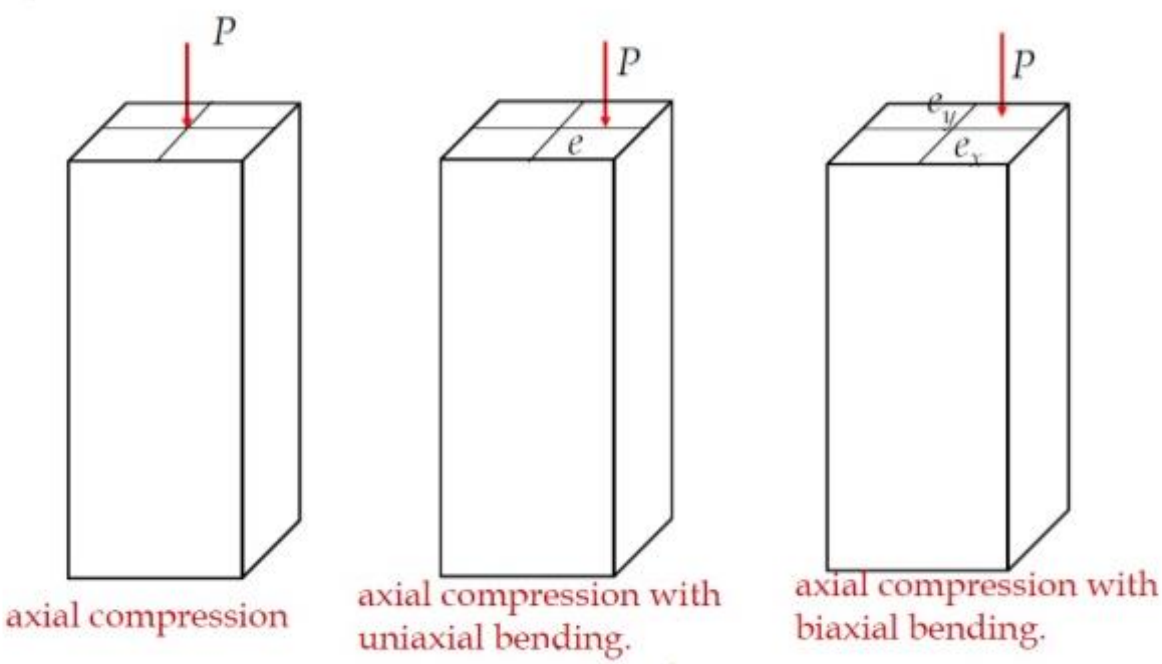
3) Types of loading

   Column may be classified as following, based on type of loading

   a) Axially loaded column
   b) Eccentrically loaded columns

      a) Axially loaded column: The column which are subjected to loads actiong along the longitudinal axis or centroid of the column section is called axially loaded columns. Axially loaded column is subjected to direct compressive stress only and no bending stress develops anywhere in the column section.
      b) Eccentrically loaded columns: Eccentrically loaded columns are those columns in which the loads do not act on the longitudinal axis of the column. They are

subjected to direct compressive stress and bending stress both. Eccentrically loaded columns may be subjected to uniaxial bending or biaxial bending depending upon the line of action of load, with respect to the two axis of the column section.



4) Slenderness ratio

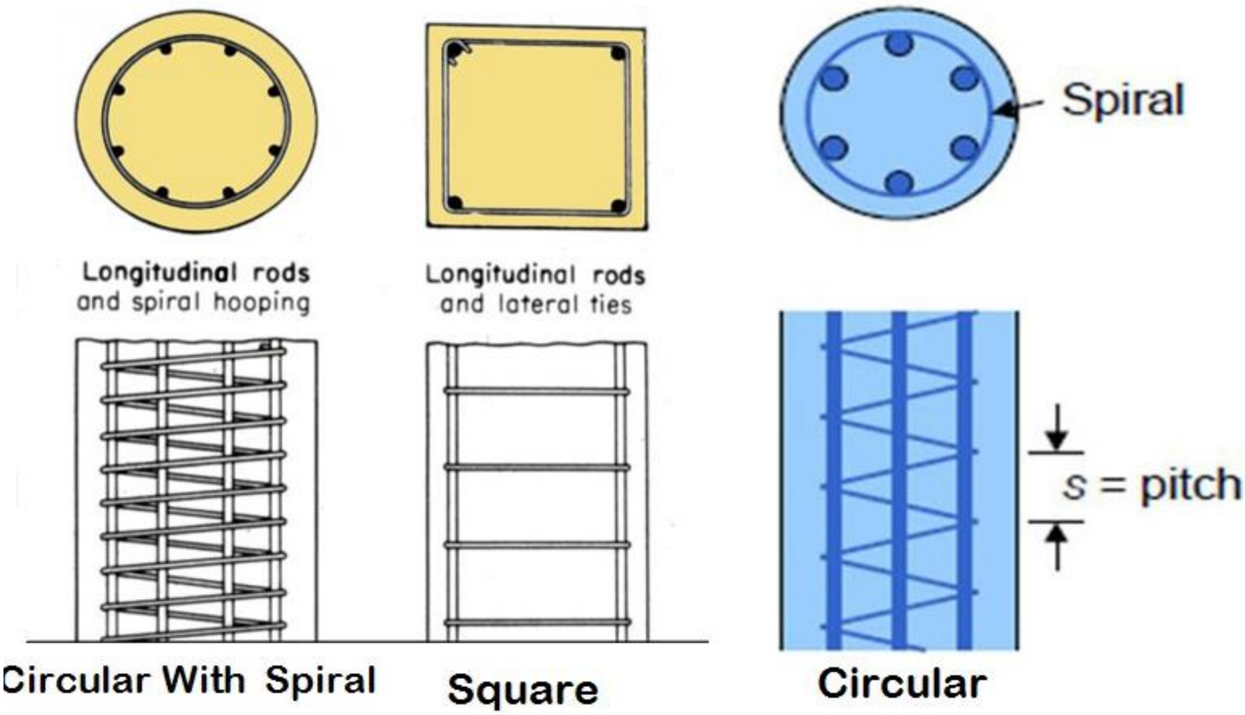
   The slenderness ratio of a compression member is defined as the ratio of effective length to the least lateral dimension. The column are classified as following two types depending upon the slenderness ratio.

a) Short column
b) Long column
   a) Short Column: The column is considered as short when the slenderness ratio of column i.e ratio of effective length to its lateral dimension $\left(\dfrac{L_{eff}}{Least\ lateral\ dimension} \leq 12\right)$ is less than or equal to 12.
   b) Long column: The column is considered as long when the slenderness ratio of column i.e ratio of effective length to its lateral dimension $\left(\dfrac{L_{eff}}{Least\ lateral\ dimension} > 12\right)$ is greater than or equal to 12.

5) Types of lateral reinforcement

   An R.C.C column has longitudinal and lateral reinforcement. They can also be classified according to the the manner in which the longitidinal steel is laterally supported or tied.

a) Column with longitudinal steel and lateral ties : In this type of arrangement the longitudinal bars are tied laterally at suitable internals with the help of ties.
b) Column with longitudinal steel and spiral ties: The longitudinal bars are tied continuously with the help of a spiral reinforcement. The columns with helical or spiral reinforcement are better in providing lateral support to bars as compared to links thus they increse the vuckling resistance and ductility of the column.



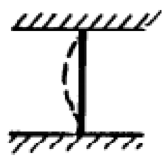
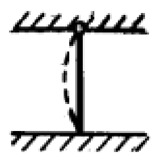
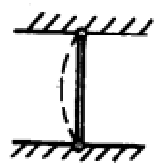
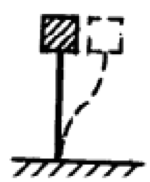
Effective length of column: The effective length of column is defined as that length of the column which takes lart in buckling under the action of loads. This is also defined as the length between the point of contraflexure of the buckled column.

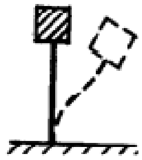
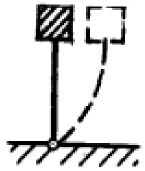
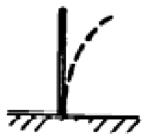
The deflected shape depends upon the types of end supports or degree of end restraints. The design of column is done on the basis of effective length.

(IS 456:2000, Table No: 28, P. No: 94)

Table 28 Effective Length of Compression Members

(*Clause* E-3)

Degree of End Restraint of Compression Members	Symbol	Theoretical Value of Effective Length	Recommended Value of Effective Length
(1)	(2)	(3)	(4)
Effectively held in position and restrained against rotation in both ends		0.50 l	0.65 l
Effectively held in position at both ends, restrained against rotation at one end		0.70 l	0.80 l
Effectively held in position at both ends, but not restrained against rotation		1.00 l	1.00 l
Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position		1.00 l	1.20 l

Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position		—	1.50 l
Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position		2.00 l	2.00 l
Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end		2.00 l	2.00 l

NOTE — l is the unsupported length of compression member.

Unsupported length of column: The unsupported length of column is clear length or height between the floor and the lower level of the ceiling.

Reinforcemet in column: Concrete is strong in compression. Thus a column can be made up of plain concrete but it is always advisable to use R.C.C. columns instead of plain concrete columns.

There are two types of reinforcement provided in a R.C.C. Column

a) Longitudinal reinforcement
b) Transverse reinforcement

a) Longitudinal reinforcement : The longitudinal reinforcement consist of steel bars are placed longitudinally in a column. It is also called as main steel. The functions of longitudinal reinforcement are as follows
   1) To share the compressive loads along with concrete, thus reducing the overall size of the column and leaving more unstable area.
   2) To resist tensile stresses developed due to any moment or accidental eccentricity.
   3) To impart ductility to the column
   4) To reduce the effect of creep and shrinkage due to continuous constant loading applied for a long time.

b) Transverse reinforcement

The transverse reinforcement is provided along the lateral direction of the column in the form of ties or spirals enclosing the main steel. The function of transverse steel are as following

1) To hold the longitudinal bars in position
2) To prevent buckling of the main longitudinal bars.
3) To resist diagonal tension caused due to transverse shear development because of any moments or load.
4) To impart ductility to the column
5) To prevent longitudinal splitting or bulging out of concrete by confining it in the core

Type I: Analysis of axially loaded column

Design Procedure

Given Data

STEP 1: To find Area of Concrete (Ac)
Area of Concrete =Total Gross Area – Area of longitudinal Steel
Ac = Ag- Asc
STEP 2: To find load carrying capacity of column (P)

For Short Column $\left(\frac{L_{eff}}{Least\ lateral\ dimension} < 12 \right)$

P= $6_{cc} x A_c + 6_{sc} x A_{sc}$ ---------------(IS 456-2000, P. No: 81, C. No: B-3)

For Long Column $\left(\frac{L_{eff}}{Least\ lateral\ dimension} > 12 \right)$

$$C_r = \left(1.25 - \frac{L_{eff}}{48b} \right) - - - - - - - - (IS\ \ 456:2000,\ \ P\ No:81,\ C.No:B-3.3)$$

P= $C_r x \left[6_{cc} x A_c + 6_{sc} x A_{sc} \right]$

1) A short column 400 mm X 600 mm in section is reinforced with 10 bars of 25 mm diameter. Find load carrying capacity of column . Use M_{25} & Fe 415. Use WSM
   Solution:
   Given Data
   Width of column = b = 400 mm
   Depth of column= D= 600 mm

$\phi = 25$ mm

No of bar = 10

Cross Sectional area of longitudinal Steel $= A_{sc} = 10 \text{ x } \frac{\pi}{4} \text{ X } \phi^2$

$A_{sc} = 10x\frac{\pi}{4} \text{ X } 25^2 = 4908.73\ mm^2$

M_{25}, $6_{cc} = 6$ N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , $6_{sc} = 190$ N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find Area of Concrete (Ac)

Area of Concrete =Total Gross Area – Area of longitudinal Steel

Ac = Ag- Asc =(400 x 600) -4908.73 =235.09 X 10^3 mm^2

STEP 2: To find load carrying capacity of column (P)

$P = 6_{cc} x A_c + 6_{sc} x A_{sc}$ ---------------(IS 456-2000, P. No: 81, C. No: B-3)

P= 6 x 235.09 x 10^3 + 190 x 4908.73

P=2343.20 x10^3N

P=2343.20 KN

2) A short column 400 mm X 500 mm in section is reinforced with 8 bars of 20 mm diameter. Find load carrying capacity of column. Use M_{20} & Fe 415. Use WSM

Solution:

Given Data

Width of column = b = 400 mm

Depth of column= D= 500 mm

$\phi = 20$ mm

No of bar = 8

Cross Sectional area of longitudinal Steel $= A_{sc} = 8 \text{ x } \frac{\pi}{4} \text{ X } \phi^2$

$A_{sc} = 8x\frac{\pi}{4} \text{ X } 20^2 = 2513.27\ mm^2$

M_{20}, $6_{cc} = 5$ N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , $6_{sc} = 190$ N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find Area of Concrete (Ac)

Area of Concrete =Total Gross Area – Area of longitudinal Steel

A_c = Ag- Asc =(400 x 500) -2513.27 =197.48 X 10^3 mm^2

STEP 2: To find load carrying capacity of column (P)

$P = 6_{cc} x A_c + 6_{sc} x A_{sc}$---------------(IS 456-2000, P. No: 81, C. No: B-3)

$P = 5 \times 197.48 \times 10^3 + 190 \times 2513.27$

$P = 1464.95 \times 10^3 N$

P=1464.95 KN

3) A column 350 mm X 500 mm in cross section is reinforced with 8 bars of 20 mm diameter, floor to floor height is 3.7 m & depth of beam is 0.5 m. Find load carrying capacity of column. Use M_{20} & Fe 415. Use WSM

Solution:

Given Data

Width of column = b = 350 mm

Depth of column= D= 500 mm

$\phi = 20$ mm

No of bar = 8

Cross Sectional area of longitudinal Steel $=A_{sc} = 8 \times \frac{\pi}{4} \times \phi^2$

$A_{sc} = 8x\frac{\pi}{4} \text{ X } 20^2 = 2513.27\ mm^2$

Effective length $=L_{eff}$ = Floor to floor height – Depth of Beam

Effective length $=L_{eff}$ =3.7-0.5 =3.2 m =3200 mm

Hence $\frac{L_{eff}}{Least\ lateral\ dimension} = \frac{3200}{350}$ =9.142<12

Thus the column is short

M_{20}, 6_{cc} = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6_{sc} = 190 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find Area of Concrete (Ac)

Area of Concrete =Total Gross Area – Area of longitudinal Steel

A_c = Ag- Asc =(350 x 500) -2513.27 =172.486 X 10^3 mm^2

STEP 2: To find load carrying capacity of column (P)

$P = 6_{cc} x A_c + 6_{sc} x A_{sc}$---------------(IS 456-2000, P. No: 81, C. No: B-3)

$P = 5 \times 172.486 \times 10^3 + 190 \times 2513.27$

$P = 1339.95 \times 10^3 N$

P=1339.95 KN

4) A column 300 mm X 450 mm in cross section is reinforced with 6 bars of 20 mm diameter. The column is 4m long and is effectively held in position and restrained against rotation at both ends. Find load carrying capacity of column. Use M_{15} & Fe 250. Use WSM

Solution:

Given Data
Width of column = b = 300 mm
Depth of column= D= 450 mm
ϕ = 20 mm
No of bar = 6
Cross Sectional area of longitudinal Steel =A_{sc} =6 x $\frac{\pi}{4}$ X ϕ^2

$A_{sc} = 6x\frac{\pi}{4}$ X $20^2 = 1884.95\ mm^2$

L=4m
Effectively held in position and restrained against rotation at both ends. (Both ends fixed) **(P. no:94, Table No:28)**
Effective length =L_{eff} =0.65x L=0.65 x4=2.6 m =2600 mm

Hence $\frac{L_{eff}}{Least\ lateral\ dimension} = \frac{2600}{300}$ =8.667<12
Thus the column is short
M_{15}, 6_{cc} = 4 N/mm^2 (IS 456:2000, Table No: 21, P No:81)
Fe 250 , 6_{sc} = 130 N/mm^2 (IS 456:2000, Table No: 22, P No:82)
STEP 1: To find Area of Concrete (Ac)
Area of Concrete =Total Gross Area – Area of longitudinal Steel
A_c = Ag- Asc =(300 x 450) -1884.95 =133.115 X 10^3 mm^2
STEP 2: To find load carrying capacity of column (P)

P= $6_{cc}xA_c + 6_{sc}xA_{sc}$---------------(IS 456-2000, P. No: 81, C. No: B-3)
P= 4 x 133.11 x 10^3 + 130 x 1884.95
P=777.50 x10^3N
P=777.50 KN

5) A circular column having diameter 400 mm is reinforced with 8 bars of 16 mm diameter. The column is 3m long and is effectively held in position at both ends but not restrained against rotation. Find load carrying capacity of column. Use M_{25} & Fe 415. Use WSM
Solution:
Given Data
Diameter of column = D= 400 mm
ϕ = 16 mm
No of bar = 8
Cross Sectional area of longitudinal Steel =A_{sc} =8 x $\frac{\pi}{4}$ X ϕ^2

$A_{sc} = 8x\frac{\pi}{4}$ X $16^2 = 1608.49\ mm^2$

L=3 m
Effectively held in position at both ends but not restrained against rotation
(Both end Hinged) **(P. no:94, Table No:28)**
Effective length =L_{eff} = L=3 m =3000 mm

Hence $\frac{L_{eff}}{Least\ lateral\ dimension} = \frac{3000}{400}$ =7.5<12

Thus the column is short

M_{25}, 6_{cc} = 6 N/mm^2 (IS 456:2000, Table No: 21, P No:81)
Fe 415 , 6_{sc} = 190 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find Area of Concrete (Ac)

Area of Concrete =Total Gross Area – Area of longitudinal Steel

A_c = Ag- Asc

$$A_c = \frac{\pi}{4} \text{ X } D^2 - A_{sc}$$

$$A_c = \frac{\pi}{4} \text{ X } 400^2 - 1608.49$$

$$A_c = 124.055x10^3 \text{ mm}^2$$

STEP 2: To find load carrying capacity of column (P)

P= $6_{cc}xA_c + 6_{sc}xA_{sc}$---------------(IS 456-2000, P. No: 81, C. No: B-3)

P=6 x 124.055 x 10^3 + 190 x 1608.49

P=1049.94 x10^3N

P=1049.94KN

6) A circular column having diameter 500 mm is reinforced with 8 bars of 16 mm diameter. The column is 8 m long and is effectively held in position at both ends but not restrained against rotation. Find load carrying capacity of column. Use M_{20} & Fe 415. Use WSM

Solution:
Given Data
Diameter of column = D= 500 mm
ϕ= 16 mm
No of bar = 8

Cross Sectional area of longitudinal Steel =A_{sc} =8 x $\frac{\pi}{4}$ X ϕ^2

$A_{sc} = 8x\frac{\pi}{4} \text{ X } 16^2 = 1608.49\ mm^2$

L=8 m
Effectively held in position at both ends but not restrained against rotation
(Both end Hinged) **(P. no:94, Table No:28)**
Effective length =L_{eff} = L=8 m =8000 mm

Hence $\frac{L_{eff}}{\text{Least lateral dimension}} = \frac{8000}{500} = 16 > 12$

Thus the column is long

Reduction Factor = C_r (IS 456:2000, P No:81, C. No:B-3.3)

$$C_r = \left(1.25 - \frac{L_{eff}}{48b}\right)$$

$$C_r = \left(1.25 - \frac{8000}{48x500}\right)$$

$$C_r = 0.9166$$

M_{25}, 6_{cc} = 5 N/ N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6_{sc} = 190 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find Area of Concrete (Ac)

Area of Concrete =Total Gross Area – Area of longitudinal Steel

A_c = Ag- Asc

$$A_c = \frac{\pi}{4} \text{ X } D^2 - A_{sc}$$

$$A_c = \frac{\pi}{4} \text{ X } 500^2 - 1608.49$$

$$A_c = 194.741x10^3 \text{ mm}^2$$

STEP 2: To find load carrying capacity of column (P)

P= C_r x $\left[6_{cc}xA_c + 6_{sc}xA_{sc}\right]$---------------(IS 456-2000, P. No: 81, C. No: B-3)

P=0.9166 x $\left[5 \text{ x } 194.741 \text{ x } 10^3 + 190 \text{ x } 1608.49\right]$

P=1172.62 x10^3N

P=1172.62 KN

Type II: Design of axially loaded column

Design Procedure

Given Data

STEP 1:: To find size of column

Assuming area of steel = 1 % of gross area

$$A_{sc} = 1\% \text{ of } A_g$$

$$A_{sc} = \frac{1}{100} \text{ x } A_g$$

$$A_{sc} = 0.01 \text{ x } A_g ------------- (1)$$

Area of concrete

$A_c = A_g - A_{sc}$

$A_c = A_g - 0.01 \text{ x } A_g$ From equation (1)

$A_c = 0.99 \text{ x } A_g -------------- (2)$

P= $б_{cc}$xA_c + $б_{sc}$xA_{sc}---------------(IS 456-2000, P. No: 81, C. No: B-3) For Square and rectangular column

P=1.05 x [$б_{cc}$xA_c + $б_{sc}$xA_{sc}]---------------(IS 456-2000, P. No: 81, C. No: B-3.2) For Circular column

From equation (1) & (2)

$A_g = ?$

Assuming square column

B x D = A_g

B=D (square column)

B x B = A_g

$B^2 = A_g$

B=?

STEP 2: Check for minimum eccentricity (IS 456-2000, P. No: 42, C. No: 25.4)

$$e_{x\min} = \left[\frac{l_x}{500} + \frac{D}{30}\right] \text{ or 20 mm (Which is greater)}$$

$$\frac{e_{x\min}}{D} \leq 0.05 \text{ (OK)}$$

$$e_{y\min} = \left[\frac{l_y}{500} + \frac{B}{30}\right] \text{ or 20 mm (Which is greater)}$$

$$\frac{e_{y\min}}{B} \leq 0.05 \text{ (OK)}$$

STEP 3: To find area of longitudinal reinforcement

From equation (1)

A_{sc} = 0.01 x A_g

Assume diameter of bar = Φ =

$$\text{Number of bars} = \frac{Asc}{(\pi/4) \text{ x } \phi^2}$$

STEP 4:: Check for A_{sc} (IS 456-2000, P. No: 48, C. No: 26.5.3.1)

A_{sc} (Provided) = ?

$$A_{sc} \text{ (Provided) } > \frac{0.8}{100} x \text{ B x D (OK)}$$

$$A_{sc} \text{ (Provided) } < \frac{6}{100} x \text{ B x D (OK)}$$

STEP 5: Design of lateral ties (IS 456-2000, P. No: 49, C. No: 26.5.3.2 – c)

a) Diameter

$$\text{Diameter} = \left(\frac{1}{4} x \text{ Diameter of main bar}\right) \text{ or 6 mm (Which is greater)}$$

b) Spacing For Square or Rectangular Column

i) Least lateral dimension

ii) 16 x Daimeter of bar

iii) 300 mm

Taking least value of i, ii and iii

For circular Column

i) 75 mm

ii) $\frac{1}{6} x$ Core Diameter >25 mm

$$= \frac{1}{6} x \left[\text{D-(2xClear Cover)}\right] > 25mm$$

Assuimng clear cover =?

Taking least value of i and ii

1) Design a short column to carry axial load of 2000 KN over unsupported length of 3.4 m. Use M_{15} & Fe 415. Use WSM

Solution:

Given Data

Axial Load= P= 2000 KN= 2000x 10^3 N

Assume Unsupported length =Effective Length = 3.4 m=3400 mm

M_{15}, 6cc = 4 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6sc = 190 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find size of column

Assuming area of steel = 1 % of gross area

$$A_{sc} = 1\% \text{ of } A_g$$

$$A_{sc} = \frac{1}{100} \text{ x } A_g$$

$$A_{sc} = 0.01 \text{ x } A_g ------------- (1)$$

Area of concrete

$A_c = A_g - A_{sc}$

$A_c = A_g - 0.01 \text{ x } A_g$ From equation (1)

$A_c = 0.99 \text{ x } A_g --------------(2)$

P= $6_{cc} x A_c + 6_{sc} x A_{sc}$ ---------------(IS 456-2000, P. No: 81, C. No: B-3)

From equation (1) & (2)

$2000 \text{ x } 10^3 = 4 \text{ x } 0.99 \, A_g + 190 \text{ x } 0.01 \, A_g$

$2000 \text{ x } 10^3 = 5.86 A_g$

$A_g = 314.296 x 10^3 \text{ mm}^2$

Assuming square column

B x D = A_g

B=D (square column)

B x B = A_g

$B^2 = A_g = 314.296 \text{ x } 10^3$

B=584.20 mm $\cong$ 590mm

B=D=590 mm

STEP 2: Check for minimum eccentricity (IS 456-2000, P. No: 42, C. No: 25.4)

$e_{x\min} = \left[\frac{l_x}{500} + \frac{D}{30}\right]$ or 20 mm (Which is greater)

$e_{x\min} = \left[\frac{3400}{500} + \frac{590}{30}\right]$ or 20 mm

$e_{x\min} = 26.46$ mm or 20 mm

$e_{x\min} = 26.46$ mm

$\frac{e_{x\min}}{D} = \frac{26.46}{590} = 0.0448 \leq 0.05$ (OK)

$e_{y\min} = \left[\frac{l_y}{500} + \frac{B}{30}\right]$ or 20 mm (Which is greater)

$e_{y\min} = \left[\frac{3400}{500} + \frac{590}{30}\right]$ or 20 mm

$e_{y\min} = 26.46$ mm or 20 mm

$e_{y\min} = 26.46$ mm

$\frac{e_{y\min}}{B} = \frac{26.46}{590} = 0.0448 \leq 0.05$ (OK)

Provide size of column 590 mm x 590 mm

STEP 3: To find area of longitudinal reinforcement

From equation (1)

A_{sc} = 0.01 x A_g =0.01 x (590 x 590)=3481 mm^2

Assume diameter of bar = Φ = 25 mm

Number of bars = $\dfrac{Asc}{(\pi/4)\text{ x }\phi^2} = \dfrac{3481}{(\pi/4)\text{ x }25^2}$ =7.09 ≅ 8

Providing 8 bars of 25 mm diameter

STEP 4: Check for A_{sc} (IS 456-2000, P. No: 48, C. No: 26.5.3.1)

$$A_{sc}\text{ (Provided) }=8x\frac{\pi}{4}\text{ X }25^2=3926.99mm^2$$

$$A_{sc}\text{ (Provided) }>\frac{0.8}{100}x\text{ B x D}$$

$$3926.99>\frac{0.8}{100}x\text{ 590 x 590}$$

$$3926.99\text{ mm}^2>2784.8\text{ mm}^2\text{ (OK)}$$

$$A_{sc}\text{ (Provided) }<\frac{6}{100}x\text{ B x D}$$

$$3926.99<\frac{6}{100}x\text{ 590 x 590}$$

$$3926.99\text{ mm}^2<20886\text{ mm}^2\text{ (OK)}$$

Providing 8 bars of 25 mm diameter

STEP 5: Design of lateral ties (IS 456-2000, P. No: 49, C. No: 26.5.3.2 – c)

a) Diameter

$$\text{Diameter}=\left(\frac{1}{4}x\text{ Diameter of main bar}\right)\text{ or 6 mm (Which is greater)}$$

$$\text{Diameter}=\left(\frac{1}{4}x\text{ 25}\right)\text{ or 6 mm}$$

Diameter=6.25 or 6 mm

Diameter=6.25 ≅ 8*mm*

providing 8 mm ϕ of lateral ties

b) Spacing

i) Least lateral dimension = 590 mm

ii) 16 x Daimeter of bar =16 x25 =400 mm

iii) 300 mm

Taking least value of i, ii and iii

Spacing =300 mm

Providing 8 mm ϕ lateral ties @ 300 mm C/C

2) Design a short column to carry axial load of 1000 KN over unsupported length of 3 m. Use M_{20} & Fe 250. Use WSM

Solution:

Given Data

Axial Load= P= 1000KN= 1000 x 10^3 N

Assume Unsupported length =Effective Length = 3 m=3000 mm

M_{20}, 6cc = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 250 , 6sc = 130 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find size of column

Assuming area of steel = 1 % of gross area

$$A_{sc} = 1\% \text{ of } A_g$$

$$A_{sc} = \frac{1}{100} \text{ x } A_g$$

$$A_{sc} = 0.01 \text{ x } A_g -------------(1)$$

Area of concrete

$$A_c = A_g - A_{sc}$$

$$A_c = A_g - 0.01 \text{ x } A_g \quad \text{From equation } (1)$$

$$A_c = 0.99 \text{ x } A_g -------------(2)$$

$P = 6_{cc} x A_c + 6_{sc} x A_{sc}$ ---------------(IS 456-2000, P. No: 81, C. No: B-3)

From equation (1) & (2)

$$1000 x\ 10^3 = 5 \text{ x } 0.99\ A_g + 130 \text{ x } 0.01\ A_g$$

$$1000 \text{ x } 10^3 = 6.25 A_g$$

$$A_g = 160 x 10^3 \text{ mm}^2$$

Assuming square column

B x D = A_g

B=D (square column)

B x B = A_g

$B^2 = A_g = 160 \text{ x } 10^3$

B=400 mm

B=D=400 mm

STEP 2: Check for minimum eccentricity (IS 456-2000, P. No: 42, C. No: 25.4)

$$e_{x\min} = \left[\frac{l_x}{500} + \frac{D}{30} \right] \text{ or 20 mm (Which is greater)}$$

$$e_{x\min} = \left[\frac{3000}{500} + \frac{400}{30} \right] \text{ or 20 mm}$$

$$e_{x\min} = 19.33 \text{ mm or 20 mm}$$

$$e_{x\min} = 20 \text{ mm}$$

$$\frac{e_{x\min}}{D} = \frac{20}{400} = 0.05 \le 0.05 \text{ (OK)}$$

$$e_{y\min} = \left[\frac{l_y}{500} + \frac{B}{30} \right] \text{ or 20 mm (Which is greater)}$$

$$e_{y\min} = \left[\frac{3000}{500} + \frac{400}{30} \right] \text{ or 20 mm}$$

$$e_{y\min} = 19.33 \text{ mm or 20 mm}$$

$$e_{y\min} = 20 \text{ mm}$$

$$\frac{e_{y\min}}{B} = \frac{20}{400} = 0.05 \le 0.05 \text{ (OK)}$$

Provide size of column 400 mm x 400 mm

STEP 3: To find area of longitudinal reinforcement

From equation (1)

A_{sc} = 0.01 x A_g =0.01 x (400 x 400)=1600 mm^2

Assume diameter of bar = Φ = 20 mm

Number of bars = $\dfrac{Asc}{(\pi/4) \text{ x } \phi^2} = \dfrac{1600}{(\pi/4) \text{ x } 20^2}$ =5.092 $\cong$ 6

Providing 6 bars of 20 mm diameter

STEP 4: Check for A_{sc} (IS 456-2000, P. No: 48, C. No: 26.5.3.1)

$$A_{sc} \text{ (Provided) } = 6x\frac{\pi}{4} \text{ X } 20^2 = 1884.95 mm^2$$

$$A_{sc} \text{ (Provided) } > \frac{0.8}{100} x \text{ B x D}$$

$$1884.95 > \frac{0.8}{100} x \text{ 400 x 400}$$

$$1884.95 \text{ mm}^2 > 1280 \text{ mm}^2 \text{ (OK)}$$

$$A_{sc} \text{ (Provided) } < \frac{6}{100} x \text{ B x D}$$

$$1884.95 < \frac{6}{100} x \text{ 400 x 400}$$

$$1884.95 \text{ mm}^2 < 9600 \text{ mm}^2 \text{ (OK)}$$

Providing 6 bars of 20 mm diameter

STEP 5: Design of lateral ties (IS 456-2000, P. No: 49, C. No: 26.5.3.2 – c)

a) Diameter

$$\text{Diameter}=\left(\frac{1}{4}\,x \text{ Diameter of main bar}\right) \text{ or 6 mm (Which is greater)}$$

$$\text{Diameter}=\left(\frac{1}{4}\,x\ 20\right) \text{ or 6 mm}$$

Diameter=5 or 6 mm

Diameter=6

providing 6 mm ϕ of lateral ties

b) Spacing

i) Least lateral dimension = 400 mm

ii) 16 x Daimeter of bar =16 x20 =400 mm

iii) 300 mm

Taking least value of i, ii and iii

Spacing =300 mm

Providing 6 mm ϕ lateral ties @ 300 mm C/C

3) Design a short rectangular column to carry axial load of 900 KN. The length of column is 3.2 m, the column is effectively held in position at both ends but not restrained against rotation at one end. Use M_{20} & Fe 250. Use WSM

Solution:

Given Data

Axial Load= P= 900KN= 900 x 10^3 N

L=3.2 m

The column is effectively held in position at both ends but not restrained against rotation at one end (One ends fixed and other hinged) **(P. no:94, Table No:28)**

L_{eff}=0.8 x L=0.8 x 3.2=2.560 m=2560 mm

M_{20}, 6cc = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 250 , 6sc = 130 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find size of column

Assuming area of steel = 1 % of gross area

$$A_{sc} = 1\% \text{ of } A_g$$

$$A_{sc} = \frac{1}{100} \text{ x } A_g$$

$$A_{sc} = 0.01 \text{ x } A_g \text{ -------------- } (1)$$

Area of concrete

$A_c = A_g - A_{sc}$

$A_c = A_g - 0.01 \text{ x } A_g$ From equation (1)

$A_c = 0.99 \text{ x } A_g --------------(2)$

P= 6_{cc}xA_c + 6_{sc}xA_{sc}---------------(IS 456-2000, P. No: 81, C. No: B-3)

From equation (1) & (2)

$900\text{x } 10^3 = 5 \text{ x } 0.99 \text{ A}_g + 130 \text{ x } 0.01 \text{ A}_g$

$900 \text{ x } 10^3 = 6.25\text{A}_g$

$\text{A}_g = 144x10^3 \text{ mm}^2$

Assuming B=400 mm

B x D = A_g

B=400 mm

400 x D = 144 x 10^3

D=360 mm $\cong$ 425mm

Note : Take value D more B=400 mm for satisfaction of check

STEP 2: Check for minimum eccentricity (IS 456-2000, P. No: 42, C. No: 25.4)

$e_{x\min} = \left[\frac{l_x}{500} + \frac{D}{30} \right]$ or 20 mm (Which is greater)

$e_{x\min} = \left[\frac{2560}{500} + \frac{425}{30} \right]$ or 20 mm

$e_{x\min} = 19.28$ mm or 20 mm

$e_{x\min} = 20$ mm

$\frac{e_{x\min}}{D} = \frac{20}{425} = 0.0470 \leq 0.05$ (OK)

$e_{y\min} = \left[\frac{l_y}{500} + \frac{B}{30} \right]$ or 20 mm (Which is greater)

$e_{y\min} = \left[\frac{2650}{500} + \frac{400}{30} \right]$ or 20 mm

$e_{y\min} = 18.633$ mm or 20 mm

$e_{y\min} = 20$ mm

$\frac{e_{y\min}}{B} = \frac{20}{400} = 0.05 \leq 0.05$ (OK)

Provide size of column 400 mm x 425 mm

STEP 3: To find area of longitudinal reinforcement

From equation (1)

A_{sc} = 0.01 x A_g =0.01 x (400 x 425)=1700 mm^2

Assume diameter of bar = Φ = 20 mm

Number of bars = $\frac{Asc}{(\pi/4) \text{ x } \phi^2} = \frac{1700}{(\pi/4) \text{ x } 20^2}$ =5.411 ≅ 6

Providing 6 bars of 20 mm diameter

STEP 4: Check for A_{sc} (IS 456-2000, P. No: 48, C. No: 26.5.3.1)

A_{sc} (Provided) $=6x\frac{\pi}{4}$ X $20^2 = 1884.95mm^2$

A_{sc} (Provided) $> \frac{0.8}{100} x$ B x D

$1884.95 > \frac{0.8}{100} x$ 400 x 425

1884.95 $\text{mm}^2 > 1360 \text{ mm}^2$ (OK)

A_{sc} (Provided) $< \frac{6}{100} x$ B x D

$1884.95 < \frac{6}{100} x$ 400 x 425

1884.95 $\text{mm}^2 < 10200 \text{ mm}^2$ (OK)

Providing 6 bars of 20 mm diameter

STEP 5: Design of lateral ties (IS 456-2000, P. No: 49, C. No: 26.5.3.2 – c)

a) Diameter

Diameter=$\left(\frac{1}{4} x \text{ Diameter of main bar}\right)$ or 6 mm (Which is greater)

Diameter=$\left(\frac{1}{4} x \ 20\right)$ or 6 mm

Diameter=5 or 6 mm

Diameter=6

providing 6 mm ϕ of lateral ties

b) Spacing

i) Least lateral dimension = 400 mm

ii) 16 x Daimeter of bar =16 x20 =400 mm

iii) 300 mm

Taking least value of i, ii and iii

Spacing =300 mm

Providing 6 mm ϕ lateral ties @ 300 mm C/C

4) Design a circular column to carry axial load of 1200 KN. The effective length of column is 3.4 m . Use M_{25} & Fe 415. Use WSM

Solution:

Given Data

Axial Load= P= 1200KN= 1200 x 10^3 N

L_{eff}= 3.4 m=3400 mm

M_{25}, 6cc = 6 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6sc = 190 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find size of column

Assuming area of steel = 1 % of gross area

$A_{sc} = 1\% \text{ of } A_g$

$A_{sc} = \frac{1}{100} \text{ x } A_g$

$A_{sc} = 0.01 \text{ x } A_g --------------(1)$

Area of concrete

$A_c = A_g - A_{sc}$

$A_c = A_g - 0.01 \text{ x } A_g \quad \text{From equation } (1)$

$A_c = 0.99 \text{ x } A_g --------------(2)$

P= 1.05 x$\left[6_{cc}\text{x}A_c + 6_{sc}\text{x}A_{sc}\right]$---------------(IS 456-2000, P. No: 81, C. No: B-3.2)

From equation (1) & (2)

$1200\text{x } 10^3 = 1.05 \text{ x}\left[6 \text{ x } 0.99 \text{ } A_g + 190 \text{ x } 0.01 \text{ } A_g\right]$

$1200 \text{ x } 10^3 = 8.232 A_g$

$A_g = 145.77 x 10^3 \text{ mm}^2$

The column is circular

$Ag = 145.77 \text{ x} 10^3 mm^2$

$\frac{\pi}{4} D^2 = 145.77 \text{ x} 10^3$

$D = 430.81 \text{ mm } \cong 450 \text{ mm}$

STEP 2: Check for minimum eccentricity (IS 456-2000, P. No: 42, C. No: 25.4)

$e_{x\min} = \left[\frac{l_x}{500} + \frac{D}{30}\right]$ or 20 mm (Which is greater)

$e_{x\min} = \left[\frac{3400}{500} + \frac{450}{30}\right]$ or 20 mm

$e_{x\min} = 21.8$ mm or 20 mm

$e_{x\min} = 21.8$ mm

$\frac{e_{x\min}}{D} = \frac{21.8}{450} = 0.0484 \le 0.05$ (OK)

Provide Diameter of column 450 mm

STEP 3: To find area of longitudinal reinforcement

From equation (1)

A_{sc} = 0.01 x A_g =0.01 x $\frac{\pi}{4}D^2$ =0.01 x $\frac{\pi}{4}x450^2$ =1590.43 mm^2

Assume diameter of bar = Φ = 20 mm

Number of bars = $\frac{Asc}{(\pi/4)\text{ x }\phi^2} = \frac{1590.43}{(\pi/4)\text{ x }20^2}$ =5.0625 ≅ 6

Providing 6 bars of 20 mm diameter

STEP 4: Check for A_{sc} (IS 456-2000, P. No: 48, C. No: 26.5.3.1)

A_{sc} (Provided) $=6x\frac{\pi}{4}$ X $20^2 = 1884.95mm^2$

A_{sc} (Provided) $> \frac{0.8}{100} x \frac{\pi}{4} xD^2$

$1884.95 > \frac{0.8}{100} x \frac{\pi}{4} x450^2$

1884.95 mm^2 $>$ 1272.345mm^2 (OK)

A_{sc} (Provided) $< \frac{6}{100} x \frac{\pi}{4} xD^2$

$1884.95 < \frac{6}{100} x \frac{\pi}{4} x450^2$

1884.95mm^2 $<$ 9542.58 mm^2 (OK)

Providing 6 bars of 20 mm diameter

STEP 5: Design of helical reinforcement (IS 456-2000, P. No: 49, C. No: 26.5.3.2 – d)

a) Diameter

Diameter=$\left(\frac{1}{4} x \text{ Diameter of main bar}\right)$ or 6 mm (Which is greater)

Diameter=$\left(\frac{1}{4} x\ 20\right)$ or 6 mm

Diameter=5*mm* or 6 mm

Diameter=6 mm

providing 6 mm ϕ of lateral ties

b) Pitch

i) 75 mm

ii) $\frac{1}{6} x$ Core Diameter >25 mm

$=\frac{1}{6} x \left[\text{D-(2xClear Cover)}\right] > 25mm$

Assuimng clear cover =40 mm

$=\frac{1}{6} x \left[\text{450-(2x40)}\right] = 61.67mm > 25mm$

Taking least value of i and ii

Pitch =61.67 mm $\cong$ 60 mm

Providing 6 mm ϕ spiral@ 60 mm C/C

5) Design a circular column to carry axial load of 2500 KN. The effective length of column is 3.5 m . Use M_{25} & Fe 415. Use WSM

Solution:

Given Data

Axial Load= P= 2500KN= 2500 x 10^3 N

L_{eff}= 3.5 m=3500 mm

M_{25}, 6cc = 6 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6sc = 190 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find size of column

Assuming area of steel = 1 % of gross area

$A_{sc} = 1\% \text{ of } A_g$

$A_{sc} = \frac{1}{100} \text{ x } A_g$

$A_{sc} = 0.01 \text{ x } A_g --------------(1)$

Area of concrete

$A_c = A_g - A_{sc}$

$A_c = A_g - 0.01 \text{ x } A_g \quad \text{From equation } (1)$

$A_c = 0.99 \text{ x } A_g --------------(2)$

P= 1.05 x$\left[6_{cc} x A_c + 6_{sc} x A_{sc}\right]$---------------(IS 456-2000, P. No: 81, C. No: B-3.2)

From equation (1) & (2)

2500x 10^3= 1.05 x$\left[6 \text{ x } 0.99 \text{ } A_g + 190 \text{ x } 0.01 \text{ } A_g\right]$

2500 x 10^3=8.232A_g

$A_g = 303.69 x 10^3 \text{ mm}^2$

The column is circular

$\text{Ag}=303.69x10^3 mm^2$

$\frac{\pi}{4}D^2 = 303.69x10^3$

$D = 621.82\text{mm} \cong 650 \text{ mm}$

STEP 2: Check for minimum eccentricity (IS 456-2000, P. No: 42, C. No: 25.4)

$e_{x\min} = \left[\frac{l_x}{500}+\frac{D}{30}\right]$ or 20 mm (Which is greater)

$e_{x\min} = \left[\frac{3500}{500}+\frac{650}{30}\right]$ or 20 mm

$e_{x\min} = 28.66$ mm or 20 mm

$e_{x\min} = 28.66$ mm

$\frac{e_{x\min}}{D} = \frac{28.66}{650} = 0.0441 \leq 0.05$ (OK)

Provide Diameter of column 650 mm

STEP 3: To find area of longitudinal reinforcement

From equation (1)

$A_{sc} = 0.01 \text{ x } A_g = 0.01 \text{ x } \frac{\pi}{4}D^2 = 0.01 \text{ x } \frac{\pi}{4}x650^2 = 3318.30 \text{ mm}^2$

Assume diameter of bar = Φ = 25 mm

Number of bars = $\frac{Asc}{(\pi/4) \text{ x } \phi^2} = \frac{3318.30}{(\pi/4) \text{ x } 25^2} = 6.75 \cong 8$

Providing 8 bars of 25 mm diameter

STEP 4: Check for A_{sc} (IS 456-2000, P. No: 48, C. No: 26.5.3.1)

A_{sc} (Provided) $=8x\frac{\pi}{4} \text{ X } 25^2 = 3926.99mm^2$

A_{sc} (Provided) $> \frac{0.8}{100}x \frac{\pi}{4}xD^2$

$3926.99 > \frac{0.8}{100}x \frac{\pi}{4}x650^2$

$3926.99 \text{ mm}^2 > 2654.64\text{mm}^2$ (OK)

A_{sc} (Provided) $< \frac{6}{100}x\frac{\pi}{4}xD^2$

$3926.99 < \frac{6}{100}x \frac{\pi}{4}x650^2$

$3926.99 \text{ mm}^2 < 19909.84 \text{ mm}^2$ (OK)

Providing 8 bars of 25 mm diameter

STEP 5: Design of helical reinforcement (IS 456-2000, P. No: 49, C. No: 26.5.3.2 – d)

a) Diameter

Diameter= $\left(\frac{1}{4}x \text{ Diameter of main bar}\right)$ or 6 mm (Which is greater)

Diameter= $\left(\frac{1}{4}x\ 25\right)$ or 6 mm

Diameter=6.25 mm or 6 mm

Diameter= $6.25 \cong 8mm$

providing 8 mm ϕ of lateral ties

b) Pitch

i) 75 mm

ii) $\frac{1}{6}x$ Core Diameter >25 mm

$= \frac{1}{6}x\ [\text{D-(2xClear Cover)}] > 25mm$

Assuimng clear cover =40 mm

$= \frac{1}{6}x\ [650\text{-}(2\text{x}40)] = 95mm > 25mm$

Taking least value of i and ii

Pitch =75 mm

Providing 8 mm ϕ spiral@ 75 mm C/C

Type III: Analysis of eccentrically loaded column

Design Procedure

Given Data

1) A R.C. column 400 mm X 400 mm is reinforced with 4 bars of 25 mm diameter, placed at a cover of 50 mm to the centre of steel bars. Determine the maximum and minimum stresses in concrete if the column is subjected to a load of 400 KN at an eccentricity of 50 mm about one of the axes. Also Check whether the section is safe or not. Use M_{15} and m= 19.

   Solution:

   Given Data:

   B= 400 mm

   D= 400 mm

   Eccentricity =e= 50 mm

   Effective Cover =d'= 50 mm

   m=19

$$A_{sc} = 4x\frac{\pi}{4} \text{ X } 25^2 = 1963.50 \; mm^2$$

M_{15}, 6cbc = 5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

M_{15}, 6cc = 4 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

STEP 1: Equivalent area of concretes (A_e) (IS 456:2000, P No:83, C No. B-4)

$$A_e = A_c + 1.5 \text{ m } A_{sc}$$

$$A_e = \left(A_g - A_{sc}\right) + 1.5 \text{ m } A_{sc}$$

$$A_e = A_g - A_{sc} + 1.5 \text{ m } A_{sc}$$

$$A_e = A_g + A_{sc}(1.5 \text{ m } -1)$$

$$A_e = (400x400) + 1963.50x(1.5 \text{ x}19 -1)$$

$$A_e = 213993.25 \; mm^2$$

STEP 2: Equivalent moment of inertia about the centroidal axix x-x (I_e)

$$I_e = \frac{BD^3}{12} + (1.5 \text{ m } -1)A_{sc}\left(\frac{D}{2} - d'\right)^2$$

$$I_e = \frac{400x400^3}{12} + (1.5 \text{ x } 19 -1)1963.50\left(\frac{400}{2} - 50\right)^2$$

$$I_e = 3348.25 \text{ x } 10^6 \text{ mm}^4$$

STEP 3: Calculated direct compressive stress in concrete ($6_{cc,\,cal}$)

$$6_{cc,cal} = \frac{P}{A_e}$$

$$6_{cc,cal} = \frac{400x10^3}{213996.25} = 1.87 \text{ N/mm}^2$$

STEP 4: Calculated bending compressive stress in concrete ($6_{cbc,\,cal}$)

$$6_{cbc,cal} = \frac{M}{Z} = \frac{Pe}{I_e/y} = \frac{Pe}{I_e}\frac{D}{2}$$

$$6_{cbc,cal} = \frac{400x10^3 x50}{3348.25x10^6} x \frac{400}{2}$$

$$6_{cbc,cal} = 1.19 \text{ N/mm}^2$$

STEP 5: Maximum and minimum stresses in concrete

Maximum stress = 6_{max} = $6_{cc,\,cal+}$ $6_{cbc,\,cal}$

6_{max} = $6_{cc,\,cal+}$ $6_{cbc,\,cal}$

6_{max} =1.87+1.19=3.06 N/mm^2

Minimum stress = 6_{min}= $6_{cc, cal}$- $6_{cbc, cal}$

6_{min} = $6_{cc, cal}$ - $6_{cbc, cal}$

6_{min} =1.87 - 1.19=0.68 N/mm^2

STEP 6: Check the section (IS 456:2000, P No:83, C No. B-4)

$$\frac{6_{cc,cal}}{6_{cc}}+\frac{6_{cbc,cal}}{6_{cbc}}\leq 1$$

$$\frac{1.87}{4}+\frac{1.19}{5}\leq 1$$

$0.71 \leq 1$ Column section is safe

2) A R.C. column 400 mm X 600 mm is reinforced with 6 bars of 20 mm diameter, placed at a cover of 40 mm from top edge & 6 similar bars at the same cover from the bottom edge. Determine the maximum load on the section, which can be applied at a distance of 80 mm, from the centre line, if the compressive stress in concrete is not exceed 7 N/mm^2. Use M_{20}

Solution:

Given Data:

Solution:

Given Data:

B= 400 mm

D= 600 mm

Eccentricity =e= 80 mm

Effective Cover =d'= 40+(20/2)=50 mm

$$A_{sc} = 12x\frac{\pi}{4}\ \text{X}\ 20^2 = 3769.91\ mm^2$$

6_{max} =7N/mm^2

M_{20}, 6cbc = 7 N/mm^2 (IS 456:2000, Table No: 21, P No:81

M_{20}, 6cc =5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

$$m = \frac{280}{3\ \text{x}\ 6_{cbc}} = \frac{280}{3\ \text{x}\ 7} = 13.33$$

STEP 1: Equivalent area of concretes (A_e)

$A_e = A_c + 1.5\ m\ A_{sc}$

$A_e = \left(A_g - A_{sc}\right) + 1.5\ m\ A_{sc}$

$A_e = A_g - A_{sc} + 1.5\ m\ A_{sc}$

$A_e = A_g + A_{sc}(1.5\ m - 1)$

$A_e = (400x600) + 3769.91x(1.5\ x13.33 - 1)$

$A_e = 311609.44\ mm^2$

STEP 2: Equivalent moment of inertia about the centroidal axix x-x (I_e)

$$I_e = \frac{BD^3}{12} + (1.5\ m - 1)A_{sc}\left(\frac{D}{2} - d'\right)^2$$

$$I_e = \frac{400x600^3}{12} + (1.5\ x\ 13.33 - 1)x13769.91\left(\frac{600}{2} - 50\right)^2$$

$$I_e = 11675.59\ x\ 10^6\ mm^4$$

STEP 3: Calculated maximum load on the section

$$6_{max} = \frac{P}{A_e} + \frac{M}{Z}$$

$$6_{max} = \frac{P}{A_e} + \frac{Pe}{I_e}\frac{D}{2}$$

$$7 = \frac{P}{311609.44} + \frac{Px80}{11675.59x10^6}x\frac{600}{2}$$

$P = 1329.61x10^3$ N

$P = 1329.61$ KN

3) A R.C. column 300 mm X 300 mm is reinforced with 4 bars of 25 mm diameter placed at effective cover of 50 mm. Find eccentricity , the line of trust may pass along the YY axis without causing tension in concrete. Take m=18

Solution:

Given Data:

B= 300 mm

D= 300 mm

Effective Cover =d'=50 mm

$A_{sc} = 4x\frac{\pi}{4}\ X\ 25^2 = 1963.50\ mm^2$

m=18

STEP 1: Equivalent area of concretes (A_e)

$$A_e = A_c + 1.5\ m\ A_{sc}$$
$$A_e = \left(A_g - A_{sc}\right) + 1.5\ m\ A_{sc}$$
$$A_e = A_g - A_{sc} + 1.5\ m\ A_{sc}$$
$$A_e = A_g + A_{sc}(1.5\ m - 1)$$
$$A_e = (300x300) + 1963.5x(1.5\ x18\ -1)$$
$$A_e = 141051\ mm^2$$

STEP 2: Equivalent moment of inertia about the centroidal axix x-x (I_e)

$$I_e = \frac{BD^3}{12} + (1.5\ m - 1)A_{sc}\left(\frac{D}{2} - d'\right)^2$$
$$I_e = \frac{300x300^3}{12} + (1.5\ x\ 18\ -1)\ x1963.5\left(\frac{300}{2} - 50\right)^2$$
$$I_e = 1185.51\ x\ 10^6\ mm^4$$

STEP 3: If tension in concrete should be just avoided the direct stress and bending stress should be equal for the section

$$6_{cc.cal} = 6_{cbc.cal}$$
$$\frac{P}{A_e} = \frac{M}{Z}$$
$$\frac{P}{A_e} = \frac{Pe}{I_e}\frac{D}{2}$$
$$\frac{1}{A_e} = \frac{e}{I_e}\frac{D}{2}$$
$$\frac{1}{141051} = \frac{e}{1185.51x10^6}\frac{300}{2}$$
$$e = 56.03mm$$

Footing

Introduction: The foundation of your house is a critical part of its structure as it helps distribute the load and minimizes distress against the foundation soil movement, thereby keeping the building stable and secure. Hence, foundations are critical to the structural safety of a building .Depending on the depth of the soil in which the foundation is made, there are two types of foundation used in constructing buildings :

Classification of footing

Isolated Footing: Isolated footing is described as the footing that is offered underneath the column to spread the loads securely towards/to the bed soil. That sort of footing is utilized to assist single columns as well as at the time the columns are organized approximately at significant distance. That is highly reasonable sort of footing. Various well-known forms in the footings plan are: 1) Rectangular Isolated Footing 2) Square Isolated Footing 3) Slope Footing.

Combined Footing: That structure supports the load of 2 or additional columns. Moreover, they are built in trapezoidal or rectangular in form. That footing is required at the time isolated construction overlays/overlaps. Also, low soil bearing is the major causes of obtaining the united/combined footing positioned.

Various famous forms in the footings' plan are: 1) Trapezoidal Combined Footing 2) Rectangular Combined Footing 3) Elliptical Combined Footing.

Strip Footing or Continuous or Wall Footing: A strip footing or wall footing is a concrete's continuous strip that caters to distribute the load-bearing wall's weight over/across a soil's region/area. Moreover, it is the part of a shallow foundation.

Strap Footing: It is a sort of combined footing wherein 2 separated/isolated columns are linked through a strap beam. These footings shift the loads out of/from external column to the internal one, forming bending movement as well as share force within/in strap beams.

Mat or Raft Footing: A raft foundation, even known as a mat foundation, is basically a continuous slab situating/lying on the soil that reaches over the whole footprint of the structure/building, thus assisting the structure as well as shifting its mass/weight towards the land/ground.

Pile Footing: It is needed with regard to low bearing soil to obtain the utmost assistance for the structure. Furthermore, piles appear/come in perpendicular/vertical structure/arrangement that are pierced/drilled into the land/ground for obtaining the required withstand capacity out of/from the deeper/deep soil layer. Moreover, the foundation is situated on piles that remain distinct or in cluster underneath the soil. The foundation's load is passed/transmitted to the deep/deeper soil through those piles.

Well Foundation: Well foundation is a sort of deep foundation that is usually offered beneath the water level with regard to bridges. Furthermore, well or Cassions have been in utilization with regard to bridges' foundations as well as additional structures from Roman as well as Mughal periods.

Design Considerations

(a) **Minimum nominal cover** (cl. 26.4.2.2 of IS 456)

The minimum nominal cover for the footings should be more than that of other structural elements of the superstructure as the footings are in direct contact with the soil. Clause 26.4.2.2 of IS 456 prescribes a minimum cover of 50 mm for footings. However, the actual cover may be even more depending on the presence of harmful chemicals or minerals, water table etc.

(b) Thickness at the edge of footings

(cls. 34.1.2 and 34.1.3 of IS 456) The minimum thickness at the edge of reinforced and plain concrete footings shall be at least 150 mm for footings on soils and at least 300 mm above the top of piles for footings on piles, as per the stipulation in cl.34.1.2 of IS 456. For plain concrete pedestals, the angle α between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane shall be determined from the following expression (cl.34.1.3 of IS 456)

$$\tan \alpha \le 0.9 \sqrt{\frac{100 q_0}{f_{ck}} + 1}$$

where q_0 = calculated maximum bearing pressure at the base of the pedestal in N/mm^2, and fck = characteristic strength of concrete at 28 days in N/mm^2.

(c) Bending moments (cl. 34.2 of IS 456)

1. It may be necessary to compute the bending moment at several sections of the footing depending on the type of footing, nature of loads and the distribution of pressure at the base of the footing. However, bending moment at any section shall be determined taking all forces acting over the entire area on one side of the section of the footing, which is obtained by passing a vertical plane at that section extending across the footing (cl.34.2.3.1 of IS 456).

2. The critical section of maximum bending moment for the purpose of designing an isolated concrete footing which supports a column, pedestal or wall shall be: (i) at the face of the column, pedestal or wall for footing supporting a concrete column, pedestal or reinforced concrete wall, and halfway between the centre-line and the edge of the wall, for footing under masonry wall (Fig.11.28.10). This is stipulated in cl.34.2.3.2 of IS 456.

d) Shear force (cl. 31.6 and 34.2.4 of IS 456) Footing slabs shall be checked in one-way or two-way shears depending on the nature of bending. If the slab bends primarily in one-way, the footing slab shall be checked in one-way vertical shear. On the other hand, when the bending is primarily two-way, the footing slab shall be checked in two-way shear or punching shear. The respective critical sections and design shear strengths are given below:

1. One-way shear (cl. 34.2.4 of IS 456)

One-way shear has to be checked across the full width of the base slab on a vertical section located from the face of the column, pedestal or wall at a distance equal to (i) effective depth of the footing slab in case of footing slab on soil, and

ii) half the effective depth of the footing slab if the footing slab is on piles. The design shear strength of concrete without shear reinforcement is given in Table 19 of cl.40.2 of IS 456

2. Two-way or punching shear

(cls.31.6 and 34.2.4) Two-way or punching shear shall be checked around the column on a perimeter half the effective depth of the footing slab away from the face of the column or pedestal. The permissible shear stress, when shear reinforcement is not provided, shall not exceed $k\tau_c$, where $k_s = (0.5 + \beta c)$, but not greater than one, β c being the ratio of short side to long side of the column, and $\tau_c = 0.25(fck)^{1/2}$ in limit state method of design, as stipulated in cl.31.6.3 of IS 456. Normally, the thickness of the base slab is governed by shear. Hence, the necessary thickness of the slab has to be provided to avoid shear reinforcement.

(e) Bond (cl.34.2.4.3 of IS 456)

The critical section for checking the development length in a footing slab shall be the same planes as those of bending moments in part (c) of this section. Moreover, development length shall be checked at all other sections where they change abruptly. The critical sections for checking the development length are given in cl.34.2.4.3 of IS 456, which further recommends to check the anchorage requirements if the reinforcement is curtailed, which shall be done in accordance with cl.26.2.3 of IS 456.

(f) Tensile reinforcement

(cl.34.3 of IS 456) The distribution of the total tensile reinforcement, calculated in accordance with the moment at critical sections, as specified in part (c) of this section, shall be done as given below for one-way and two-way footing slabs separately. (i) In one-way reinforced footing slabs like wall footings, the reinforcement shall be distributed uniformly across the full width of the footing i.e., perpendicular to the direction of wall. Nominal distribution reinforcement shall be provided as per cl. 34.5 of IS 456 along the length of the wall to take care of the secondary moment, differential settlement, shrinkage and temperature effects.

(ii) In two-way reinforced square footing slabs, the reinforcement extending in each direction shall be distributed uniformly across the full width/length of the footing.

(iii) In two-way reinforced rectangular footing slabs, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing slab. In the short direction, a central band equal to the width of the footing shall be marked along the length of the footing, where the portion of the reinforcement shall be determined as given in the equation below. This portion of the reinforcement shall be distributed across the central band:

Design Procedure:

Given Data

STEP 1: To find design constant

i) Modular Ratio (m)

$$m = \frac{280}{3 \text{ x } 6_{\text{cbc}}}$$

ii) Neutral Axis depth factor (k)

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}}$$

iii) Lever arm factor (j)

$$j = 1 - \frac{k}{3}$$

iv) Moment resisting factor (Q)

$$\text{Q}= \frac{1}{2} 6_{\text{cbc}} \text{jk}$$

STEP 2:: To find size of footing

Axial Load = P

Assuming self weight of footing =10 % of Axial load

Assuming self weight of footing $P_1 == \frac{10}{100} \text{xP}$

Total load= $P+P_1$

Area of footing =A= $\frac{\text{Total load}}{SBC}$

STEP 3: To find net upward pressure

Net upward pressure= $q_0 = \frac{Axial\ \text{Load}}{A_{provided}}$

STEP 4: To find bending Moment

The maximum bending moment act at the face of column

Distance= $\frac{L-D}{2}$

$$M = \frac{(q_0 B)\, x (Distance)^2}{2}$$

STEP 5: Check for depth

Equating maximum BM to resisting moment

$M = QBd^2$

$d = ?$ (Increse depth upto half of actual for correct check in shear)

Assuming effective cover =d'=50 mm

Depth of footing $= D_f = d + d'$

STEP 6: To find area of steel

M=T(jxd)

$$A_{st} = \frac{M}{6_{st}\, jd}$$

Assuming ϕ=? mm

$$Number \text{ of bar} = \frac{Ast}{(\pi/4) \text{ x } \phi^2}$$

STEP 7: Check for one way shear

The critical section for one way shear at a distance ‘d’ from the face of column

$$\text{Distance} = \left(\frac{L-D}{2}\right) - d$$

a) Nominal shear stress (τ_v)

$$\tau_v = \frac{V}{Bd} = \frac{q_0 \ x \text{ Shaded Area}}{Bd}$$

b) Design shear strength of concrete ($\tau_c k$)

$$P_t = 100 \text{ x } \frac{A_{st}}{Bd}$$ (Page Number 84, Table Number 23, IS 456:2000)

To find design shear strength of concrete (τ_c)

(Page Number 84, Table Number 23, IS 456:2000)

Pt %	τ_c
0.25	**0.22**
0.3427	**?**
0.50	**0.30**

$\tau_c = ?$

k=1 (IS 456:2000. P. No:84 C. No: B-5.2.1.1) as Df=300 mm

$\tau_c k = 0.2496 x 1 = 0.2496 \text{ N/mm}^2$

Compare τ_v & $\tau_c k$

$\tau_v < \tau_c k$

Safe in one way shear

STEP 8:: Check for two way shear

The critical section for two way shear at a distance ‘d/2’ from the face of column

a) Nominal shear stress (τ_v) **(IS 456:2000, P No: 57 , C No:31.6.2.1)**

$$\tau_v = \frac{V}{B_0 d} = \frac{q_0 \ x \text{ Shaded Area}}{Perimeter \text{ x } d}$$

b) Permissible shear stress of concrete ($\tau_c k_s$) **(IS 456:2000. P. No:58 C. No:** 31.6.3)

$$k_s = (0.5 + B_c) = 0.5 + \left[\frac{B}{L}\right]$$

$$and$$

$$\tau_c = 0.16\sqrt{F_{ck}}$$

Compare τ_v & $\tau_c k_s$

$\tau_v < \tau_c k_s$

Safe in two way shear

STEP 9:: Check for development length

IS 456:2000, P. No. 81, Table No:21 & IS 456:2000, P. No. 80, C.No No:B-2.1.2

$$L_d = \frac{\phi 6_s}{4 x \tau_{bd}}$$

Length of bar available= $\frac{1}{2}(B-b) - \text{cov}\, er$

Length of bar available=L

=L>L_d (Ok)

1) Design an isolated footing of uniform thickness of a R.C. column carrying a load of 500 KN and having size 500 mm X 500 mm. The safe bearing capacity of soil may be takes as 120 KN/m^2. Use M_{20} and Fe 415. Use WSM

   Solution:

Given Data

Width of column = b = 500 mm

Depth of column= D= 500 mm

Load =P =500 KN

Safe bearing capacity of soil =120 KN/m^2

M_{20}, 6cbc = 7 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6st = 230 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find design constant

v) Modular Ratio (m)

$$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 7} = 13.33$$

vi) Neutral Axis depth factor (k)

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{13.33x\ 7}{(13.33x\ 7) + 230} = 0.2886$$

vii) Lever arm factor (j)

$$j = 1 - \frac{k}{3} = 1 - \frac{0.2886}{3} = 0.9037$$

viii) Moment resisting factor (Q)

$$Q=\frac{1}{2}6_{cbc}jk$$

$$Q=\frac{1}{2}x7x0.9037x0.2886=0.9128$$

STEP 2:: To find size of footing

Axial Load = P=500 KN

Assuming self weight of footing =10 % of Axial load

Assuming self weight of footing = $\frac{10}{100}$x500=50 KN

Total load= 500 +50 = 550 KN

Area of footing =A= $\frac{\text{Total load}}{SBC}=\frac{550}{120}=4.58m^2$

The footing is square footing

Each side of footing= L= B (Square Column)

L = B= $\sqrt{A}=\sqrt{4.58}=2.14m\cong 2.2m$

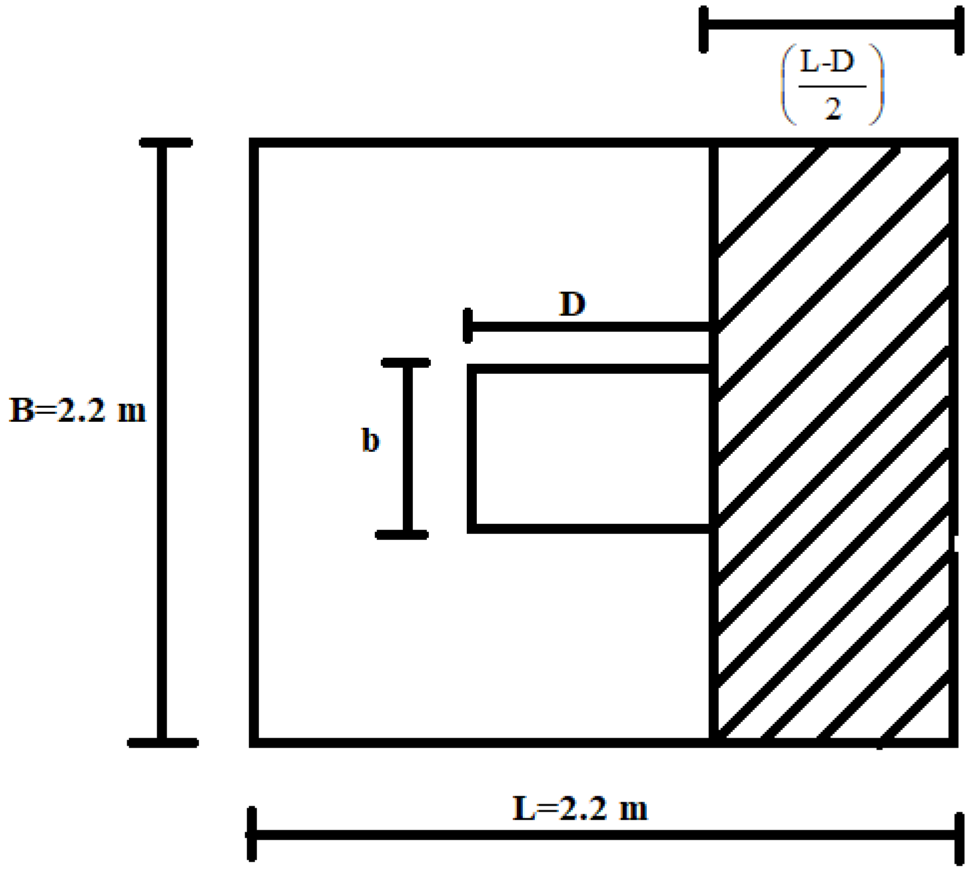
Providing size of footing = L x B= 2.2 m x 2.2m

STEP 3: To find net upward pressure

Net upward pressure= $q_0=\frac{Axial\ \text{Load}}{A_{provided}}=\frac{500}{2.2x2.2}=$103.305 KN/m^2

STEP 4: To find bending Moment

The maximum bending moment act at the face of column



Distance= $\frac{L-D}{2}=\frac{2.2-0.5}{2}=0.85m$

$$M=\frac{(q_0 B)x0.85^2}{2}=\frac{103.305x2.2x0.85^2}{2}=82.101 \text{ KNm}$$

STEP 5: Check for depth

Equating maximum BM to resisting moment

$M=QBd^2$

$82.10x10^6=0.9128x2200xd^2$

$d=202.19mm\cong 250mm$ (Increse depth upto half of actual for correct check in shear)

Assuming effective cover =d'=50 mm

Depth of footing =D_f =250+50=300 mm

STEP 6:: To find area of steel

M=T(jxd)

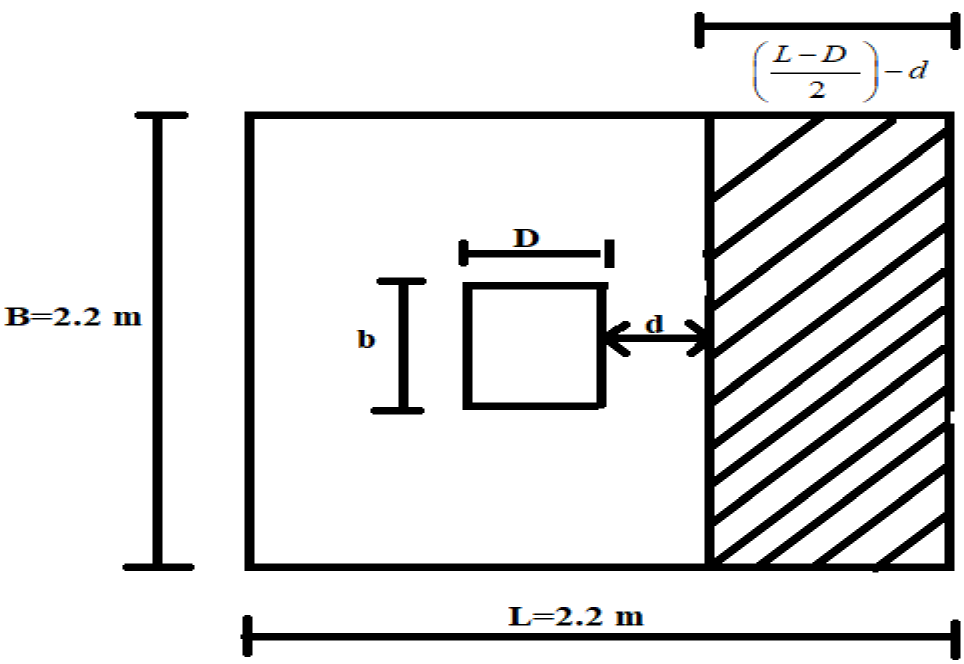
$$A_{st} = \frac{M}{6_{st}\,jd} = \frac{82.101\ x10^6}{230\ x0.9037\ x250} = 1579.99mm^2$$

Assuming ϕ=10 mm

$$Number\ \text{of bar}=\frac{Ast}{(\pi/4)\ x\ \phi^2} = \frac{1579.99}{(\pi/4)\ x\ 10^2}$$ =20.11 $\cong$ 24 (Increaes no of bar For safety of one way shear)

STEP 7: Check for one way shear

The critical section for one way shear at a distance 'd' from the face of column



$$\text{Distance}=\left(\frac{L-D}{2}\right)-d = \frac{2.2-0.5}{2} - 0.25 = 0.60m$$

c) Nominal shear stress (τ_v)

$$\tau_v = \frac{V}{Bd} = \frac{q_0\ x\ \text{Shaded Area}}{Bd} = \frac{103.305x(0.60x2.2)}{2.2x0.25} = 247.932KN/m^2$$

$$\tau_v = \frac{247.932x10^3}{10^6} = 0.2479N/mm^2$$

d) Design shear strength of concrete ($\tau_c k$)

$$P_t = 100 \text{ x } \frac{A_{st}}{Bd}$$ (Page Number 84, Table Number 23, IS 456:2000)

$$P_t = 100 \text{ x } \frac{24x\frac{\pi}{4}x10^2}{2200 \text{ x } 250} = 100x\frac{1884.95}{2200 \text{ x } 250} = 0.3427$$

To find design shear strength of concrete (τ_c)

(Page Number 84, Table Number 23, IS 456:2000)

Pt %	τ_c
0.25	**0.22**
0.3427	**?**
0.50	**0.30**

$$\tau_c = 0.22 + \left[\frac{(0.30-0.22)}{(0.5-0.25)} \text{ X } (0.3427\text{-}0.25) \right] = 0.2496 \text{ N/mm}^2$$

k=1 (IS 456:2000. P. No:84 C. No: B-5.2.1.1) as Df=300 mm

$$\tau_c k = 0.2496x1 = 0.2496 \text{ N/mm}^2$$

Compare τ_v & $\tau_c k$

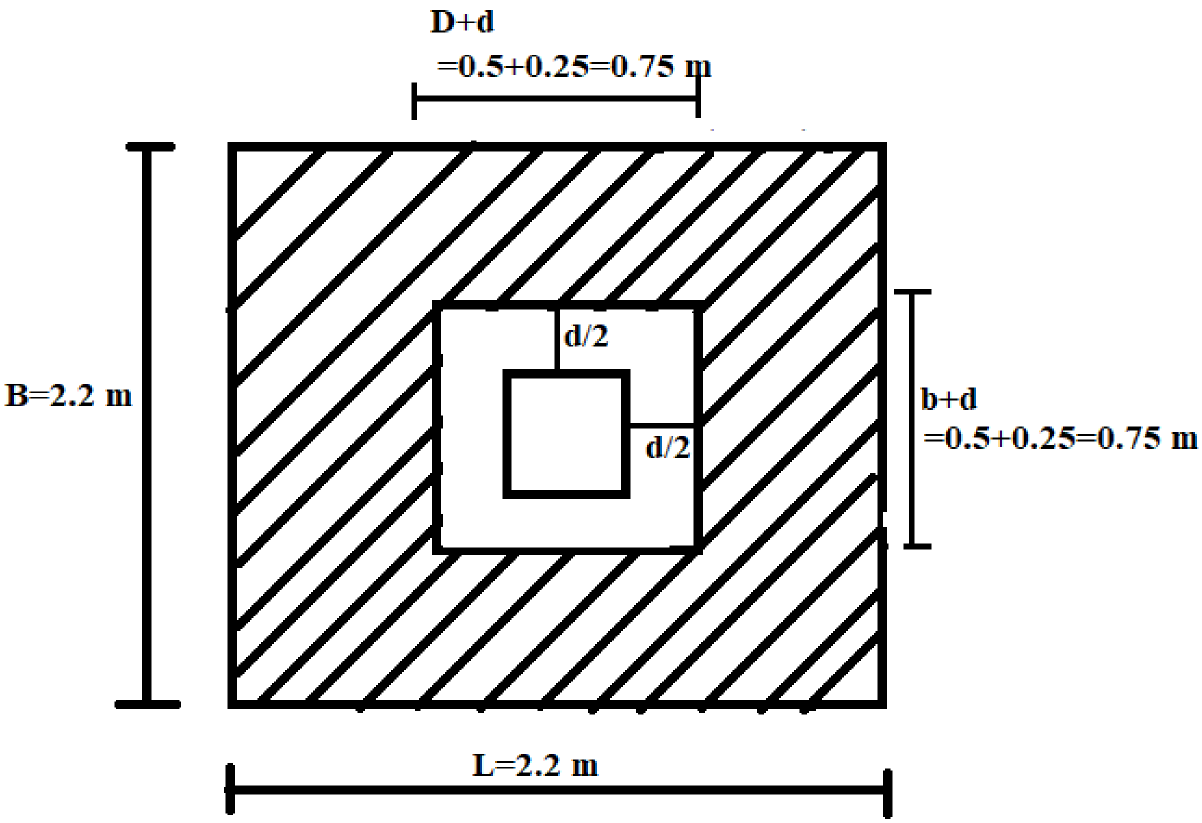
$\tau_v < \tau_c k$

$0.2479 < 0.2496$ (ok)

Safe in one way shear

STEP 8:: Check for two way shear

The critical section for two way shear at a distance ‘d/2’ from the face of column



c) Nominal shear stress (τ_v) **(IS 456:2000, P No: 57 , C No:31.6.2.1)**

$$\tau_v = \frac{V}{B_0 d} = \frac{q_0 \text{ x Shaded Area}}{Perimeter \text{ x } d} = \frac{103.305x\left[(2.2)^2 - (0.75)^2\right]}{4x0.75x0.25} = 589.182 KN/m^2$$

$$\tau_v = \frac{589.182x10^3}{10^6} = 0.5891\ N/mm^2$$

d) Permissible shear stress of concrete ($\tau_c k_s$) **(IS 456:2000. P. No:58 C. No:** 31.6.3)

$$k_s = (0.5 + B_c) = 0.5 + \left[\frac{500}{500}\right] = 1.5 > 1$$

$$Hence\ k_s = 1$$

and

$$\tau_c = 0.16\sqrt{F_{ck}} = 0.16\sqrt{20} = 0.72 \text{ N/mm}^2$$

Compare τ_v & $\tau_c k_s$

$\tau_v < \tau_c k_s$
$0.5891 < 0.72$ (ok)

Safe in two way shear

STEP 9:: Check for development length

IS 456:2000, P. No. 81, Table No:21 & IS 456:2000, P. No. 80, C.No No:B-2.1.2

$$L_d = \frac{\phi 6_s}{4x\tau_{bd}} = \frac{10x230}{4x0.8x1.6} = 449.22mm \cong 450\ mm$$

Length of bar available= $\frac{1}{2}(B-b)-\text{cov}er$

Length of bar available= $\frac{1}{2}(2200-500)-50$

=800mm >L_d (Ok)

2) Design an isolated footing of uniform thickness of a R.C. column carrying a load of 1650 KN and having size 450 mm X 450 mm. The safe bearing capacity of soil may be takes as 250 KN/m^2. Use M_{25} and Fe 415. Use WSM

Solution:

Given Data

Width of column = b = 450 mm

Depth of column= D= 450 mm

Load =P =1650 KN

Safe bearing capacity of soil =250 KN/m^2

M_{25}, 6cbc = 8.5 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6st = 230 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find design constant

i) Modular Ratio (m)

$$m = \frac{280}{3\ x\ 6_{cbc}} = \frac{280}{3\ x\ 8.5} = 10.98$$

ii) Neutral Axis depth factor (k)

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{10.98x\ 8.5}{(10.98x\ 8.5) + 230} = 0.2886$$

iii) Lever arm factor (j)

$$j = 1 - \frac{k}{3} = 1 - \frac{0.2886}{3} = 0.9037$$

iv) Moment resisting factor (Q)

$$Q=\frac{1}{2}6_{cbc}jk$$

$$Q=\frac{1}{2}x8.5x0.9037x0.2886=1.1084$$

STEP 2: To find size of footing

Axial Load = P=1650 KN

Assuming self weight of footing =10 % of Axial load

Assuming self weight of footing = $\frac{10}{100}$x1650=165 KN

Total load= 1650 +165 = 1815 KN

Area of footing =A= $\frac{\text{Total load}}{SBC}=\frac{1815}{250}=7.26m^2$

The footing is square footing

Each side of footing= L= B

L =B= $\sqrt{A}=\sqrt{7.26}=2.69m\cong 2.75m$

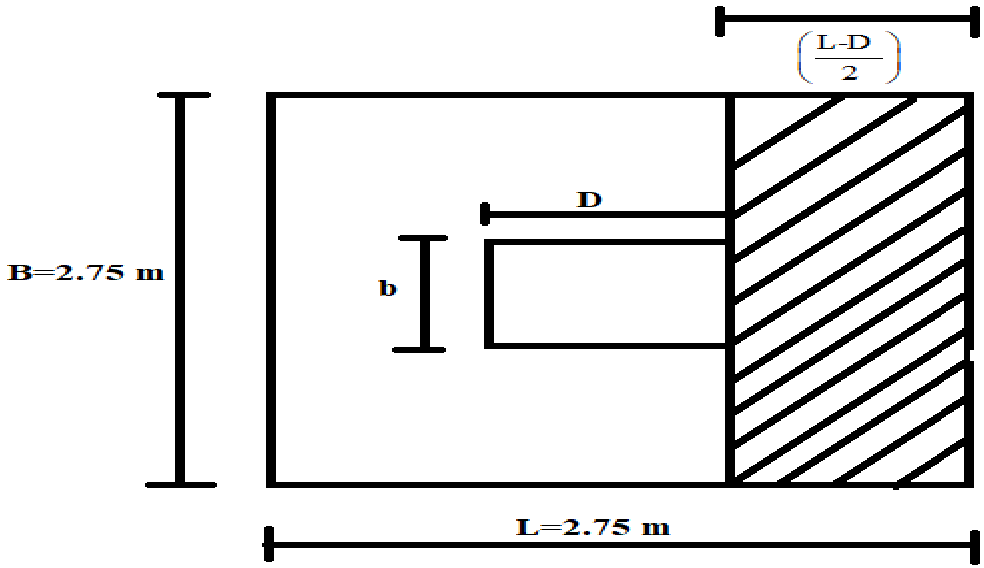
Providing size of footing = L x B= 2.75 m x 2.75m

STEP 3: To find net upward pressure

Net upward pressure= $q_0=\frac{Axial\ \text{Load}}{A_{provided}}=\frac{1650}{2.75x2.75}=218.18\ \text{KN/m}^2$

STEP 4: To find bending Moment

The maximum bending moment act at the face of column



Distance= $\frac{L-D}{2}=\frac{2.75-0.45}{2}=1.15m$

$$M=\frac{(q_0 B)x1.15^2}{2}=\frac{218.18x2.75x1.15^2}{2}=396.74 \text{ KNm}$$

STEP 5: Check for depth

Equating maximum BM to resisting moment

$M=QBd^2$

$396.74x10^6=1.1084x2750xd^2$

$d=360.77mm \cong 600mm$ (Increse depth upto half of actual for correct check in shear)

Assuming effective cover =d'=50 mm

Depth of footing =D_f =600+50=650 mm

STEP 6: To find area of steel

M=T(jxd)

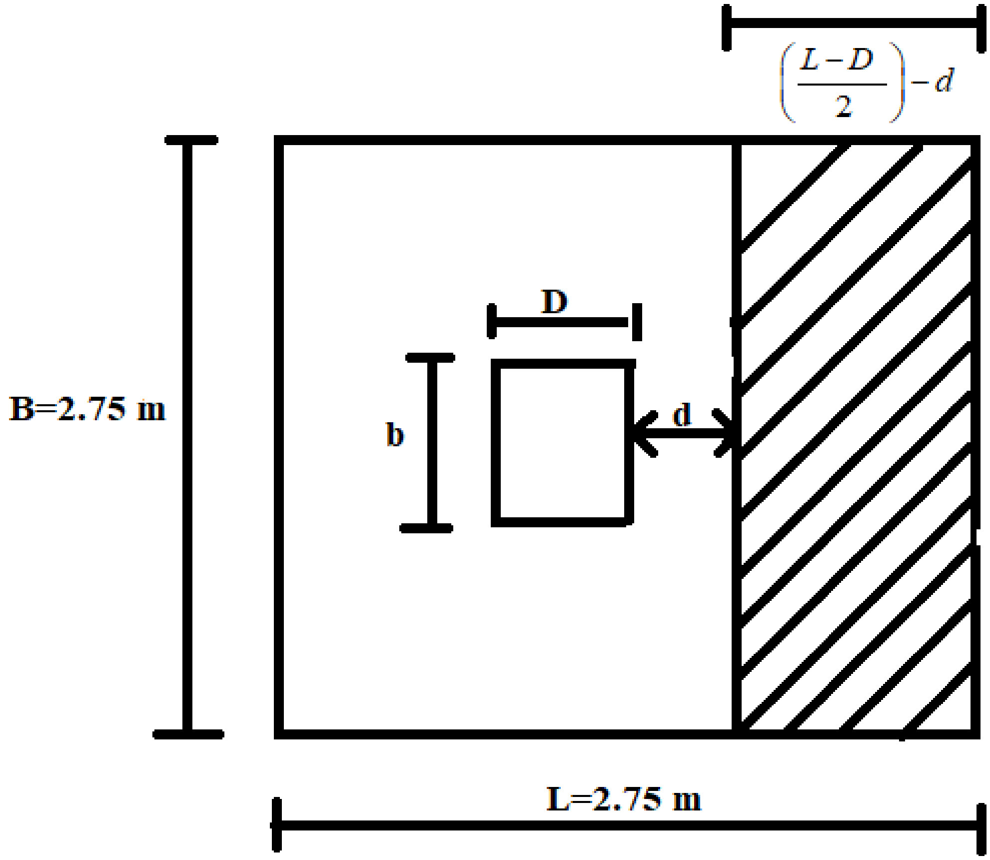
$$A_{st}=\frac{M}{6_{st}\,jd}=\frac{396.74 \text{ x}10^6}{230 \text{ x}0.9037 \text{ x}600}=3181.28mm^2$$

Assuming ϕ=16 mm

$$Number \text{ of bar}=\frac{Ast}{(\pi/4) \text{ x } \phi^2}=\frac{3181.28}{(\pi/4) \text{ x } 16^2} = 15.88 \cong 16$$

STEP 7: Check for one way shear

The critical section for one way shear at a distance ‘d’ from the face of column



Distance= $\left(\frac{L-D}{2}\right)-d=\frac{2.75-0.45}{2}-0.6=0.4m$

a) Nominal shear stress (τ_v)

$$\tau_v=\frac{V}{Bd}=\frac{q_0\ x\ \text{Shaded Area}}{Bd}=\frac{218.18x(0.4x2.75)}{2.75x0.6}=145.45KN/m^2$$

$$\tau_v=\frac{145.45x10^3}{10^6}=0.1454N/mm^2$$

b) Design shear strength of concrete ($\tau_c k$)

$P_t=100$ x $\frac{A_{st}}{Bd}$ (Page Number 84, Table Number 23, IS 456:2000)

$$P_t=100\text{ x }\frac{16x\frac{\pi}{4}x16^2}{2750\text{ x }600}=100x\frac{3216.99}{2750\text{ x }600}=0.1949$$

To find design shear strength of concrete (τ_c)

(Page Number 84, Table Number 23, IS 456:2000)

Pt %	τ_c
0.15	**0.19**
0.1949	**?**
0.25	**0.23**

$$\tau_c = 0.19 + \left[\frac{(0.23 - 0.19)}{(0.25 - 0.15)} \text{ X } (0.1949\text{-}0.15) \right] = 0.2079 \text{ N/mm}^2$$

k=1 (IS 456:2000. P. No:84 C. No: B-5.2.1.1)

$$\tau_c k = 0.2079 x 1 = 0.2079 \text{ N/mm}^2$$

Compare τ_v & $\tau_c k$

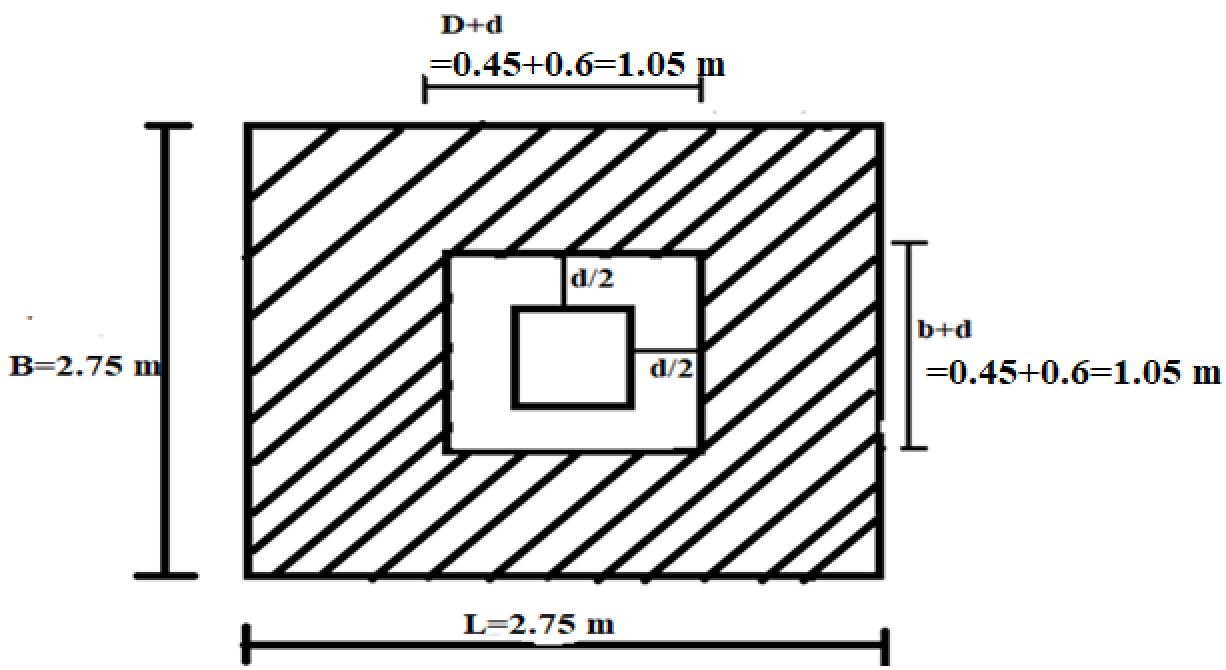
$\tau_v < \tau_c k$

$0.1454 < 0.2079$ (ok)

Safe in one way shear

STEP 8: Check for two way shear

The critical section for two way shear at a distance 'd/2' from the face of column



a) Nominal shear stress (τ_v) **(IS 456:2000, P No: 57 , C No:31.6.2.1)**

$$\tau_v = \frac{V}{B_0 d} = \frac{q_0 \ x \text{ Shaded Area}}{Perimeter \ \text{x} \ d} = \frac{218.18x\left[(2.75)^2 - (1.05)^2\right]}{4x1.05x0.6} = 559.30KN/m^2$$

$$\tau_v = \frac{559.30x10^3}{10^6} = 0.5593 \ N/mm^2$$

b) Permissible shear stress of concrete ($\tau_c k_s$) **(IS 456:2000. P. No:58 C. No:** 31.6.3)

$$k_s = (0.5 + B_c) = 0.5 + \left[\frac{450}{450}\right] = 1.5 > 1$$

$$Hence \ k_s = 1$$

and

$$\tau_c = 0.16\sqrt{F_{ck}} = 0.16\sqrt{25} = 0.8 \text{ N/mm}^2$$

Compare τ_v & $\tau_c k_s$

$$\tau_v < \tau_c k_s$$
$$0.5593 < 0.8 \ \text{(ok)}$$

Safe in two way shear

STEP 9: Check for development length

IS 456:2000, P. No. 81, Table No:21 & IS 456:2000, P. No. 80, C.No No:B-2.1.2

$$\text{L}_\text{d} = \frac{\phi 6_\text{s}}{4\text{x}\tau_\text{bd}} = \frac{16\text{x}230}{4\text{x}0.9\text{x}1.6} = 638.88\text{mm} \cong 640 \text{ mm}$$

$$\text{Length of bar available} = \frac{1}{2}(B-b) - cover$$

$$\text{Length of bar available} = \frac{1}{2}(2750-450) - 50$$

$$= 1100\text{mm} > \text{L}_\text{d} \quad \text{(Ok)}$$

3) Design rectangular isolated footing of uniform thickness of a R.C. column carrying a load of 750 KN and having size 400 mm X 600 mm. The safe bearing capacity of soil may be takes as 150KN/m^2. Use M_{20} and Fe 415. Use WSM

Solution:

Given Data

Width of column = b = 400 mm

Depth of column= D= 600 mm

Load =P =750 KN

Safe bearing capacity of soil =150 KN/m^2

M_{20}, 6cbc = 7 N/mm^2 (IS 456:2000, Table No: 21, P No:81)

Fe 415 , 6st = 230 N/mm^2 (IS 456:2000, Table No: 22, P No:82)

STEP 1: To find design constant

i) Modular Ratio (m)

$$m = \frac{280}{3 \text{ x } 6_{cbc}} = \frac{280}{3 \text{ x } 7} = 13.33$$

ii) Neutral Axis depth factor (k)

$$k = \frac{m\ 6_{cbc}}{m\ 6_{cbc} + 6_{st}} = \frac{13.33x\ 7}{(13.33x\ 7) + 230} = 0.2886$$

iii) Lever arm factor (j)

$$j = 1 - \frac{k}{3} = 1 - \frac{0.2886}{3} = 0.9037$$

iv) Moment resisting factor (Q)

$$Q = \frac{1}{2} 6_{cbc} jk$$

$$Q = \frac{1}{2} x7x0.9037x0.2886 = 0.9128$$

STEP 2: To find size of footing

Axial Load = P=750 KN

Assuming self weight of footing =10 % of Axial load

Assuming self weight of footing = $\frac{10}{100}$x750=75 KN

Total load= 750 +75 = 825 KN

Area of footing =A= $\frac{\text{Total load}}{SBC} = \frac{825}{150} = 5.5m^2$

For rectangular footing

$$\frac{L_f}{B_f} = \frac{L}{B}$$

$$\frac{L_f}{B_f} = \frac{600}{400}$$

$$L_f = \frac{600}{400} B_f$$

$$L_f = 1.5 B_f$$

Area of footing= L_f x B_f=A

1.5 B_f x B_f= A

1.5 B^2_f=A=5.5

B_f =1.91 m $\cong$ 2 m

$L_f = 1.5B_f = 1.5x2 = 3m$

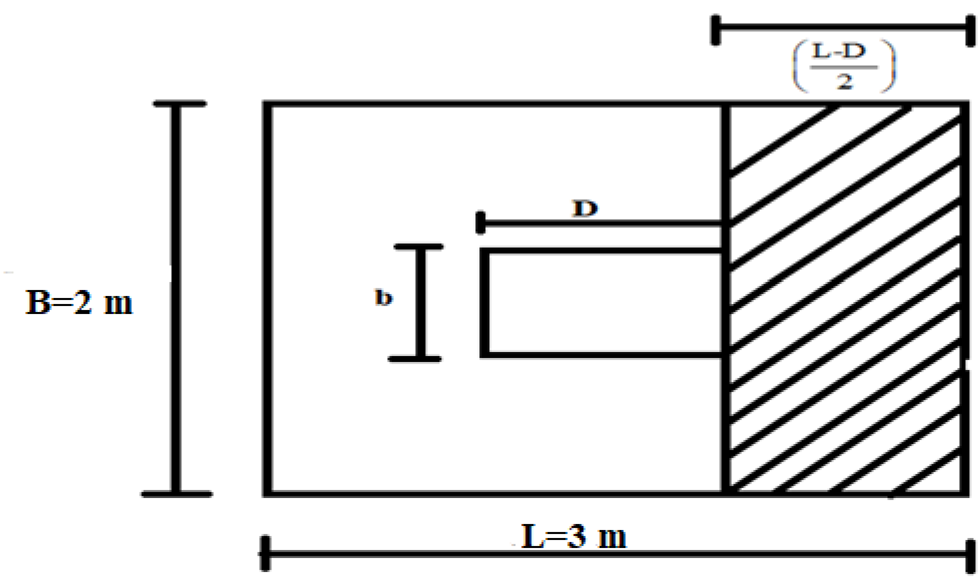
Providing size of footing = L_f x B_f= 3 m x 2 m

STEP 3: To find net upward pressure

Net upward pressure= $q_0 = \dfrac{Axial \text{ Load}}{A_{provided}} = \dfrac{750}{3x2} = 125 \text{ KN/m}^2$

STEP 4: To find bending Moment

The maximum bending moment act at the face of column



Distance= $\dfrac{L-D}{2} = \dfrac{3-0.6}{2} = 1.2m$

$$M = \frac{(q_0 B)x1.2^2}{2} = \frac{125x2x1.2^2}{2} = 180 \text{ KNm}$$

STEP 5: Check for depth

Equating maximum BM to resisting moment

$M = QBd^2$

$180x10^6 = 0.9128x2000xd^2$

$d = 314mm \cong 500mm$ (Increse depth upto half of actual for correct check in shear)

Assuming effective cover =d'=50 mm

Depth of footing =D_f =500+50=550 mm

Distance= $\frac{B-b}{2} = \frac{2-0.4}{2} = 0.8m$

$M = \frac{(q_0 L)x0.8^2}{2} = \frac{125x3x0.8^2}{2} = 120$ KNm

180 KNm $<$ 120 KNm (The effective depth found above has to be checked for shear)

STEP 6: To find area of steel

Along X-X

M=T(jxd)

$A_{st} = \frac{M}{6_{st}\,jd} = \frac{180 \text{ x}10^6}{230 \text{ x}0.9037 \text{ x}500} = 1732.00mm^2$

Assuming ϕ=10 mm

$Number$ of bar=$\frac{Ast}{(\pi / 4) \text{ x } \phi^2} = \frac{1732}{(\pi / 4) \text{ x } 10^2}$ =22.05 $\cong$ 23

Along Y-Y

M=T(j-d)

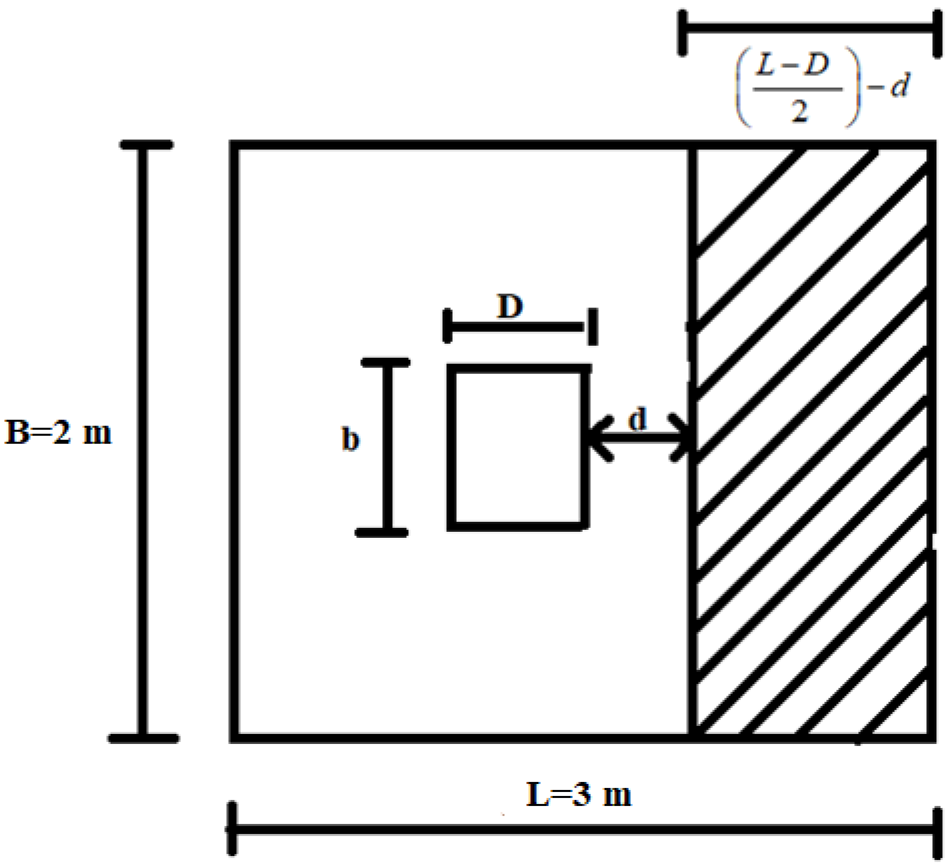
$A_{st} = \frac{M}{6_{st}\,jd} = \frac{120 \text{ x}10^6}{230 \text{ x}0.9037 \text{ x}500} = 1154.67mm^2$

Assuming ϕ=10 mm

$Number$ of bar=$\frac{Ast}{(\pi / 4) \text{ x } \phi^2} = \frac{1154.67}{(\pi / 4) \text{ x } 10^2}$ =14.70 $\cong$ 15

STEP 7: Check for one way shear

The critical section for one way shear at a distance ‘d’ from the face of column



Distance= $\left(\dfrac{L-D}{2}\right)-d=\dfrac{3-0.6}{2}-0.5=0.7m$

a) Nominal shear stress (τ_v)

$$\tau_v=\frac{V}{Bd}=\frac{q_0\ x\ \text{Shaded Area}}{Bd}=\frac{125x(0.7x2.0)}{2x0.5}=175KN/m^2$$

$$\tau_v=\frac{175x10^3}{10^6}=0.175N/mm^2$$

b) Design shear strength of concrete ($\tau_c k$)

$P_t=100$ x $\dfrac{A_{st}}{Bd}$ (Page Number 84, Table Number 23, IS 456:2000)

$$P_t=100\text{ x }\frac{23x\frac{\pi}{4}x10^2}{2000\text{ x }500}=100x\frac{1806.41}{2000\text{ x }500}=0.1806$$

To find design shear strength of concrete (τ_c)

(Page Number 84, Table Number 23, IS 456:2000)

Pt %	τ_c
0.15	**0.18**
0.1806	**?**
0.25	**0.22**

$$\tau_c = 0.15 + \left[\frac{(0.22-0.15)}{(0.25-0.15)} \text{ X } (0.1806\text{-}0.15)\right] = 0.19224 \text{ N/mm}^2$$

k=1 (IS 456:2000. P. No:84 C. No: B-5.2.1.1)

$$\tau_c k = 0.19224x1 = 0.19224 \text{ N/mm}^2$$

Compare τ_v & $\tau_c k$

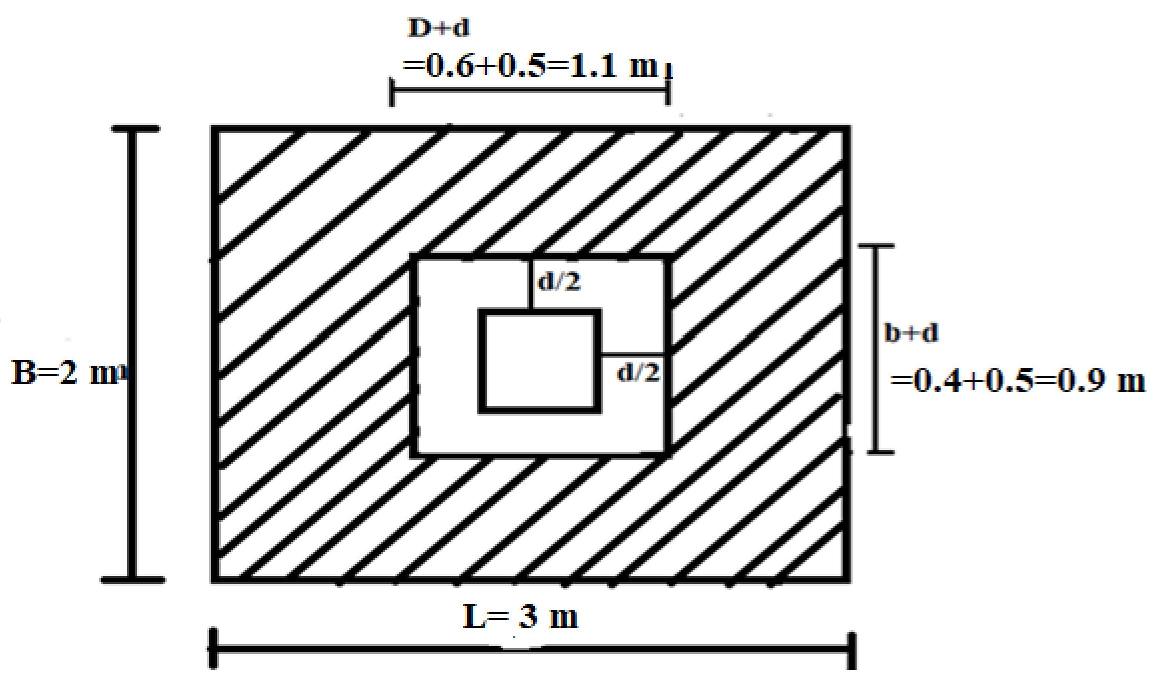
$\tau_v < \tau_c k$

$0.175 < 0.19224 \quad \text{(ok)}$

Safe in one way shear

STEP 8:: Check for two way shear

The critical section for two way shear at a distance ‘d/2’ from the face of column



a) Nominal shear stress (τ_v) **(IS 456:2000, P No: 57 , C No:31.6.2.1)**

$$\tau_v = \frac{V}{B_0 d} = \frac{q_0 \ x \text{ Shaded Area}}{Perimeter \text{ x } d} = \frac{125x\left[(3x2)-(1.1x0.9)\right]}{\left[\left(2x1.1\right)+(2x0.9)\right]x0.5} = 312.625KN/m^2$$

$$\tau_v = \frac{312.625x10^3}{10^6} = 0.3126 \ N/mm^2$$

b) Permissible shear stress of concrete ($\tau_c k_s$) **(IS 456:2000. P. No:58 C. No:** 31.6.3)

$$k_s = (0.5+B_c) = 0.5+\left[\frac{400}{600}\right] = 1.1666 > 1$$

$$Hence \ k_s = 1$$

$$and$$

$$\tau_c = 0.16\sqrt{F_{ck}} = 0.16\sqrt{20} = 0.72 \text{ N/mm}^2$$

Compare τ_v & $\tau_c k_s$

$\tau_v < \tau_c k_s$

$0.3126 < 0.72$ (ok)

Safe in two way shear

STEP 9: Check for development length

IS 456:2000, P. No. 81, Table No:21 & IS 456:2000, P. No. 80, C.No No:B-2.1.2

$$\text{L}_\text{d} = \frac{\phi 6_\text{s}}{4\text{x}\tau_{\text{bd}}} = \frac{10\text{x}230}{4\text{x}0.8\text{x}1.6} = 449.21\text{mm} \cong 450 \text{ mm}$$

Length of bar available= $\frac{1}{2}(B-b)-\text{cov}\,er$

Length of bar available= $\frac{1}{2}(2000-400)-50$

$=750\text{mm} > \text{L}_\text{d}$ (Ok)

Introduction

Limit State Approach

The disparity between the behaviour predicted by elastic analysis and that observed in practise has led to the use of a theory based on the conditions present in an actual structure at various stages of loading for reinforced concrete design.

To be more specific, the Limit State Method of R. C. Structure Design is based on limiting the state of the system.

The deflection and cracking in the functioning state (loads); (ii) stresses and strains in the failure or "collapse" condition (loads). Aside from the limiting states listed above, a few structures that perform unique duties should adhere to the limit states that apply to them.

Objectives:

You will be able to apply the basic ideas involved in the analysis and design of R. C. Structures using the Limit State Method after completing this unit. As a result, the following are the unit's goals:

1) Enumeration of various 'Limit States' considered while designing R. C. Structures,
2) Determination of Design Values of stresses in concrete as well as in steel,
3) Determination of Design Values of Loads at Collapse and at Serviceability Limit States,
4) Determination of Permissible Deflection of Service State (Loads),
5) Determination of Permissible width of Cracks in concrete at Service State (Loads), and Method of Analysis of Structures and their Components.

LIMIT STATE METHOD :- The limit state design originated form the ultimate design. The object of design based on limit state concept , the structure will not become unserviceable in its life times. The important limit state which must be examine for the design of structure are

a) Limit State of collapse

b) Limit State of serviceability

c) Limit State of durability

a) LIMIT STATE OF COLLAPSE :- IS 456 On page number 67 and clause number: 35.2

The limit state of collapse is reach when the structure as a whole or part of structure collapse. The collapse may be of one or more member.

There are following types of limit state

A) Flexure

B) Shear

C) Compression

D) Torsion

b) LIMIT STATE OF SERVICIABILITY :- IS 456 On page number 67 and clause number: 35.3

This limit relates the performance or behavior of structure and based on the causes affecting serviceability of structure and limit state of serviceability having following type

1) Limit State of Deflection

2) Limit State of Cracking

3) Limit State of Vibration

1) Limit State of Deflection :- IS 456 On page number 67 and clause number: 35.3.1

The excessive deflection causes numbers of problems expanding the appearance of structure. The feeling lack of safety . The actual deflection should be less than permissible deflection.

2) Limit State of Cracking :- IS 456 On page number 67 and clause number: 35.3.2

The excessive cracking is spoil the appearance of structure. It create leakage problem and corrosion of steel. The maximum permissible crack should taken as IS code.

3) Limit State of Vibration :- The vibration reduce the life of structure and should be given in IS code.

c) Limit State of durability :- It is related the durability of structure against the action and forces of nature like rainwater.

Example:- fire, chemical action etc

FACTOR OF SAFETY :- (ϒ)

The factor of safety which gives the margined at strength for safety can be define as the ratio of strength of member to the force acting on the member. It is denoted by **ϒ.**

$$\Upsilon = R/F$$

Where ,

R = Strength Of Member

F= Force acting on the member

PARTIAL SAFETY FACTOR:- (Υ_m) IS 456 On page number 68 and clause number: 36.4

It is the strength reduction factor and it is the ratio of characteristics strength to design strength. It is denoted by Υ_m.

$$\Upsilon_m = F_{CK}/F_d$$

Where ,

F_{CK} = Grade or characteristics strength

F_d = Partial safety factor for material

Υ_m depend upon type of material

For limit state of collapse

for concrete Υ_m = 1.5

for steel Υ_m = 1.15

PARTIAL SAFETY FACTOR FOR LOADS :- IS 456 On page number 68 and clause number: 36.4

It is the ratio design load to the characteristics load. It is denoted by Υ_m.

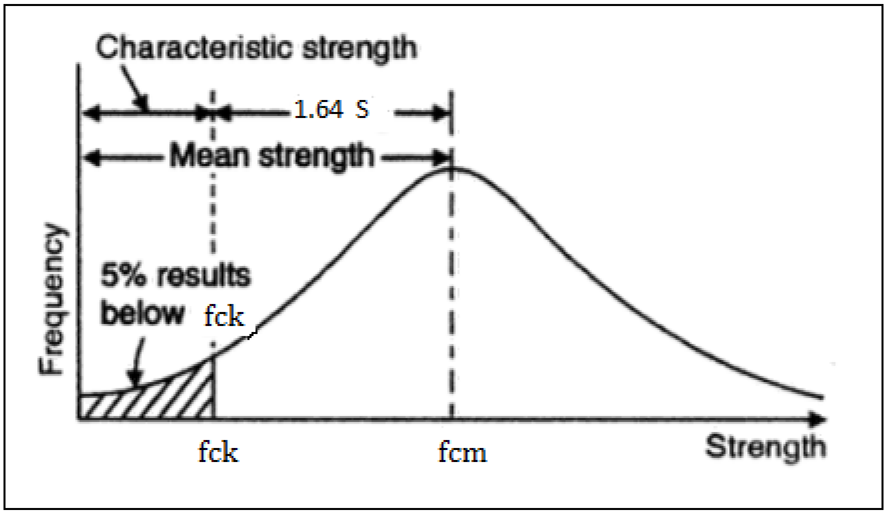
$$\Upsilon_m = F_d/F$$

Where ,

F = Characteristics load

F_d = Partial safety factor for material

CHARACTERISTICS STRENGTH :- Characteristics strength of material is that value of material whose strength not more than 5% of result are expected to fall below. It means the characteristics strength has 95% accuracy.



Where ,

F_{CK} = Characteristics strength

F_m = Mean strength

$$F_m = \frac{\sum F}{n}$$

Where ,

n= Number of observations

S= Standard deviation

$$S = \sqrt{\frac{\sum \Delta^2}{n-1}}$$

$\Delta = F - F_m$

$F_{CK} = F_m - 1.64\ S$

NUMERICALS

1. Find the Characteristics of concrete from test are given below

28.7, 30.4, 31.7, 29.3, 28.5, 29.2, 30.3, 32.5, 31.5, 34.3, 32.8, 33.8, 34.7, 32.9, 33.8, 32.7, 30.9, 32.6, 33.4, 32.2

Solution:- F_m = Mean strength

$$F_m = \frac{\sum F}{n}$$

F_m= 636.1/ 20 = 31.805

n=20

Sr No	F	Δ= F - F_m	Δ^2
1	28.7	28.7-31.805 = -3.106	9.641
2	30.4	30.4-31.805 = -1.405	1.970
3	31.7	31.7-31.805=-0.105	0.0110
4	29.3	29.3-31.805=-2.505	6.275
5	28.5	28.5-31.805=-3.305	10.923
6	29.5	29.5-31.805=-2.605	6.786
7	30.3	30.3-31.805=-1.505	2.265
8	32.5	32.5-31.805=0.695	0.483
9	31.5	31.5-31.805=-0.305	0.0930
10	34.3	34.3-31.805=2.495	6.220
11	32.8	32.8-31.805=-0.995	0.990
12	33.8	33.8-31.805=1.995	3.98
13	34.7	34.7-31.805=2.895	8.380
14	32.9	32.9-31.805=1.095	1.199
15	33.8	33.8-31.805=1.995	3.980
16	32.7	32.7-31.805=0.895	0.801
17	30.9	30.9-31.805=-0.905	0.819
18	32.6	32.6-31.805=0.795	0.632
19	33.4	33.4-31.805=1.595	2.542
20	32.2	32.2-31.805=0.395	0.156
			= 68.106

S= Standard deviation

$$S = \sqrt{\frac{\sum \Delta^2}{n-1}} = \sqrt{\frac{68.108}{20-1}}$$

S= 1.893

F_{CK} = F_m - 1.64 S

F_{CK} = 31.805-1.64 X 1.893 = 28.70 N/mm^2

2. Find the Characteristics of steel from test are given below

438, 502, 651, 369, 242, 578, 195, 450, 471, 495, 510, 575, 391, 427, 580, 622, 468, 543, 493, 400

Solution:- F_m = Mean strength

$$F_m = \frac{\sum F}{n}$$

$F_m = 9400/20 = 470$

n=20

Sr No	F	$\Delta = F - F_m$	Δ^2
1	438	438 - 470 = -32	1024
2	502	502 - 470 = 32	1024
3	651	651 -470 = 181	32761
4	369	369 -470 = -101	10201
5	242	242 -470 = -228	51984
6	578	578 - 470 =108	11664
7	195	195-470 =-275	75625
8	450	450 -470 =-20	400
9	471	471 -470 = 1	1
10	495	495 -470 = 25	625
11	510	510 -470 = 40	1600
12	575	575 - 470 = 105	11025
13	391	391 -470= -79	6241
14	427	427 -470 = -43	1849
15	580	580 -470 = 110	12100
16	622	622 -470 = 151	23104
17	468	468 -470 = -2	4
18	543	543 -470 = 73	5329
19	493	493 -470 =23	529
20	400	400 -470 =-70	4900
			= 251990

S= Standard deviation

$$S = \sqrt{\frac{\sum \Delta^2}{n-1}} = \sqrt{\frac{251990}{20-1}}$$

S= 115.16

$F_y = F_m - 1.64\ S$

$F_y = 470 - 1.64 \times 115.16 = 281.14\ N/mm^2$

STRUCTURAL ELEMENTS :- Following are structural elements used in any type of building.

1) Slab

2) Beam

3) Column

4) Footing

1) Slab :- A slab is two dimensitional structural elements having very small thickness compare to its length and width . It used to provided the covering the building. The slab carrying distributed load by flexure . A slab may be supported by beam and wall.

There are two types of slab

a) One Way Slab

b) Two Way Slab

Classification of Slab

i) According to shape

ii) According to spanning direction

iii) According to support

iv) According to uses

v) According to practical design

2) BEAM :- Beam are member that are subjected to bending. Bending causes compressive as well as tensile strength in the same cross section depending upon the position of practice and type of end support.

In simply supported beam , top half of cross section is under compression and the bottom half under tension.

3) COLUMN :- Column are vertical compression member used to transfer loads of the structure such as buildings, factory , cinema balcony, auditorium halls , flow of frame building etc to the foundation below.

TYPES OF COLUMN:-

i) Column with longitudinal steel and with lateral ties or spirals

ii) Composite columns with rolled steel specimen induced in concrete

iii) Concrete filled steel tabular column.

column may of any shape like rectangular, circular, hexagonal etc

4) FOOTING :- Footing is that part of building whose function is to distribute the load of the structure to the soil supporting the structure. The foundation distribute the load over a larger area at o uniform rate so that the pressure on the soil does not exceed . Its allowable bearing capacity, foundation increases the stability of structure.

TYPES OF FOOTING:-

i) Spread Footing

ii) Combined Footing

iii) Eccentrically Loaded Footing

iv) Raft or Mat Foundation

v) Pile Foundation

LOADS ON STRUCTURE :- IS 456 On page number 32,33 and clause number: 19

Following load which are acting in any type of construction

1) Dead Load

2) Live Load

3) Wind Load

4) Impact Load

5) Earthquake Load

1) Dead Load :- Dead load are permanent and stationary load which are transferred to the structure thought there their life span. Dead load is primaralily due to self weight of structural member. Permanent partition wall fixed permanent equipment and weight of different material. The dead load of material are given in IS 875-PART -I.

2) Live Load :- Live load are either moveable or moving load without any impact. These are assumed to be produced by the intended use or occupancy of the building including weight of movable portion. The live load of material are given in IS 875-PART -II.

3) Wind Load :- Wind load basically horizontal load causes by movement of air. Wind load is required to be consider in the design specially when the height of building exceeds the two times of dimensions transferred to expose surface. The wind load of material are given in IS 875-PART -III.

4) Impact Load :- Imposed load caused by vibrator or impact or acceleration of person walking, produce live load but soldiers marching or frame supporting lifts produced impact load. Thus impact load is equal to imposed load incremental by some percentage depending on the intensity of impact.

5) Earthquake Load :- Earthquake load are horizontal load caused by earthquake and shall be calculated in accordance with IS 1983.

STRUCTURAL PROPERTIES OF CONCRETE:- IS 456 On page number 15,16,17,18,28,29

Following are structural properties of concrete

1) Grade Of Concrete

2) Compressive Strength

3) Tensile Strength

4) Creep

5) Shrinkage

6) Short Term Modulus Of Elasticity

7) Long Term Modulus Of Elasticity

8) Modular Ratio

9) Poissions Ratio

10) Durability

11) Unit Weight Of Concrete

12) Stress-Strain Curve

1) Grade Of Concrete :- Concrete is known by its grade which is design as M_{20}, M_{25}, M_{30} etc in which M stand concrete mix and number 20,25,30 etc denotes specified compressive strength of 150 mm size cube expressed in N/mm^2 .

2) Compressive Strength :- Like load, the strength of concrete is also quantity which varies considerably for some concrete mix , therefore a single representative value known as characteristics strength, is arrived at using statistical probabilistic principles.

3) Tensile Strength :- The estimate of flexural tensile strength or the modulus of rupture or the cracking strength of concrete from cube compressive strength is obtained from the relation

$F_{cr} = 0.7 \times \sqrt{F_{ck}}$ in N/mm^2

4) Creep :- Plastic deformation under sustained load is called creep.

5) Shrinkage:- The property of diminishing in volume during the process of drying and hardening is called shrinkage.

6) Short Term Modulus Of Elasticity :- According to code ,short term modulus of elasticity of concrete is given by

$E_c = 5000 \text{ X } \sqrt{F_{ck}}$ in N/mm^2

7) Long Term Modulus Of Elasticity:- The long term modulus of elasticity of concrete takes into account the effect of creep and shrinkage.

8) Modular Ratio:- It is the ratio of modulus of elasticity of steel to modulus of elasticity of concrete.

Modular ration $m = E_s / E_c$

9) Poissions Ratio:- Poissons ratio varies between 0.1 for high strength concrete and 0.2 for weak mixes. It is normally taken 0.15 for strength design and 0.2 for serviceability criteria.

10) Durability:- Durability of concrete is its ability to resist its disintegration and decay.

11) Unit Weight Of Concrete: - The unit weight of reinforced concrete depends on percentage of reinforcement, type of aggregates, amount of voids and varies from 23 KN/m^3 to 26 KN/m^3 .The unit weight of plain and reinforced concrete, as specified by IS:456 are 24 KN/m^3 and 25 KN/m^3 respectively.

12) Stress-Strain Curve:- The typical idealized stress- strained curved adopted by IS code is shown in Figure.

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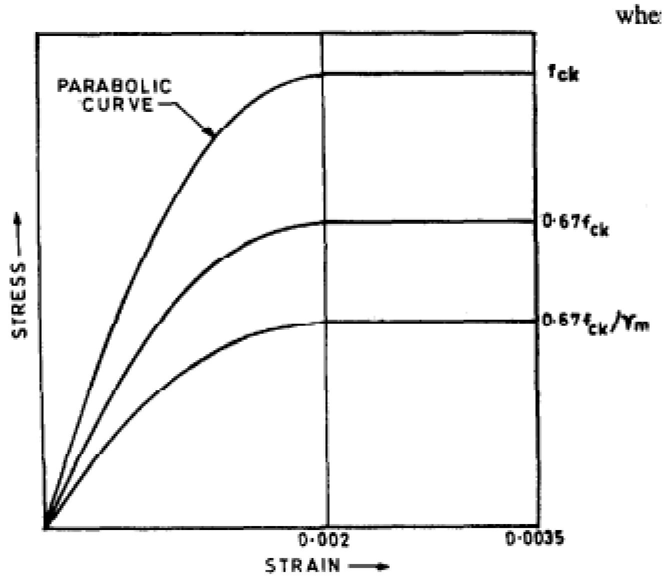


FIG. 21 STRESS-STRAIN CURVE FOR CONCRETE

REDISRTUBUTION OF MOMENT AND IS CODE PROVISIONS:- IS 456 On page number 68 and clause number: 37.1.1

The feature of limit state design for collapse is the distribution of moment in statically indeterminate beam. The redistribution of moment causes the rotation of plastic hinges in proportion to amount of redistribution and therefore it demand that a section should posses that much rotation capacity but R.C. member the rotation capacity is required to be limited serviceability consideration.

IS CODE PROVISIONS FOR REDISRTUBUTION OF MOMENT:- IS 456 On page number 68 and clause number: 37.1.1

a) Equilibirum between the interal forces and the external loads is maintained.

b) The ultimate moment of resistance provided at any section of a member is not less than 70 percent of the moment at that section obtained from an elastic maximum moment diagram covering all appropriate combinations of loads.

c) The elastic moment at any section in a member due to a particular combination of loads shall not be reduced by more than 30 percent of the numerically largest moment given anywhere by the elastic maximum moments diagram for the particular member, covering all appropriate combination of loads.

d) At sections where the moment capacity after redistribution is less than that from the elastic maximum moment diagram, the following relationship shall be satisfied:

$(Xu/d) + (\delta M/100) \leq 0.6$

where

Xu = depth of neutral axis,
d= effective depth, and
δM = percentage reduction in moment.

e) In structures in which the structural frame provides the lateral stability, the reductions in moment allowed shall be restricted to 10 percent for structures over 4 storeys in height.

SIGNIFICANCE OF DELFECTION:- Deflection form one of the important criteria of accepting the performance a structure under service load. The time dependant factor like creep and shrinkage further increases in deflection under sustained load. This may result in excessive deflection which may not be detrimental to the safety of structure but may considerbly the serviceability or utility of structure and also the load distribution among the interconnected member.

TYPES OF DELFECTION:- IS 456 On page number 88

a) Short term deflection

b) Long term deflection

c) Total deflection

a) Short term deflection :- It is due to initial elastic deformation of the member due to load and permanent imposed load under service condition.

b) Long term deflection :- It is caused due to creep and shrinkage under sustained load and additional short term deflection due to temporary live load.

c) Total deflection :- Total deflection includes short term deflecting and long term deflection this quantity required overall control.

IS CODE PROVISIONS FOR DEFLECTION:- IS 456 On page number 37 and clause number 23.2

a) The final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from the as-cast level of the , supports of floors, roofs and all other horizontal members, should not normally exceed span/250.

b) The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes should not normally exceed span/350 or 20 mm whichever is less.

CRACKING:- Cracking is an important phenomenon special R.C. members. Study of behavior of R.C members can not be complete without study of cracking. Micro cracks occur in R.C member even before the applications of loads due to environmental effect of temperature and dryness by a way of thermal strength and shrinkage.

CAUSES OF CRACKING:-

1. Creep
2. Change in moisture
3. Elastic deformation
4. Foundation Settlement
5. Shrinkage

6. Thermal variation
7. Vegetation growth
8. Corrosion of reinforcement
9. Volumetric change
10. Stress concentration due to curtailment and splicing of bar
11. Direct stress and flexural stress due to bending and shear
12. Different moment of support

MECHANICS OF CRACKING:- Visible cracking is generated initiated internal micro cracks due to volumetric change or flexural micro cracks. The micro cracks are surface crack which are not visible expect by close examination one micro cracks have form, a sight increase in the external forces causes these cracks to open up suddenly to measurable width.

EFFECT OF CRACKIMG:-

1. It mark the appearance of exposed surface.
2. It create lot of maintenance problems.
3. It deduce stiffness of the member, hence increases deflection.
4. It create feeling of lack of safety.
5. It reduce the imperviousness creating leakage problem in roof, tank, walls etc.
6. It lead to corrosion of steel and hence reduction in strength and durability of structure.

CLASSIFICATION AND TYPE OF CRACKS:-

Cracks in R.C are classified into two main categories

1. Crack due to external forces and resulting structural action and deformation :- flexural cracking due to bending included cracks due to shear and torsion, normal cracks due to direct tension.

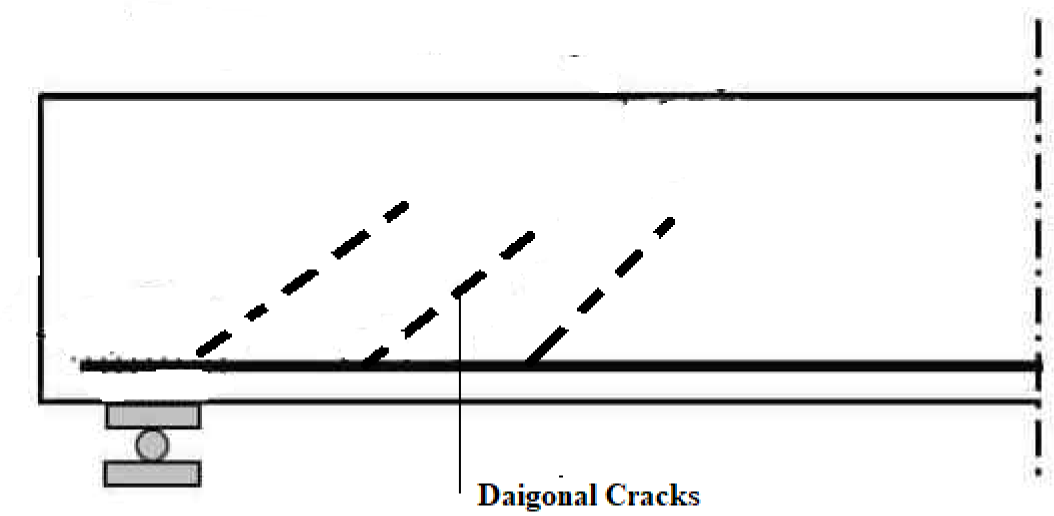
2. Cracks which are independent of load and which are caused due to restrained drying shrinkage of temperature expansion are normally exposed surfaces.

BAR DETAILING RULES:- The rule for bar detailing are form to satisfy the serviceability requirement for cracking to meet the requirement of durability of normal structure.

They includes rule regarding

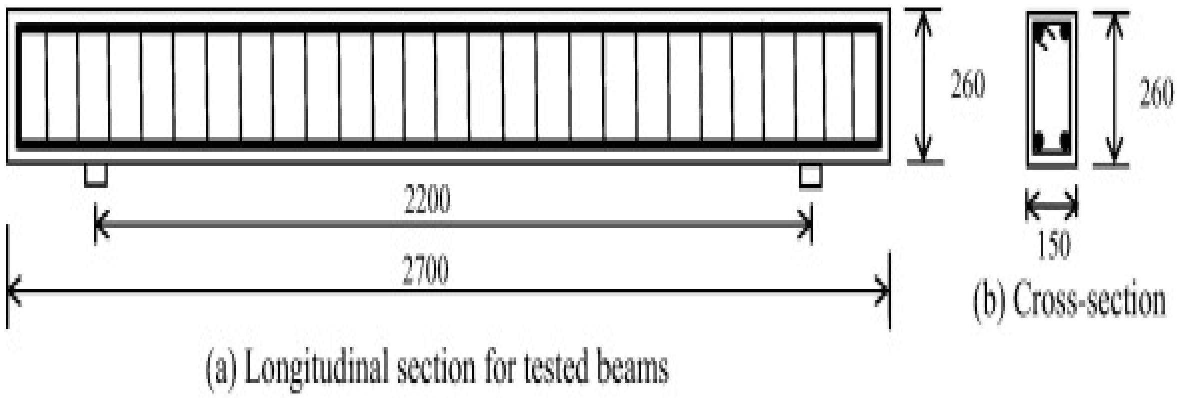
1. Cover to reinforcement
2. Reinforcement requirement
3. Spacing of reinforcement

Limit State For shear



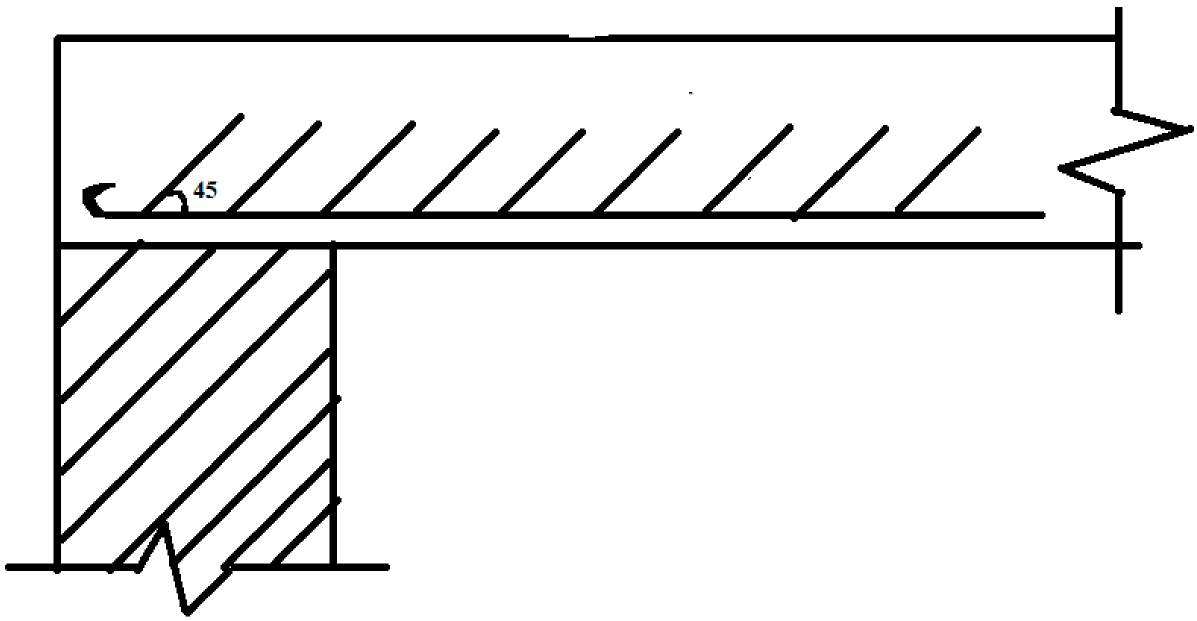
To avoid the tension cracks developed along bottom of beam. We provide longitudinal reinforcement or main beam reinforcement along the length of beam. The main reinforcement will act against tension crack along the bottom edge of the beam.

A longitudinal cracks are developed at 45^0 with horizontal along the length of beam. The tension created by diagonal cracks is known as diagonal tension. To avoid this tension, we provide the reinforcement is known as shear reinforcement.

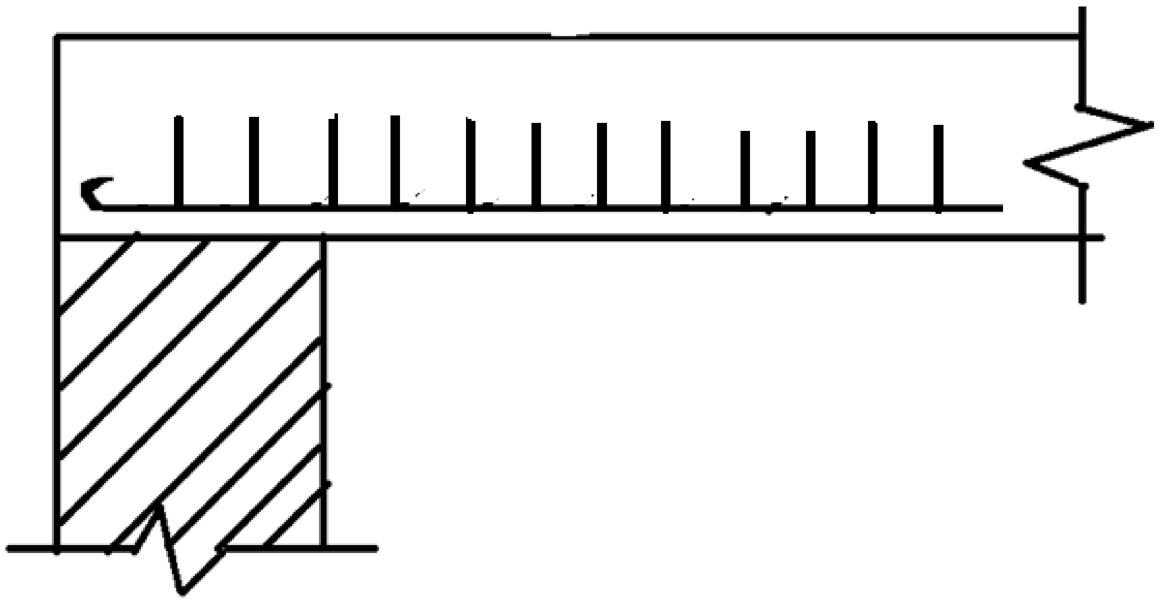


Shear failure of beam without shear reinforcement

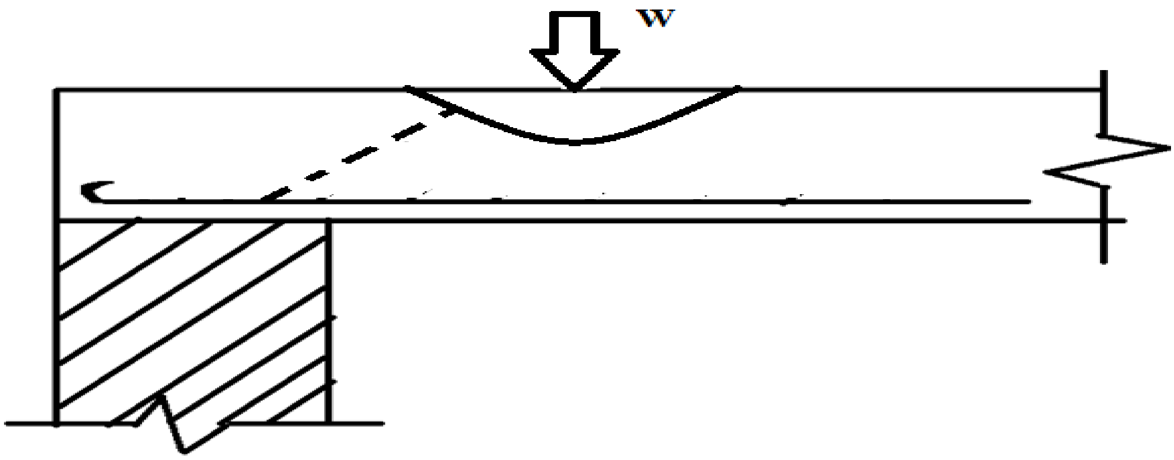
1. **Diagonal tension failure**
   This type of failure occurs when magnitude of shear force is large as compared to bending moment i.e near the support.



2. **Flexural Shear failure**
   This type of failure will occur when bending moment is larger than shear force.



3. **Diagonal compression failure**
   This type of failure takes place under the load by crushing of concrete as diagonal cracks are developed.



Shear reinforcement as per IS 456:2000

a) **Nominal Shear Stress (Page Number: 72, IS 456:2000)**

$$\tau_v = \frac{V_u}{bd}$$

τ_v = Nominal Shear Reinforcement

V_u = Shear force due to design load

b= Breadth of the section for T section

b=b_w

d= Effective depth

b) **Shear taken by concrete**

The design strength of concrete varies with the percentage of steel, different grade of concrete and denoted by τ_c. It is shear stress of concrete τ_c

Percentage of steel Pt

$P_t = 100$ x $\frac{A_{st}}{bd}$ (Page Number 73, Table Number 19, IS 456:2000)

$P_t = Percentage$ of steel

Pt %	τ_c
0.75	**0.56**
0.7527	**?**
1.00	**0.62**

$$\tau_c = 0.56 + \left[\frac{(0.62-0.56)}{(1-0.75)} \text{ X } (0.7527\text{-}0.75)\right] = 0.5606 \text{ N/mm}^2$$

c) The maximum shear stress

The nominal shear strength in concrete should not be greater than maximum shear stress from Table Number 20, page number 73. IS 456:2000

Table 20 Maximum Shear Stress, $\tau_{c\,max}$, N/mm²

(*Clauses* 40.2.3, 40.2.3.1, 40.5.1 *and* 41.3.1)

Concrete Grade	M 15	M 20	M 25	M 30	M 35	M 40 and above
$\tau_{c\,max}$, N/mm²	2.5	2.8	3.1	3.5	3.7	4.0

$\tau_{c\ \max} = 2.8\ \text{N/mm}^2$

$\tau_c = 0.496\ \ \text{N/mm}^2$

$\tau_{c\ \max} > \tau_c$

d) Minimum Shear reinforcement in the form of stirrups shall be provided such that **Page number 48 .C. No: 26.5.1.6. IS 456:2000**

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87\, fy}$$

Where

$A_{sv} =$ Total c/s area of stirrups legs effective in shear

$s_v =$ Stirrups spacing along the length of the member

b= Breadth of the beam

Fy=Charactaristics strength of stirrups should be $\leq 415\ \text{N/mm}^2$

e) Shear reinforcement is provided in 3 ways (**Page number 73 , IS 456:2000**

1) Vertical Stirrups
2) Bent up bars along with stirrups.
3) Inclined Stirrups

For vertical stirrups:

$$V_{us} = \frac{0.87\ \text{x Fy x A}_{\text{sv}}\text{x d}}{S_v}$$

Bent up bars along with stirrups.

$$V_{us} = \frac{0.87\ \text{x Fy x A}_{\text{sv}}\text{x d}}{S_v}(\sin\alpha+\cos\alpha)$$

Inclined Stirrups

$$V_{us} = 0.87\ \text{x Fy x A}_{\text{sv}}\ \text{x}\sin\alpha$$

Type I: Design of shear reinforcement

Given Data

STEP 1: **To find nominal shear reinforcement. (τ_v)**

(Page Number: 72, IS 456:2000)

$\tau_v = \frac{V_u}{bd}$

τ_v = Nominal Shear Reinforcement

V_u = Shear force due to design load

b= Breadth of the section for T section

b=b_w

d= Effective depth

STEP 2: To find maximum shear stress ($\tau_{c\,\max}$)

(Page Number 73, Table Number: 20, IS 456:2000)

$\tau_{c\,\max} = ?$

$\tau_v < \tau_{c\,\max}$, The section is Safe

STEP 3: To find percentage of Steel (P_t)

$P_t = 100 \text{ x } \frac{A_{st}}{bd}$ (Page Number 73, Table Number 19, IS 456:2000)

STEP 4: To find design shear strength of concrete (τ_c)

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
-	-
-	**?**
-	-

STEP 5: Design of Shear Reinforcement is not required

$\tau_v < \tau_c$

Design of Shear Reinforcement is not required

Provide the mimimum shear reinforcement

Page number 48 .C. No: 26.5.1.6. IS 456:2000

$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87\,fy}$

Where

A_{sv} = Total c/s area of stirrups legs effective in shear

s_v = Stirrups spacing along the length of the member

b= Breadth of the beam

Fy=Charactaristics strength of stirrups should be ≤ 415 N/mm^2

STEP 6: Design of Shear Reinforcement is required ($\tau_v > \tau_c$)

There are two way for providing shear reinforcement

1) Vertical Stirrups
2) Bent up bars along with stirrups.

a) **The shear available for design**

$V_{us} = V_u - \tau_c bd$ (Page Number 73, IS 456:2000)

b) The shear taken by bent up bars

$V_{usb} = 0.87$ x Fy x A_{sv} x sin α (Page Number 73, IS 456:2000)

where

$\alpha = 45^0$

$Fy = Yield\ stress\ of\ main\ re\text{inf}\ orcement$

$A_{sv} =$ Area of bent up bars

$V_{ub} \leq \frac{V_{us}}{2}$

Shear taken by Stirrups

$V_{us1} = V_{us} - V_{ub}$

$Spacing = S_v = \frac{0.87x \text{ Fy x } A_{sv} \text{ x d}}{V_{us1}}$ (Page Number 73, IS 456:2000)

Check For spacing

i) **Sv= Calculated**

ii) **Sv= Minimum Spacing**

$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b}$ (Page Number 48, IS 456:2000)

iii) **Sv= Maximum Spacing**

$S_v = 0.75$ d or 300 whichever is less (Page Number 47, IS 456:2000)

Take least value of Sv

1. **A RC beam 230 mm x 500 mm overall depth is reinforced with 2 bars of 20 mm diameter with effective cover of 50 mm. The beam is subjected to factored shear force of 30 KN. Design shear reinforcement using vertical stirrups only. Use M_{15} and Fe 415.**

Solution :

Given Data

b = 230 mm

Overall depth= D= 500 mm

Diameter of bar = ϕ = 20 mm

No of bar = 2

Ast = 2 X $\frac{\pi}{4}$ X ϕ^2 = 2 X $\frac{\pi}{4}$ X 20^2 = 628.32 mm^2

Factored Shear Force=Vu=30 KN

Effective Cover =d'= 50 mm

Effective depth = d= Overall Depth – Effective cover

Effective depth = d= D-d'= 500-50=450 mm

M_{15} = Fck = 15 N/mm^2

Fe 415 = Fy = 415 N/mm^2

STEP 1: **To find nominal shear reinforcement. (τ_v)**

$\tau_v = \dfrac{V_u}{bd}$ (Page Number 72, IS 456:2000)

$= \dfrac{30 \text{ x } 10^3}{230 \text{ x } 450} = 0.29\ N/mm^2$

STEP 2: To find maximum shear stress ($\tau_{c\ \max}$)

(Page Number 73, Table Number: 20, IS 456:2000)

$\tau_{c\ \max} = 2.5$ N/mm^2 For F_{ck} = 15 N/mm^2

$\tau_v < \tau_{c\ \max}$

0.29<2.5, The section is Safe

STEP 3: To find percentage of Steel (P_t)

$P_t = 100 \text{ x } \dfrac{A_{st}}{bd}$ (Page Number 73, Table Number 19, IS 456:2000)

$P_t = 100 \text{ x } \dfrac{628.32}{230 \text{ x } 450} = 0.6070$

STEP 4: To find design shear strength of concrete (τ_c)

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
0.50	**0.46**
0.6070	**?**
0.75	**0.54**

$$\tau_c = 0.46 + \left[\frac{(0.54-0.46)}{(0.75-0.5)} \text{ X } (0.6070\text{-}0.5)\right] = 0.4942 \text{ N/mm}^2$$

$\tau_v < \tau_c$

0.29<0.4942

Design of Shear Reinforcement is not required

STEP 5: To find spacing of reinforcement (S_v)

Assuming 8 mm diameter , 2 legged Mild steel vertical stirrups (Fy= 250 N/mm^2)

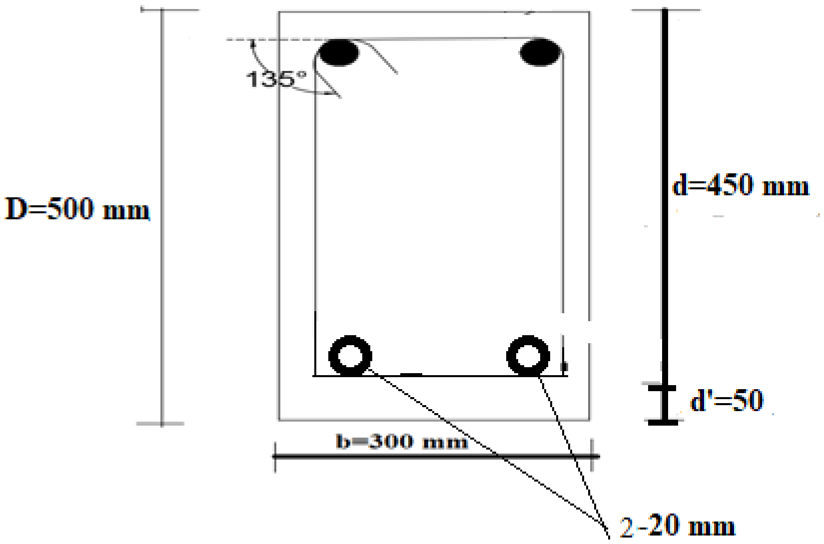
$$A_{sv} = 2x\frac{\pi}{4} \text{ X } \phi^2 = 2x\frac{\pi}{4} \text{ X } 8^2 = 100.54\ mm^2$$

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b} \quad \text{(Page Number 48, IS 456:2000)}$$

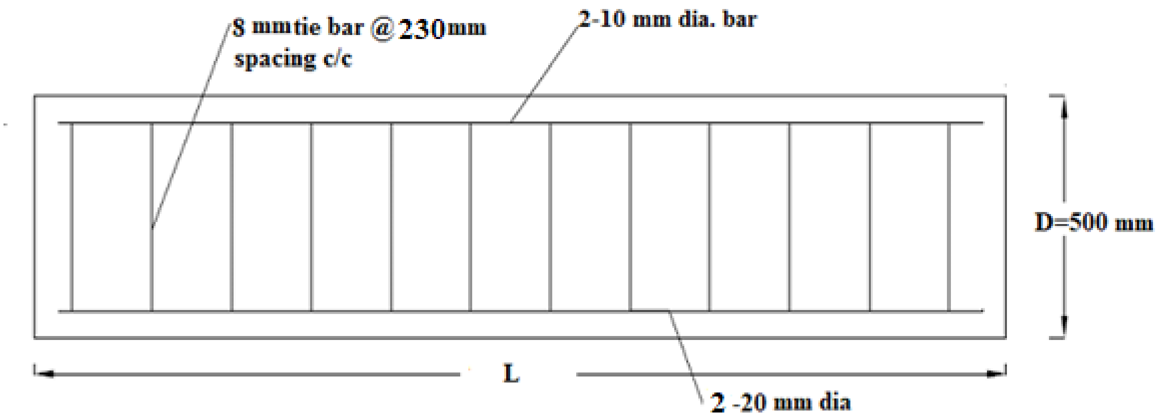
$$S_v = \frac{0.87 \text{ x } 250 \text{ x } 100.54}{0.4 \text{ x } 230} = 237.68 \; mm$$

Sv= 237.68 mm $\cong$ **230mm**

Providing 8 mm diameter with 2 legged vertical stirrups @ 230 mm c/c



a) Cross -section of beam



b) Longitudinal section of beam

2. A beam 300 mm X 1010 mm effective has a span of 7m. It is loaded with udl 45 KN/m over the entire span. The tensile steel consists of 6 bars of 22 mm diameter. Design the shear reinforcement using M_{20} and Fe 250
   Solution :
   Given Data

   b = 300 mm

Effective depth d= 1010 mm

Diameter of bar = ϕ = 22 mm

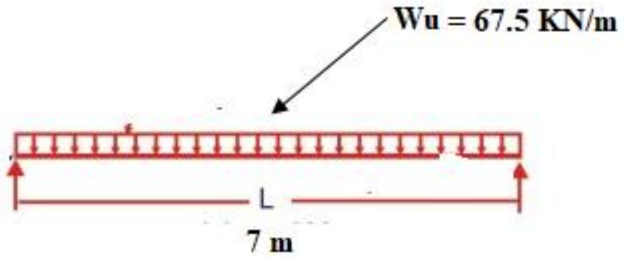
No of bar = 6

Ast = 6 X $\frac{\pi}{4}$ X ϕ^2 = 6 X $\frac{\pi}{4}$ X 22^2 = 2280.80 mm^2

L = 7 m

UDL= 45 KN/m

Factored udl =Wu=1.5 X 45 = 67.5 KN/m



Shear Force =Vu= $\frac{W_u L}{2} = \frac{67.5 \text{ X } 7}{2} = 236.25$ KN

M_{20} = Fck = 20 N/mm^2

Fe 250 = Fy = 250 N/mm^2

STEP 1: To find nominal shear reinforcement. (τ_v)

$\tau_v = \frac{V_u}{bd}$ (Page Number 72, IS 456:2000)

$= \frac{236.25 \text{ x } 10^3}{300 \text{ x } 1010} = 0.779\ N/mm^2$

STEP 2: To find maximum shear stress ($\tau_{c\ \max}$)

(Page Number 73, Table Number: 20, IS 456:2000)

$\tau_{c\ \max} = 2.8 \text{ N/mm}^2$

$\tau_v < \tau_{c\ \max}$

0.779 < 2.8, The section is Safe

STEP 3: To find percentage of Steel (P_t)

$P_t = 100 \text{ x } \frac{A_{st}}{bd}$ (Page Number 73, Table Number 19, IS 456:2000)

$P_t = 100 \text{ x } \frac{2280.80}{300 \text{ x } 1010} = 0.7527$

STEP 4: To find design shear strength of concrete (τ_c)

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
0.75	**0.56**
0.7527	**?**
1.00	**0.62**

$$\tau_c = 0.56 + \left[\frac{(0.62-0.56)}{(1-0.75)} \text{ X } (0.7527\text{-}0.75) \right] = 0.5606 \text{ N/mm}^2$$

$\tau_v > \tau_c$

0.779>0.5606

Design of Shear Reinforcement is required

STEP 5: To find Shear available for design (V_{us})

$V_{us} = V_u - \tau_c bd$ (Page Number 73, IS 456:2000)

$V_{us} = 236.25 \text{ x } 10^3 - (0.5606 \text{ x } 300 \text{ x}1010)$

$V_{us} = 66.388 \text{ x } 10^3 N = 66.388 \text{ KN}$

Assuming 6 mm diameter , 2 legged mild steel vertical stirrups (Fy= 250N/mm^2)

$$A_{sv} = 2x\frac{\pi}{4} \text{ X } \phi^2 = 2x\frac{\pi}{4} \text{ X } 6^2 = 56.55\ mm^2$$

STEP 6: To find spacing of reinforcement (S_v)

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv} \text{x d}}{V_{us}}$$ (Page Number 73, IS 456:2000)

$$S_v = \frac{0.87 \text{ x } 250 \text{ x } 56.55 \text{ x}1010}{66.388 \text{ x}10^3} = 187.12\ mm$$

Check for spacing.

i) **Sv= Calculated = 187.12 mm**

ii)

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b}$$ (Page Number 48, IS 456:2000)

$$S_v = \frac{0.87 \text{ x } 250 \text{ x } 56.55}{0.4 \text{ x } 300} = 102.50\ mm$$

iii)

$S_v = 0.75$ d or 300 mm whichever is less (Page Number 47, IS 456:2000)

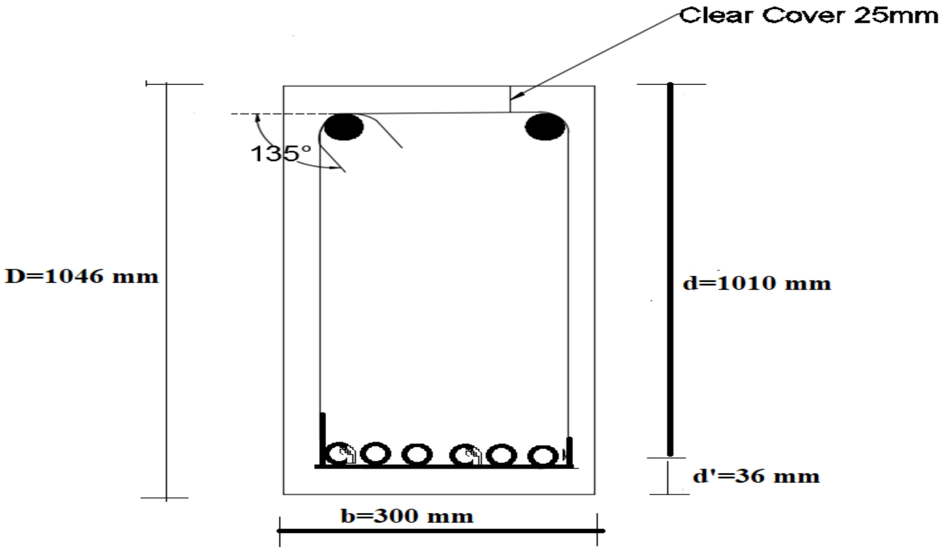
$S_v = 0.75$ x 1010 = 757.7 mm or 300 mm

$S_v = 300$ mm

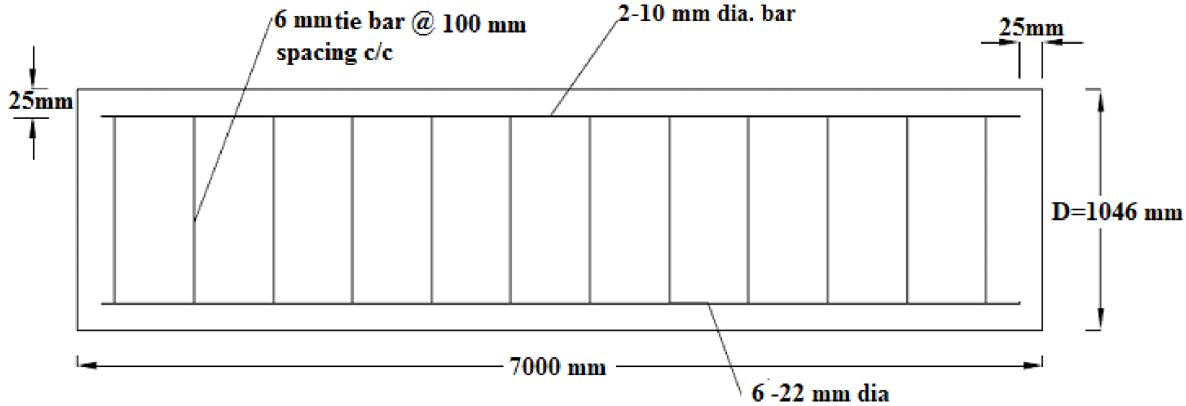
Take least value of i, ii and iii

Sv= 102.50 mm $\cong$ **100 mm**

Providing 6 mm diameter with 2 legged vertical stirrups @ 100 mm c/c



a) Cross -section of beam



b) Longitudinal section of beam

3. **Design shear reinforcement for a beam having size 300 mm X 600 mm effective carrying udl of 50 KN/m over a simply supported beam of span 7.5 m. The tensile steel consists of 4 bars of 25 mm diameter. Use M_{20} and Fe 415.**

   Solution :

   Given Data

   b = 300 mm

   d=Effective depth = 600 mm

   Diameter of bar = ϕ = 25 mm

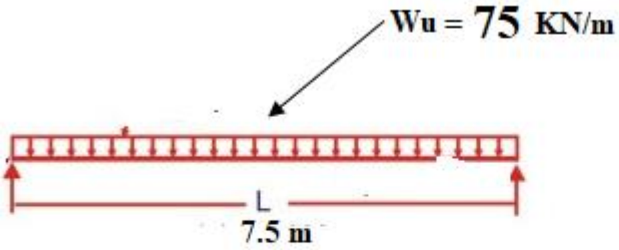
   No of bar = 4

   Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 25^2 = 1963.50 mm^2

   L = 7.5 m

   UDL= 50 KN/m

Factored udl =Wu=1.5 X 50 = 75 KN/m



Shear Force =Vu= $\frac{W_u L}{2} = \frac{75 \text{ X } 7.5}{2} = 281.25$ KN

M20 = Fck = 20 N/mm²

Fe 415= Fy = 415 N/mm²

STEP 1: **To find nominal shear reinforcement. (τ_v)**

$\tau_v = \frac{V_u}{bd}$ (Page Number 72, IS 456:2000)

$= \frac{281.25 \text{ x } 10^3}{300 \text{ x } 600} = 1.563 N/mm^2$

STEP 2: To find maximum shear stress ($\tau_{c\ \max}$)

(Page Number 73, Table Number: 20, IS 456:2000)

$\tau_{c\ \max} = 2.8 \text{ N/mm}^2$

$\tau_v < \tau_{c\ \max}$

1.563< 2.8, The section is Safe

STEP 3: To find percentage of Steel (P_t)

$P_t = 100 \text{ x } \frac{A_{st}}{bd}$ (Page Number 73, Table Number 19, IS 456:2000)

$P_t = 100 \text{ x } \frac{1963.50}{300 \text{ x } 600} = 1.09$

STEP 4: To find design shear strength of concrete (τ_c)

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
1.00	**0.62**
1.09	**?**
1.25	**0.67**

$$\tau_c = 0.62 + \left[\frac{(0.67-0.32)}{(1.25-1.00)} \text{ X } (1.09\text{-}1.00) \right] = 0.638 \text{ N/mm}^2$$

$\tau_v > \tau_c$

1.563>0.638

Design of Shear Reinforcement is required

STEP 5: To find Shear available for design (V_{us})

$V_{us} = V_u - \tau_c bd$ (Page Number 73, IS 456:2000)

$V_{us} = 281.25 \text{ x } 10^3 - (0.638 \text{ x } 300 \text{ x} 600)$

$V_{us} = 166.41 \text{ x } 10^3 N = 166.41 \text{ KN}$

Assuming 8 mm diameter , 2 legged HYSD vertical stirrups (Fy= 415 N/mm²)

$A_{sv} = 2x\frac{\pi}{4} \text{ X } \phi^2 = 2x\frac{\pi}{4} \text{ X } 8^2 = 100.54\ mm^2$

STEP 6: To find spacing of reinforcement (S_v)

$S_v = \frac{0.87 \text{ x Fy x } A_{sv} \text{x d}}{V_{us}}$ (Page Number 73, IS 456:2000)

$S_v = \frac{0.87 \text{ x } 415 \text{ x } 100.54 \text{ x}600}{166.41 \text{ x}10^3} = 130.88\ mm$

Check for spacing.

i) **Sv= Calculated = 130.88 mm**

ii)

$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b}$ (Page Number 48, IS 456:2000)

$S_v = \frac{0.87 \text{ x } 415 \text{ x } 100.54}{0.4 \text{ x } 300} = 302.50\ mm$

iii)

$S_v = 0.75$ d or 300 mm whichever is less (Page Number 47, IS 456:2000)

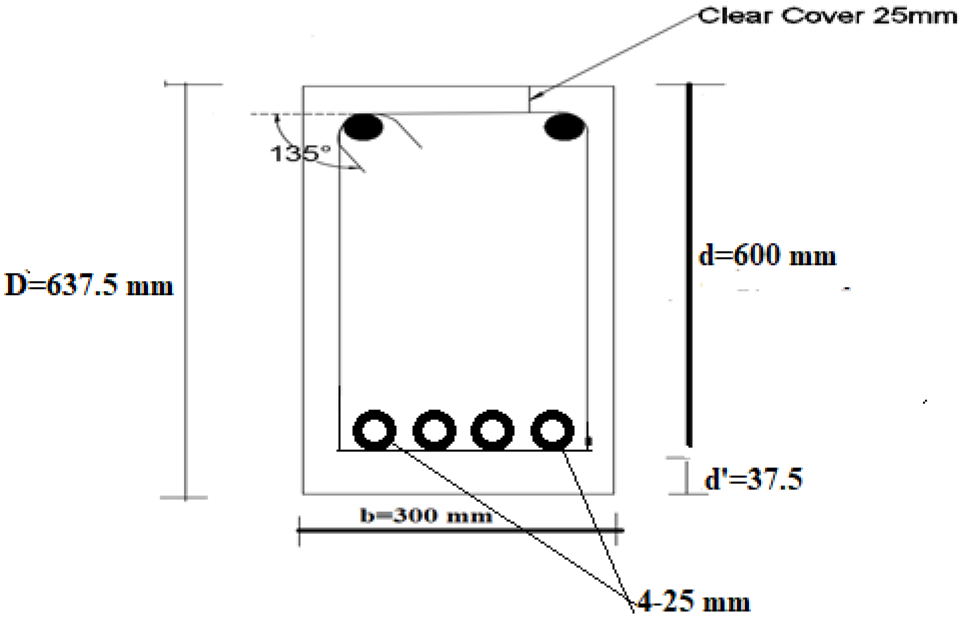
$S_v = 0.75 \text{ x } 600 = 450$ mm or 300 mm

$S_v = 300$ mm

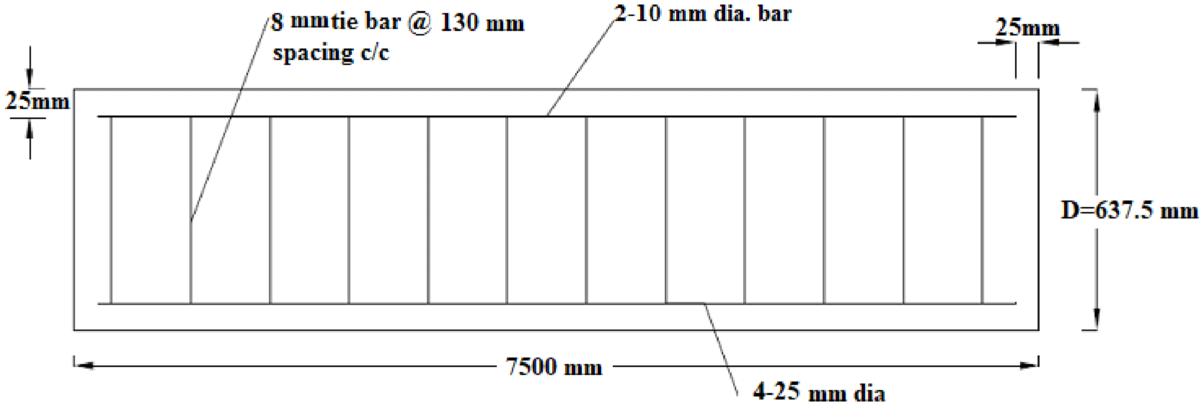
Take least value of i, ii and iii

Sv= 130.88 mm $\cong$ **130 mm**

Providing 8 mm diameter with 2 legged vertical stirrups @ 130 mm c/c



a) Cross -section of beam



b) Longitudinal section of beam

4. **A RC beam 300 mm x 500 mm overall depth is reinforced with 3 bars of 20 mm diameter with effective cover of 50 mm. The beam is subjected to shear force of 100 KN. Design shear reinforcement using vertical stirrups only. Use M_{20} and Fe250.**

   Solution :

   Given Data

   b = 300 mm

   Overall depth= D= 500 mm

   Diameter of bar = ϕ = 20 mm

   No of bar = 3

Ast = 3 X $\frac{\pi}{4}$ X ϕ^2 = 3 X $\frac{\pi}{4}$ X 20^2 = 942.47 mm^2

Shear Force = 100 KN

Factored Shear Force=Vu=1.5 x 100 = 150 KN

Effective Cover =d'= 50 mm

Effective depth = d= Overall Depth – Effective cover

Effective depth = d= D-d'= 500-50=450 mm

M_{20} = Fck = 20 N/mm^2

Fe 250 = Fy = 250 N/mm^2

STEP 1: **To find nominal shear reinforcement. (τ_v)**

$\tau_v = \frac{V_u}{bd}$ (Page Number 72, IS 456:2000)

$= \frac{150 \text{ x } 10^3}{300 \text{ x } 450} = 1.111\ N/mm^2$

STEP 2: To find maximum shear stress ($\tau_{c\ \max}$)

(Page Number 73, Table Number: 20, IS 456:2000)

$\tau_{c\ \max} = 2.8 \text{ N/mm}^2$

$\tau_v < \tau_{c\ \max}$

1.111<2.8, The section is Safe

STEP 3: To find percentage of Steel (P_t)

$P_t = 100 \text{ x } \frac{A_{st}}{bd}$ (Page Number 73, Table Number 19, IS 456:2000)

$P_t = 100 \text{ x } \frac{942.47}{300 \text{ x } 450} = 0.698$

STEP 4: To find design shear strength of concrete (τ_c)

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
0.50	**0.48**
0.698	**?**
0.75	**0.56**

$$\tau_c = 0.48 + \left[\frac{(0.56-0.48)}{(0.75-0.5)} \text{ X } (0.698\text{-}0.5) \right] = 0.543 \text{ N/mm}^2$$

$\tau_v > \tau_c$

1.111>0.543

Design of Shear Reinforcement is required

STEP 5: To find Shear available for design (V_{us})

$V_{us} = V_u - \tau_c bd$ (Page Number 73, IS 456:2000)

$V_{us} = 150 \text{ x } 10^3 - (0.543 \text{ x } 300 \text{ x}450)$

$V_{us} = 76.695 \text{ x } 10^3 N = 76.695 \text{ KN}$

Assuming 8 mm diameter , 2 legged mild steel vertical stirrups (Fy= 250N/mm^2)

$$A_{sv} = 2x\frac{\pi}{4} \text{ X } \phi^2 = 2x\frac{\pi}{4} \text{ X } 8^2 = 100.54 \; mm^2$$

STEP 6: To find spacing of reinforcement (S_v)

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv} \text{x d}}{V_{us}}$$ (Page Number 73, IS 456:2000)

$$S_v = \frac{0.87 \text{ x } 250 \text{ x } 100.54 \text{ x}450}{76.695 \text{ x}10^3} = 128.37 \; mm$$

Check for spacing.

i) **Sv= Calculated = 128.37 mm**

ii)

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b}$$ (Page Number 48, IS 456:2000)

$$S_v = \frac{0.87 \text{ x } 250 \text{ x } 100.54}{0.4 \text{ x } 300} = 182.22 \; mm$$

iii)

$S_v = 0.75$ d or 300 mm whichever is less (Page Number 47, IS 456:2000)

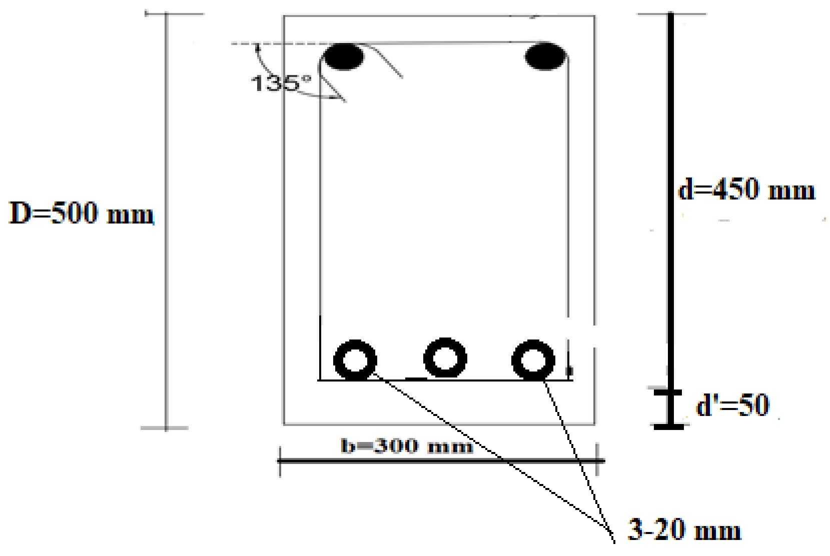
$S_v = 0.75 \text{ x } 450 = 337.5$ mm or 300 mm

$S_v = 300$ mm

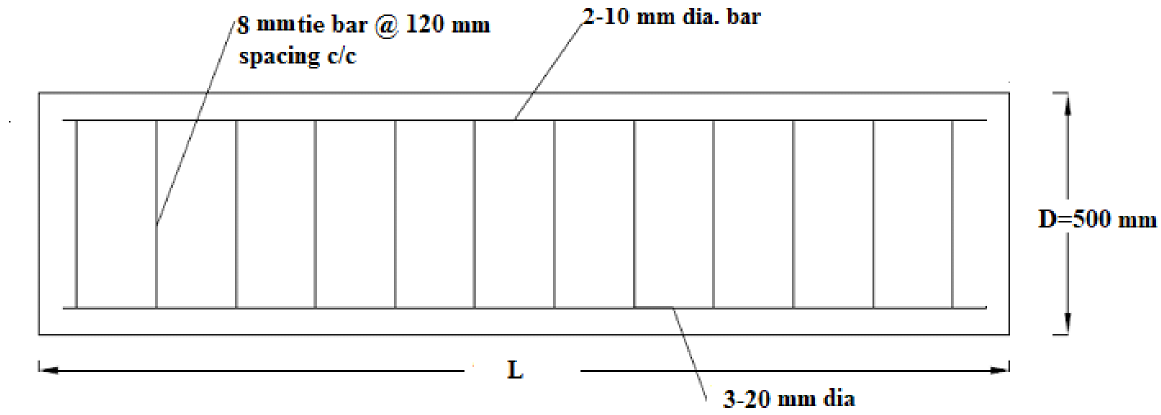
Take least value of i, ii and iii

Sv= 128.37 mm $\cong$ **120mm**

Providing 8 mm diameter with 2 legged vertical stirrups @ 120 mm c/c



c) **Cross -section of beam**



d) **Longitudinal section of beam**

5. **Design shear reinforcement using vertical stirrups and 2 bent up bars for a beam having size 350 mm X 500 mm effective carrying udl of 75 KN/m over a simply supported beam of span 7 m. The tensile steel consists of 4 bars of 25 mm diameter. Use M_{20} and Fe 415.**

Solution :

Given Data

b = 350 mm

d=Effective depth = 500 mm

Diameter of bar = ϕ = 25 mm

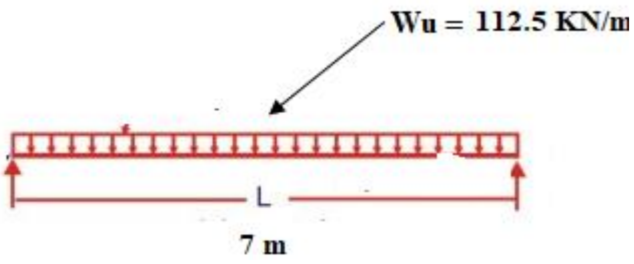
No of bar = 4

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 25^2 = 1963.50 mm^2

L = 7 m

UDL= 75 KN/m

Factored udl =Wu=1.5 X 75 = 112.5 KN/m



Shear Force =Vu= $\frac{W_u L}{2} = \frac{112.5 \text{ X } 7}{2} = 393.75$ KN

M_{20} = Fck = 20 N/mm^2

Fe 415= Fy = 415 N/mm^2

STEP 1: **To find nominal shear reinforcement. (τ_v)**

$\tau_v = \frac{V_u}{bd}$ (Page Number 72, IS 456:2000)

$= \frac{393.75 \text{ x } 10^3}{350 \text{ x } 500} = 2.25 N/mm^2$

STEP 2: To find maximum shear stress ($\tau_{c\,\max}$)

(Page Number 73, Table Number: 20, IS 456:2000)

$\tau_{c\,\max} = 2.8 \text{ N/mm}^2$

$\tau_v < \tau_{c\,\max}$

2.25< 2.8, The section is Safe

STEP 3: To find percentage of Steel (P_t)

$P_t = 100 \text{ x } \frac{A_{st}}{bd}$ (Page Number 73, Table Number 19, IS 456:2000)

$P_t = 100 \text{ x } \frac{1963.50}{350 \text{ x } 500} = 1.122$

STEP 4: To find design shear strength of concrete (τ_c)

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
1.00	**0.62**
1.122	**?**
1.25	**0.67**

$$\tau_c = 0.62 + \left[\frac{(0.67 - 0.32)}{(1.25 - 1.00)} \text{ X } (1.122\text{-}1.00) \right] = 0.6444 \text{ N/mm}^2$$

$\tau_v > \tau_c$

2.25>0.644

Design of Shear Reinforcement is required

STEP 5: To find Shear available for design (V_{us})

$V_{us} = V_u - \tau_c bd$ (Page Number 73, IS 456:2000)

$V_{us} = 393.75 \text{ x } 10^3 - (0.6444 \text{ x } 350 \text{ x} 500)$

$V_{us} = 280.98 \text{ x } 10^3 N = 280.98 \text{ KN}$

Assuming out of 280.98 KN , 50 % load is resisted by vertical stirrups (280.98/2 =140.49 KN) and 50% load is resisted by bent up bars.

Shear resisted by bent up bars

Out of 4 bars , 2 bars are bent up

$$V_{usb} = 0.87\ F_y\ A_{sv}\ \text{Sin}\alpha \text{ (Page Number 73, IS 456:2000)}$$

$$V_{usb} = 0.87 \text{ x}415 \text{ x } 2\text{x}\frac{\pi}{4}\text{x}25^2 \text{ x Sin}(45^0)$$

$$V_{usb} = 250.64 \text{ x } 10^3 = 250.64 \text{ KN}>140.49 \text{ KN}$$

Assuming 8 mm diameter , 2 legged HYSD vertical stirrups (Fy= 415 N/mm²)

$$A_{sv} = 2x\frac{\pi}{4} \text{ X } \phi^2 = 2x\frac{\pi}{4} \text{ X } 8^2 = 100.54\ mm^2$$

STEP 6: To find spacing of reinforcement (S_v)

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv} \text{x d}}{V_{us}} \text{ (Page Number 73, IS 456:2000)}$$

$$S_v = \frac{0.87 \text{ x } 415 \text{ x } 100.54 \text{ x}500}{140.49 \text{ x}10^3} = 129.19\ mm$$

Check for spacing.

i) Sv= Calculated = 129.19 mm

ii)

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b} \text{ (Page Number 48, IS 456:2000)}$$

$$S_v = \frac{0.87 \text{ x } 415 \text{ x } 100.54}{0.4 \text{ x } 350} = 259.28\ mm$$

iii)

$$S_v = 0.75 \text{ d or 300 mm whichever is less (Page Number 47, IS 456:2000)}$$

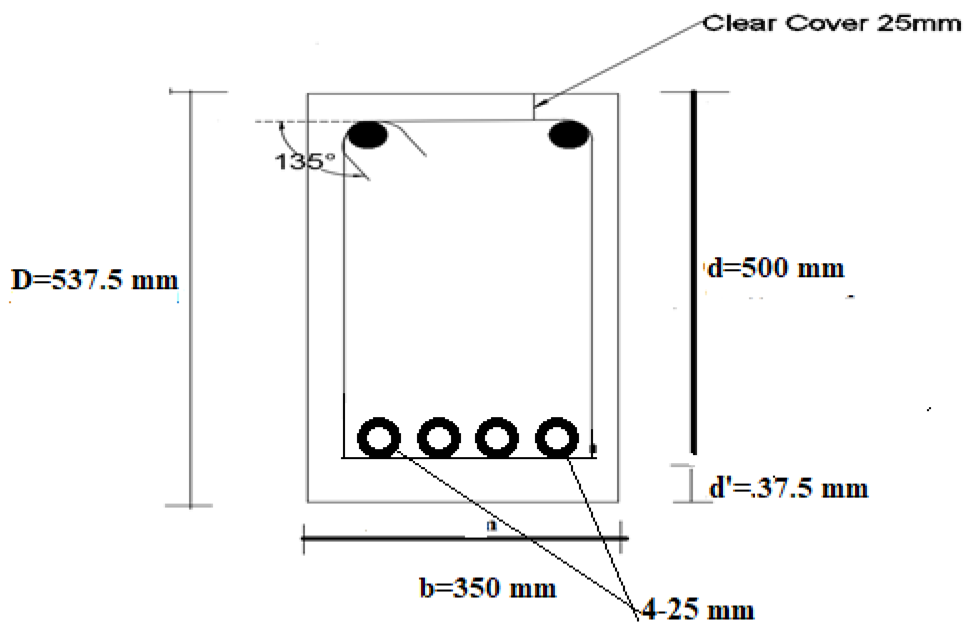
$$S_v = 0.75 \text{ x } 500 = 375 \text{ mm or } 300 \text{ mm}$$

$$S_v = 300 \text{ mm}$$

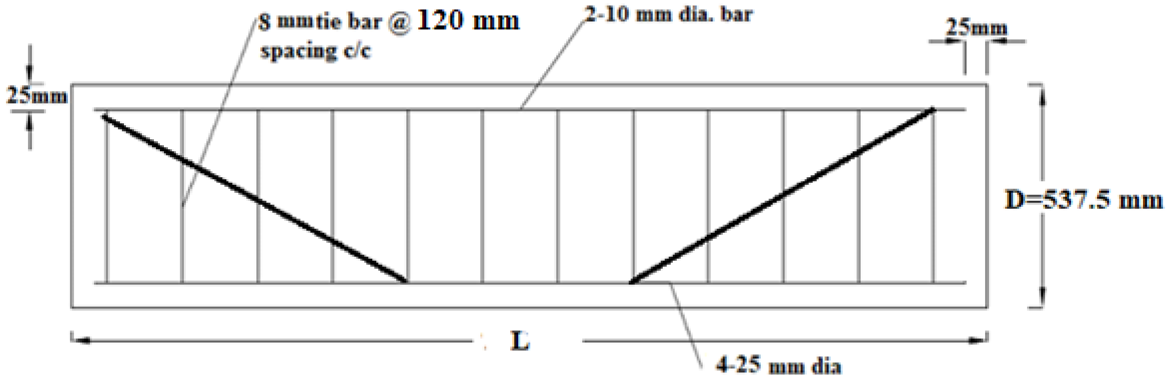
Take least value of i, ii and iii

Sv= 129.19 mm $\cong$ **120 mm**

Providing 8 mm diameter with 2 legged vertical stirrups @ 120 mm c/c



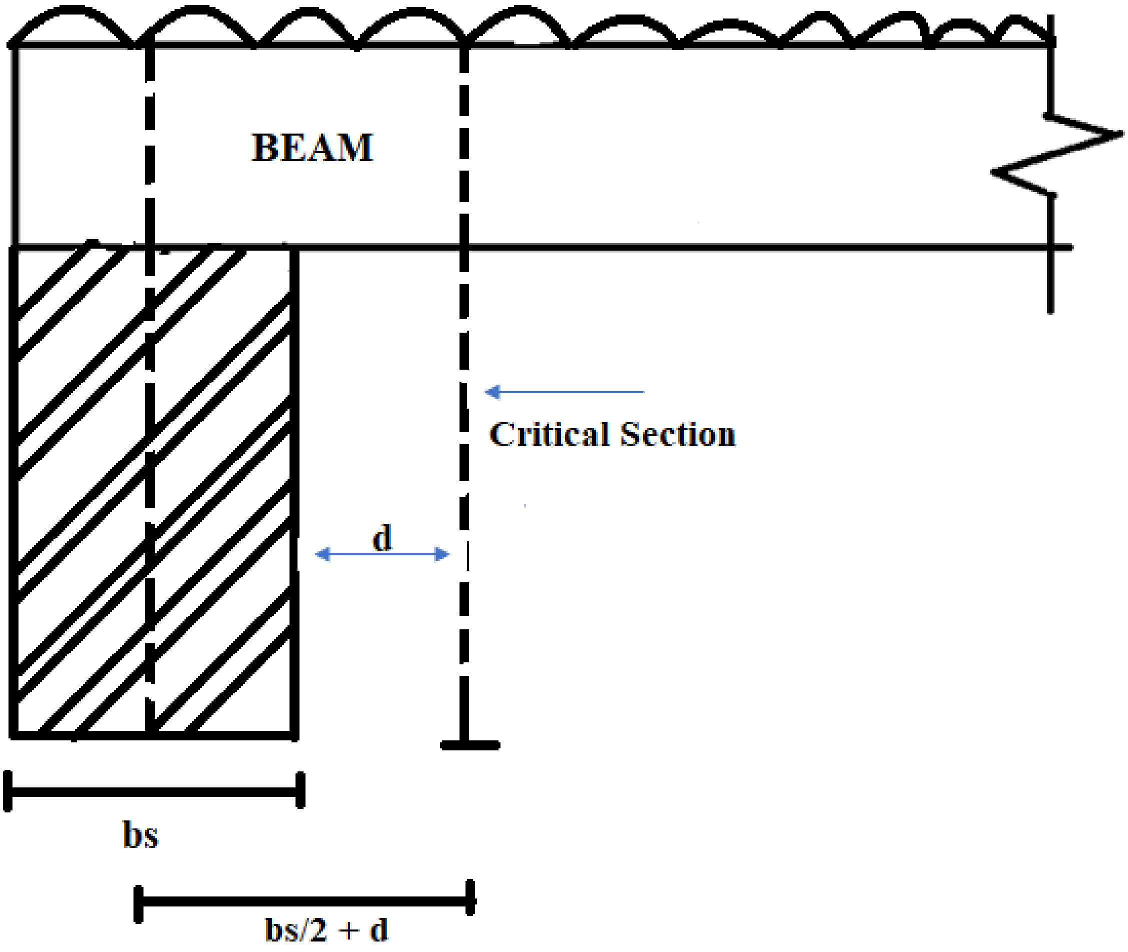
a) Cross -section of beam

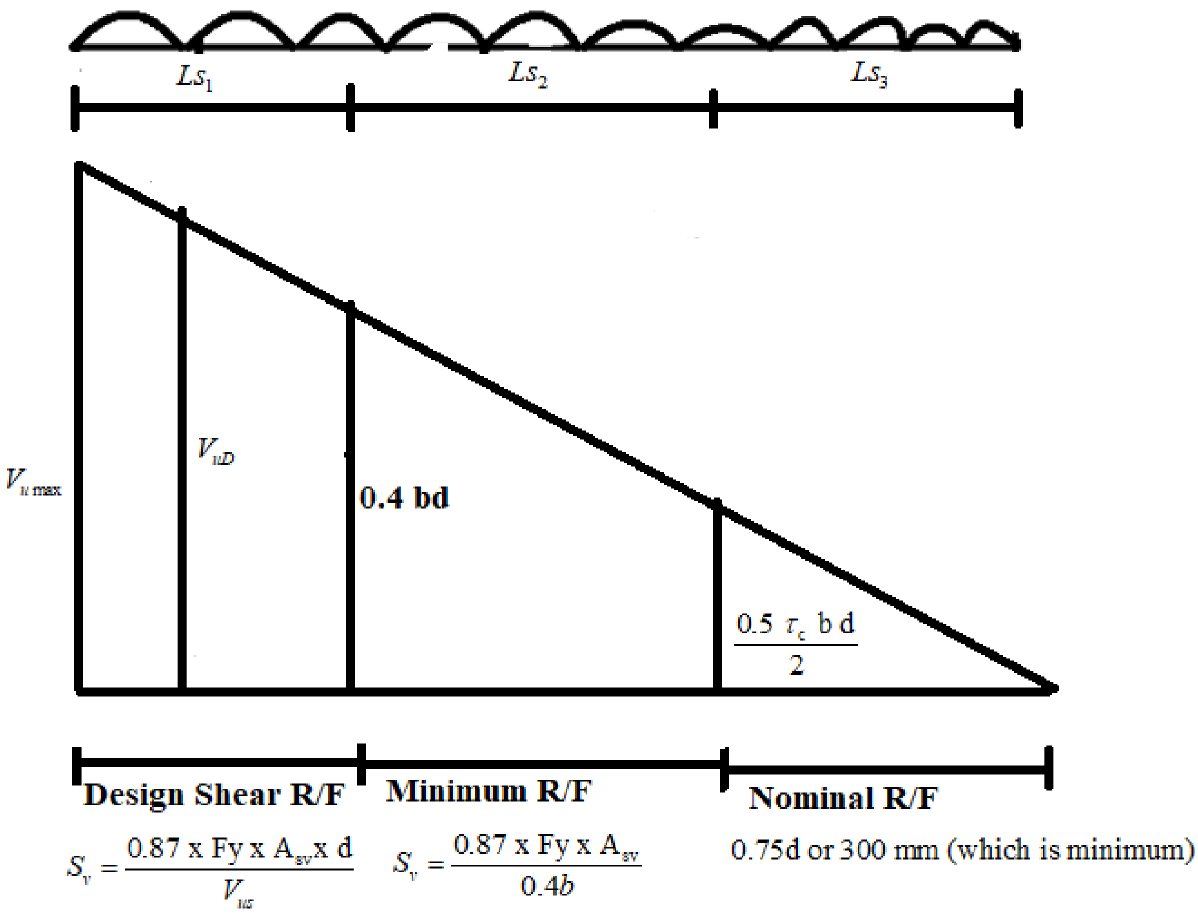


b) Longitudinal section of beam

Limit State of Collapse (Shear)

Zone of Shear Reinforcement





Design Procedure

Solution: Given Data

Width of Beam =b

Overall depth =D

Effective cover= d'

Effective depth d= D-d'

Width of Support= b_s

Diameter of bar = ϕ

No of bar = N

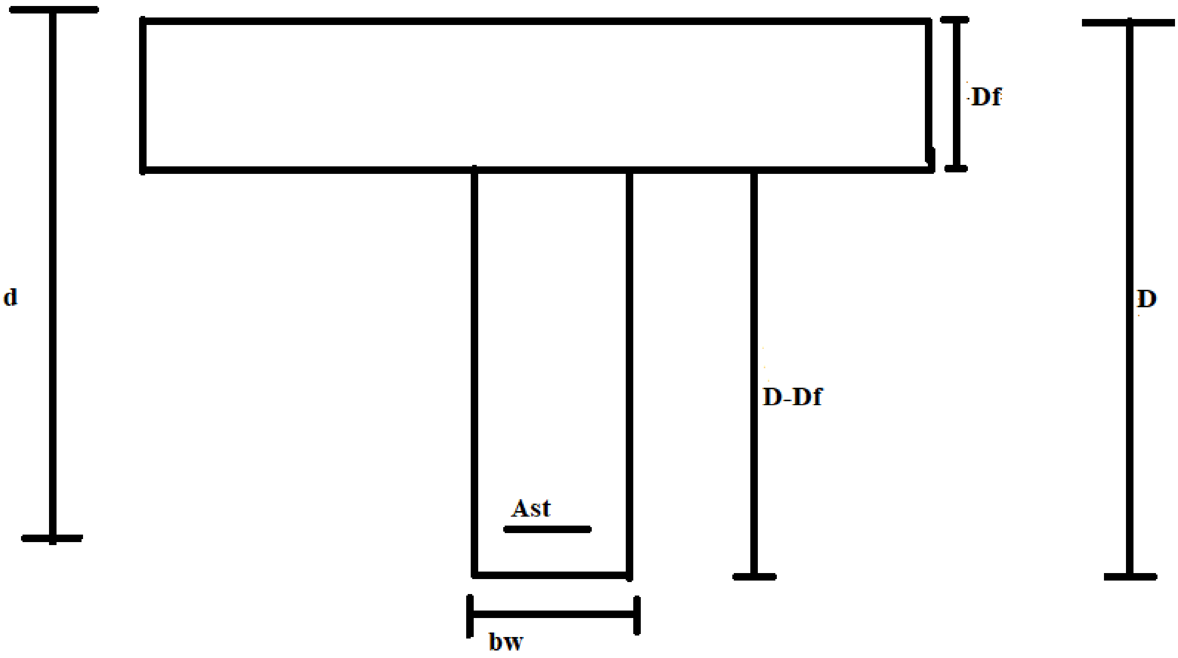
Ast = N X $\frac{\pi}{4}$ X ϕ^2

The thickness of flange= D_f=

L =

superimposed load =

Fck , Fy



STEP 1: Loading

a) Superimposed Load= Given KN/m
b) Self weight of beam = A_w x Density of Concrete
= ? KN/m

Total Load= W= ? KN/m

Total Factored Load= Wu=W x1.5= ? KN/m

STEP 2: Maximum Shear Force

$$V_{u\max} = \frac{W_u L}{2}$$

STEP 3: To find Design shear force V_{uD}

$$V_{uD} = V_{u\max} - \mathrm{W_u}\left(\frac{b_s}{2} + d\right)$$

STEP 4: **Shear taken by concrete**

The design strength of concrete varies with the percentage of steel, different grade of concrete and denoted by τ_c. It is shear stress of concrete τ_c

Percentage of steel Pt

$P_t = 100$ x $\frac{A_{st}}{bd}$ (Page Number 73, Table Number 19, IS 456:2000)

To find design shear strength of concrete (τ_c **)**

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
1.75	**0.75**
1.883	**?**
2.00	**0.79**

$$\tau_c = 0.75 + \left[\frac{(0.79 - 0.75)}{(2.00 - 1.75)} \text{ X } (1.883\text{-}1.75) \right] = 0.7713 \text{ N/mm}^2$$

Shear resisted by concrete

$V_{uc} = \tau_c bd$ (Page Number 73, IS 456:2000)

Maximum Shear resisted by concrete

$V_{ucMax} = \tau_{c\max} bd$ (Page Number 73, IS 456:2000)

$\tau_{c\max}$ calculated from T.No:20, Page Number 73, IS 456:2000

$V_{uD} < V_{ucMax}$

The section is safe

STEP 5: The ultimate shear resistance of RC member with minimum stirrups

$V_{urMin} = 0.4bd + \tau_c bd$

Compare V_{uD} and V_{urMin}

$V_{uD} > V_{urMin}$

Design of shear reinforcement is required

STEP 6: Shear force for design

$V_{us} = V_{uD} - \tau_c bd$

Zone I: Providing the design shear R/F

Assume *diameter of bar* , and grade of steel

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv} \text{x d}}{V_{us}} < 0.75d \text{ or 300 mm (which is less)}$$

$$Ls_1 = \frac{V_{u\max} - V_{ur\min}}{Wu} \text{ (upto this 1}^{st} \textit{spacing} \text{ is provided)}$$

Zone III: Providing nominal shear R/F

Because Zone II is depend upon Zone III and it is calculated as

$S_v = 0.75d$ or 300 mm (which is less)

$$Ls_3 = \frac{0.5 \ \tau_c \text{ b d}}{Wu}$$

Zone II: Providing the minimum shear R/F

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b} < 0.75\text{d or 300 mm (which is minimum) (page no: 48, IS 456:2000)}$$

$$Ls_2 = \frac{L}{2} - L_{S1} - L_{S3}$$

1) An RC beam of size 230 mm X 750 mm deep over all supported over a span of 10 m. It carries superimposed load of 32 KN/m. The thickness of flange is 140 mm, the tension reinforcement consists of 6 bars of 25 mm diameter, the width of support is 230 mm. Design the shear reinforcement using vertical stirrups and find the different zone of shear reinforcement, the effective cover is 70 mm. Use M_{20} and Fe 415.

Solution: Given Data

b = 230 mm

Overall depth =D= 750 mm

Effective cover= d'= 70 mm

Effective depth d= D-d'=750-70 = 680mm

Width of Support= b_s=230 mm

Diameter of bar = ϕ = 25 mm

No of bar = 6

Ast = 6 X $\frac{\pi}{4}$ X ϕ^2 = 6 X $\frac{\pi}{4}$ X 25^2 =2945.24 mm^2

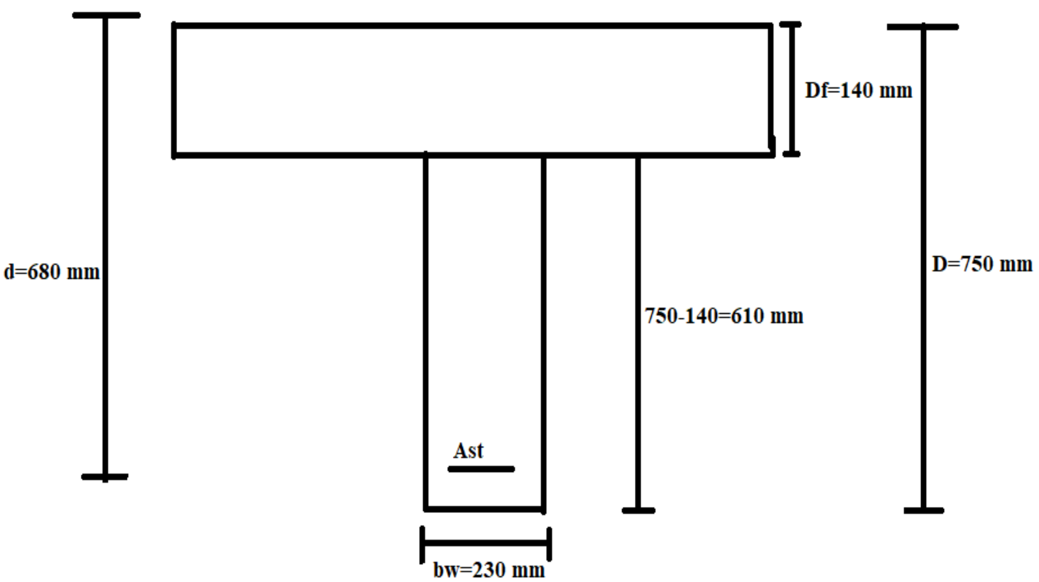
The thickness of flange= D_f=140 mm

L = 10 m

superimposed load = 32 KN/m

M_{20} = Fck = 20 N/mm^2

Fe 415 = Fy = 415 N/mm^2



STEP 1: Loading

a) Superimposed Load= 32 KN/m

b) Self weight of beam = A_w x Density of Concrete
= 0.23 x 0.61 x 25 = 3.51 KN/m

Total Load= W=35.51 KN/m

Total Factored Load= Wu=35.51 x1.5= 53.261 KN/m

STEP 2: Maximum Shear Force

$$V_{u\max} = \frac{W_u L}{2} = \frac{53.261 \text{ X } 10}{2} = 266.31 \text{ KN}$$

STEP 3: To find Design shear force V_{uD}

$$V_{uD} = V_{u\max} - \text{W}_u \left(\frac{b_s}{2} + d \right)$$

$$= 266.31 - 53.261\left(\frac{0.230}{2} + 0.68 \right)$$

$$V_{uD} = 223.97 \text{ KN}$$

STEP 4: **Shear taken by concrete**

The design strength of concrete varies with the percentage of steel, different grade of concrete and denoted by τ_c. It is shear stress of concrete τ_c

Percentage of steel Pt

$$P_t = 100 \text{ x } \frac{A_{st}}{bd}$$ (Page Number 73, Table Number 19, IS 456:2000)

$$P_t = 100 \text{ x } \frac{2945.24}{230 \text{ x } 680} = 1.883$$

To find design shear strength of concrete (τ_c)

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
1.75	**0.75**
1.883	**?**
2.00	**0.79**

$$\tau_c = 0.75 + \left[\frac{(0.79 - 0.75)}{(2 - 1.75)} \text{ X (1.883-1.75)} \right] = 0.7713 \text{ N/mm}^2$$

Shear resisted by concrete

$V_{uc} = \tau_c bd$ (Page Number 73, IS 456:2000)

$V_{uc} = 0.7713 \text{ x } 230 \text{ x } 680 = 120.63 \text{ x } 10^3 N = 120.63 \text{ KN}$

Maximum Shear resisted by concrete

$V_{ucMax} = \tau_{c\max} bd$ (Page Number 73, IS 456:2000)

$V_{ucMax} = 2.8$ x 230 x 680 =437.92 x $10^3 N = 437.92$ KN

$\tau_{c\max}$ calculated from T.No:20, Page Number 73, IS 456:2000

$V_{uD} < V_{ucMax}$

$223.97 KN < 437.92 KN$

The section is safe

STEP 5: The ultimate shear resistance of RC member with minimum stirrups

$V_{urMin} = 0.4bd + \tau_c bd$

$V_{urMin} = 0.4x230$ x $680 + 0.7713$ x 230 x 680

$V_{urMin} = 183.93$ x 10^3 N=183.93 KN

Compare V_{uD} and V_{urMin}

$223.97 > 183.93$

Design of shear reinforcement is required

STEP 6: Shear force for design

$V_{us} = V_{uD} - \tau_c bd$

$V_{us} = 223.97$ x $10^3 - 0.7713$ x 230 x 680

$V_{us} = 103.34$ x 10^3 N=103.34 KN

Zone I: Providing the design shear R/F

Assume 8 mm ϕ , 2 legged mild steel stirrups

Fy = 250 N/mm^2

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv} \text{x d}}{V_{us}} < 0.75d \text{ or 300 mm (which is less)}$$

$$S_v = \frac{0.87x\ 250 \text{ x } 2\text{x} \frac{\pi}{4} x\ 8^2 x680}{103.34x10^3} < 0.75x680 = 510 \text{ or 300 mm}$$

$S_v = 144.01mm$ < 300 mm (ok)

Providing = $S_v = 140\ mm$

$$Ls_1 = \frac{V_{u\max} - V_{ur\min}}{Wu} = \frac{266.31 - 183.19}{53.261} = 1.56m \text{ (upto this 1}^{st} \textit{spacing} \text{ is provided)}$$

Zone III: Providing nominal shear R/F

Because Zone II is depend upon Zone III and it is calculated as

$S_v = 0.75d$ or 300 mm (which is less)

$S_v = 0.75x680 = 510$ or 300 mm

$S_v = 300\ mm$

Providing = $S_v = 300\ mm$

$$Ls_3 = \frac{0.5\ \tau_c \text{ b d}}{Wu} = \frac{0.5 \text{ x } 0.7713 \text{ x } 230 \text{ x } 680}{53.261x\ 10^3} = 1.132\ m$$

Zone II: Providing the minimum shear R/F

$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b}$ < 0.75d or 300 mm (which is minimum) (page no: 48, IS 456:2000)

$S_v = \frac{0.87x \ 250 \text{ x } 2\text{x} \frac{\pi}{4} x \ 8^2}{0.4x \ 230} < 0.75x680 = 510mm$ or 300 mm

$S_v = 237.66 \ mm$ < 300 mm (ok)

Providing = S_v=230 mm

$Ls_2 = \frac{L}{2} - L_{S1} - L_{S3}$

$Ls_2 = \frac{10}{2} - 1.56 - 1.132$

$Ls_2 = 2.308m$

2) A simply supported RC beam of size 250 mm X 700 mm deep over all supported over a span of 9 m. It carries superimposed load of 35 KN/m. The thickness of flange is 120 mm, the tension reinforcement consists of 6 bars of 25 mm diameter, the width of support is 250 mm. Design the shear reinforcement using vertical stirrups and find the different zone of shear reinforcement, the effective cover is 70 mm. Use M_{20} and Fe 415.

Solution: Given Data

b = 250 mm

Overall depth =D= 700 mm

Effective cover= d'= 70 mm

Effective depth d= D-d'=700-70 = 630mm

Width of Support= b_s=250 mm

Diameter of bar = ϕ = 25 mm

No of bar = 6

Ast = 6 X $\frac{\pi}{4}$ X ϕ^2 = 6 X $\frac{\pi}{4}$ X 25^2 =2945.24 mm^2

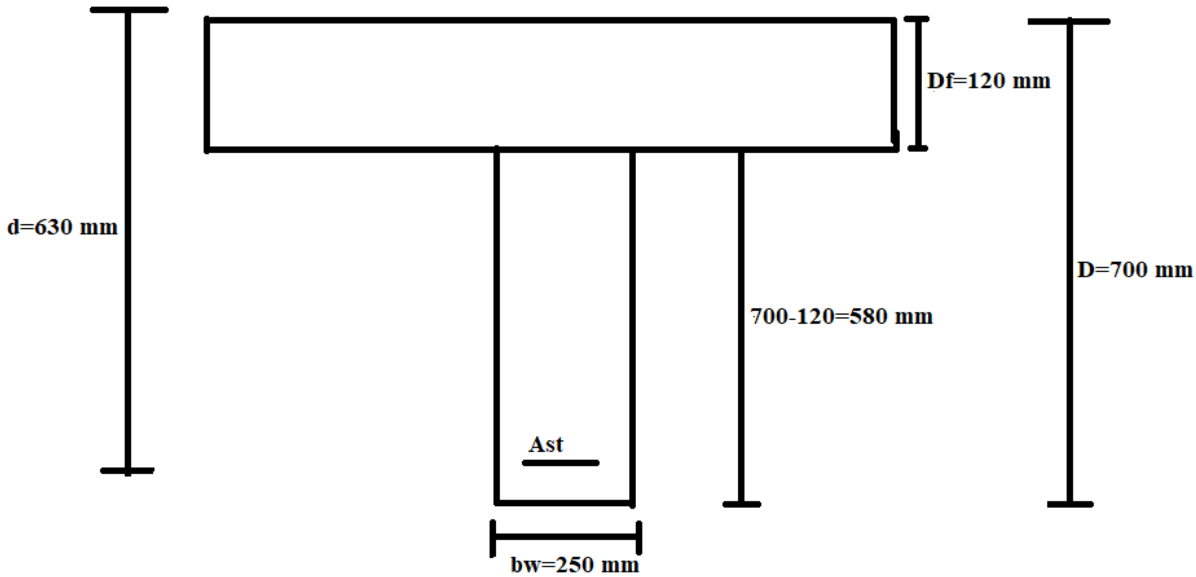
The thickness of flange= D_f=120 mm

L = 9 m

superimposed load = 35 KN/m

M_{20} = Fck = 20 N/mm^2

Fe 415 = Fy = 415 N/mm^2



STEP 1: Loading

a) Superimposed Load= 35 KN/m
b) Self weight of beam = A_w x Density of Concrete

$= 0.25 \times 0.58 \times 25 = 3.625$ KN/m

Total Load= W=38.625 KN/m

Total Factored Load= Wu=38.625 x1.5= 57.94 KN/m

STEP 2: Maximum Shear Force

$$V_{u\max} = \frac{W_u L}{2} = \frac{57.94 \text{ X } 9}{2} = 260.73 \text{ KN}$$

STEP 3: To find Design shear force V_{uD}

$$V_{uD} = V_{u\max} - \text{W}_\text{u}\left(\frac{b_s}{2} + d\right)$$

$$= 260.73 - 57.94\left(\frac{0.250}{2} + 0.63\right)$$

$$V_{uD} = 260.73 \text{ KN}$$

STEP 4: **Shear taken by concrete**

The design strength of concrete varies with the percentage of steel, different grade of concrete and denoted by τ_c. It is shear stress of concrete τ_c

Percentage of steel Pt

$$P_t = 100 \text{ x } \frac{A_{st}}{bd} \text{ (Page Number 73, Table Number 19, IS 456:2000)}$$

$$P_t = 100 \text{ x } \frac{2945.24}{250 \text{ x } 630} = 1.87$$

To find design shear strength of concrete (τ_c **)**

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
1.75	**0.75**
1.87	**?**
2.00	**0.79**

$$\tau_c = 0.75 + \left[\frac{(0.79-0.75)}{(2-1.75)} \text{ X } (1.87\text{-}1.75) \right] = 0.77 \text{ N/mm}^2$$

Shear resisted by concrete

$V_{uc} = \tau_c bd$ (Page Number 73, IS 456:2000)

$V_{uc} = 0.77$ x 250 x 630 =121.25 x $10^3 N = 121.25$KN

Maximum Shear resisted by concrete

$V_{ucMax} = \tau_{c\max} bd$ (Page Number 73, IS 456:2000)

$V_{ucMax} = 2.8$ x 250 x 630 =441 x $10^3 N = 441$ KN

$\tau_{c\max}$ calculated from T.No:20, Page Number 73, IS 456:2000

$V_{uD} < V_{ucMax}$

$216.98KN < 441KN$

The section is safe

STEP 5: The ultimate shear resistance of RC member with minimum stirrups

$V_{urMin} = 0.4bd + \tau_c bd$

$V_{urMin} = 0.4x250$ x $630 + 0.77$ x 250 x 630

$V_{urMin} = 184.28x10^3$ N= 184.28 KN

Compare V_{uD} and V_{urMin}

$216.98 > 184.28$

Design of shear reinforcement is required

STEP 6: Shear force for design

$V_{us} = V_{uD} - \tau_c bd$

$V_{us} = 216.98$ x $10^3 - 0.77$ x 250 x 630

$V_{us} = 95.71$ x 10^3 N=95.71 KN

Zone I: Providing the design shear R/F

Assume 8 mm ϕ , 2 legged mild steel stirrups

Fy = 250 N/mm^2

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv} \text{x d}}{V_{us}} < 0.75d \text{ or 300 mm (which is less)}$$

$$S_v = \frac{0.87x \text{ 250 x 2x} \frac{\pi}{4} x \ 8^2 x630}{95.71x10^3} < 0.75x680 = 510 \text{ or 300 mm}$$

$S_v = 143.93$ mm $< 0.75x630 = 472.5$ or 300 mm

$S_v = 140mm$ < 300 mm (ok)

Providing = $S_v = 140\ mm$

$$Ls_1 = \frac{V_{u\max} - V_{ur\min}}{Wu} = \frac{260.73 - 184.28}{57.94} = 1.32\ m \text{ (upto this } 1^{st} \textit{spacing} \text{ is provided)}$$

Zone III: Providing nominal shear R/F

Because Zone II is depend upon Zone III and it is calculated as

$S_v = 0.75d$ or 300 mm (which is less)

$S_v = 0.75x630 = 472.5$ or 300 mm

$S_v = 300\ mm$

Providing = $S_v = 3000\ mm$

$$Ls_3 = \frac{0.5\ \tau_c \text{ b d}}{Wu} = \frac{0.5 \text{ x } 0.77 \text{ x } 250 \text{ x } 630}{57.94x\ 10^3} = 1.05\ m$$

Zone II: Providing the minimum shear R/F

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b} < 0.75\text{d or 300 mm (which is minimum) (page no: 48, IS 456:2000)}$$

$$S_v = \frac{0.87x \text{ 250 x 2x} \frac{\pi}{4} x \ 8^2}{0.4x \ 250} < 0.75x630 = 472.5 \text{ or 300 mm}$$

$S_v = 218.65\ mm$ < 300 mm (ok)

Providing = S_v=210 *mm*

$$Ls_2 = \frac{L}{2} - L_{S1} - L_{S3}$$

$$Ls_2 = \frac{9}{2} - 1.32 - 1.105$$

$Ls_2 = 2.13m$

3) An RC beam of size 230 mm X 750 mm deep over all supported over a span of 10 m. It carries superimposed load of 30 KN/m. The thickness of flange is 140 mm, the tension reinforcement consists of 6 bars of 25 mm diameter, the width of support is 230 mm. Design the shear reinforcement using vertical stirrups and one bent up bar at 45^0. Find the different zone of shear reinforcement, the effective cover is 50 mm. Use M_{20} and Fe 415.
Solution: Given Data

b = 230 mm

Overall depth =D= 750 mm

Effective cover= d'= 50 mm

Effective depth d= D-d'=750-50 = 700mm

Width of Support= b_s=230 mm

Diameter of bar = ϕ = 25 mm

No of bar = 6

Ast = 6 X $\frac{\pi}{4}$ X ϕ^2 = 6 X $\frac{\pi}{4}$ X 25^2 =2945.24 mm^2

The thickness of flange= D_f=140 mm

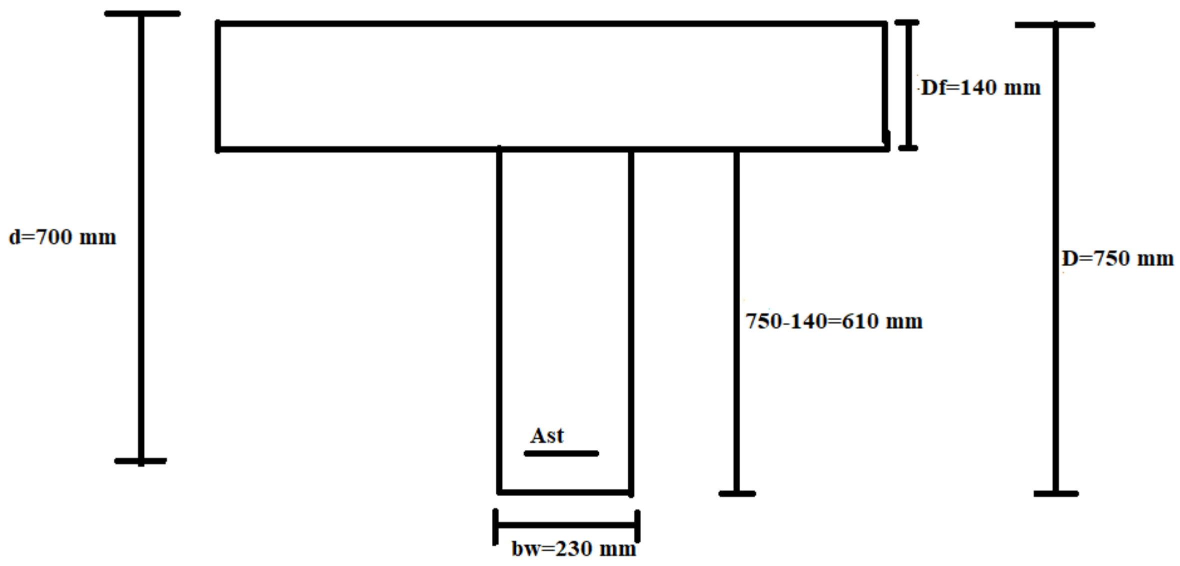
L = 10 m

$\alpha = 45^0$

superimposed load = 30 KN/m

M_{20} = Fck = 20 N/mm²

Fe 415 = Fy = 415 N/mm²



STEP 1: Loading

a) Superimposed Load= 30 KN/m
b) Self weight of beam = A_w x Density of Concrete
= 0.23 x 0.61 x 25 = 3.507 KN/m

Total Load= W=33.507 KN/m

Total Factored Load= Wu=33.507 x1.5= 50.26 KN/m

STEP 2: Maximum Shear Force

$$V_{u\max} = \frac{W_u L}{2} = \frac{50.26 \text{ X } 10}{2} = 210.34 \text{ KN}$$

STEP 3: To find Design shear force V_{uD}

$$V_{uD} = V_{u\max} - \text{ W}_u \left(\frac{b_s}{2} + d \right)$$

$$= 251.30\text{- } 50.26 \left(\frac{0.230}{2} + 0.7 \right)$$

$$V_{uD} = 210.34 \text{ KN}$$

STEP 4: **Shear taken by concrete**

The design strength of concrete varies with the percentage of steel, different grade of concrete and denoted by τ_c. It is shear stress of concrete τ_c

Percentage of steel Pt

$P_t = 100 \text{ x } \frac{A_{st}}{bd}$ (Page Number 73, Table Number 19, IS 456:2000)

$$P_t = 100 \text{ x } \frac{2945.24}{230 \text{ x } 700} = 1.524$$

To find design shear strength of concrete (τ_c **)**

(Page Number 73, Table Number 19, IS 456:2000)

Pt %	τ_c
1.50	**0.72**
1.524	**?**
1.75	**0.75**

$$\tau_c = 0.72 + \left[\frac{(0.75 - 0.72)}{(1.75 - 1.5)} \text{ X } (1.524\text{-}1.5) \right] = 0.723 \text{ N/mm}^2$$

Shear resisted by concrete

$V_{uc} = \tau_c bd$ (Page Number 73, IS 456:2000)

$V_{uc} = 0.723 \text{ x } 230 \text{ x } 700 = 116.4\text{x } 10^3 N = 116.4 \text{ KN}$

Maximum Shear resisted by concrete

$V_{ucMax} = \tau_{c\max} bd$ (Page Number 73, IS 456:2000)

$V_{ucMax} = 2.8 \text{ x } 230 \text{ x } 700 = 450.8 \text{ x } 10^3 N = 450.8 \text{ KN}$

$\tau_{c\max}$ calculated from T.No:20, Page Number 73, IS 456:2000

$V_{uD} < V_{ucMax}$

$210.34 KN < 450.8 KN$

The section is safe

STEP 5: The ultimate shear resistance of RC member with minimum stirrups

$V_{ur\,Min} = 0.4bd + \tau_c bd$

$V_{ur\,Min} = 0.4x230 \text{ x } 700 + 0.723 \text{ x } 230 \text{ x } 700$

$V_{ur\,Min} = 180.80 \text{ x } 10^3 \text{ N=180.83 KN}$

Compare V_{uD} and $V_{ur\,Min}$

$210.34 > 180.80$

*De*si*gn* of shear reinforcement is required

STEP 6: Shear force for design

$V_{us} = V_{uD} - \tau_c bd$

$V_{us} = 210.34 \text{ x } 10^3 - 0.723 \text{ x } 230 \text{ x } 700$

$V_{us} = 93.94 \text{ x } 10^3 \text{ N=93.94 KN}$

Zone I: Providing the design shear R/F

Assume 8 mm ϕ , 2 legged mild steel stirrups

$\text{Fy} = 250 \text{ N/mm}^2$

There is 1 bent up bar

$\text{V}_{\text{ub}} = 0.87 \text{ x Fy x A}_{\text{sv}} \text{ x Sin } \alpha$ (Page No: 73, IS 456:2000)

$\text{V}_{\text{ub}} = 0.87 \text{ x } 250 \text{ x } 1 \text{ x} \frac{\pi}{4} x\ 8^2 \text{ x Sin } 45^0 = 125.32 \text{ x } 10^3 N = 125.32\ KN$

Re*maining* shear for stirrups

$\text{V}_{\text{us1}} = \text{V}_{\text{us}} - \frac{\text{V}_{\text{us}}}{2} = 94.94 - \frac{94.94}{2} = 46.97 \text{ KN}$

$S_v = \frac{0.87 \text{ x Fy x A}_{\text{sv}} \text{x d}}{V_{us1}} < 0.75d$ or 300 mm (which is less)

$S_v = \frac{0.87x\ 250 \text{ x } 2\text{x} \frac{\pi}{4} x\ 8^2 x700}{46.97x10^3} < 0.75x700 = 525$ or 300 mm

$S_v = 325.861mm$ > 300 mm

Providing = $S_v = 300\ mm$

$Ls_1 = \frac{V_{u\max} - V_{ur\min}}{Wu} = \frac{251.30 - 180.80}{50.26} = 1.402\ m$ (upto this 1st *spacing* is provided)

Zone III: Providing nominal shear R/F

Because Zone II is depend upon Zone III and it is calculated as

$S_v = 0.75d$ or 300 mm (which is less)

$S_v = 0.75x700 = 525mm$ or 300 mm

$S_v = 300\ mm$

Providing = $S_v = 3000\ mm$

$Ls_3 = \frac{0.5\ \tau_c \text{ b d}}{Wu} = \frac{0.5 \text{ x } 0.723 \text{ x } 230 \text{ x } 700}{50.26x\ 10^3} = 1.158\ m$

Zone II: Providing the minimum shear R/F

$$S_v = \frac{0.87 \text{ x Fy x } A_{sv}}{0.4b} < 0.75\text{d or 300 mm (which is minimum) (page no: 48, IS 456:2000)}$$

$$S_v = \frac{0.87x \text{ 250 x 2x} \frac{\pi}{4} x \ 8^2}{0.4x \ 230} < 0.75x630 = 472.5 \text{ or 300 mm}$$

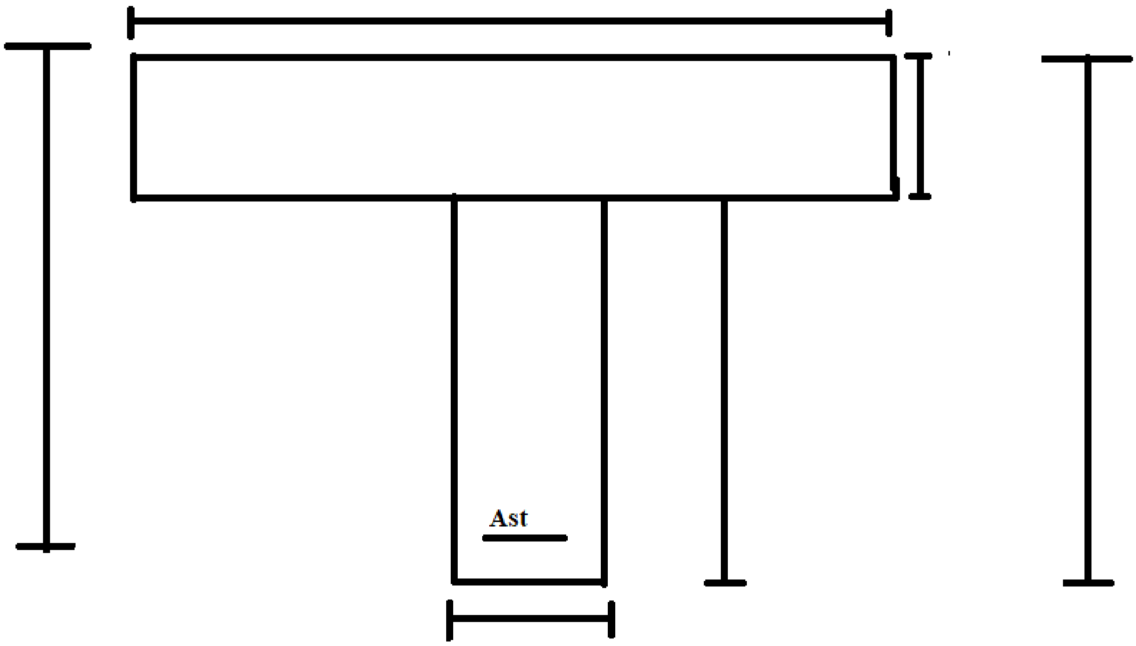
$S_v = 237.67\ mm < 300$ mm (ok)

Providing = S_v=230 *mm*

$$Ls_2 = \frac{L}{2} - L_{S1} - L_{S3}$$

$$Ls_2 = \frac{10}{2} - 1.1.402 - 1.158$$

$$Ls_2 = 2.44\ m$$



Limit States of Collapse for Bond

BOND: One of the most important assumption in the behavior of reinforced concrete structure is that there is proper ‘bond’ between concrete and reinforcing bars. The force which prevents the slippage between the two constituent materials is known as bond. In fact, bond is responsible for providing ‘strain compatibility ’ and composite action of concrete and steel. It is through the action of bond resistance that the axial stress (tensile or compressive) in a reinforcing bar can undergo variation from point to point along its length. This is required to accommodate the variation in bending moment along the length of the flexural member. When steel bars are embedded in concrete, the concrete, after setting, adheres to the surface of the bar and thus resists any force that tends to pull or push this rod. The intensity of this adhesive force bond stress. The bond stresses are the longitudinal shearing stress acting on the surface between the steel and concrete, along its length. Hence bond stress is also known as interfacial shear. Hence bond stress is the shear stress acting parallel to the reinforcing bar on the interface between the bar and the concrete.

The force which prevents the relative movement between concrete and steel is known as **bond.**

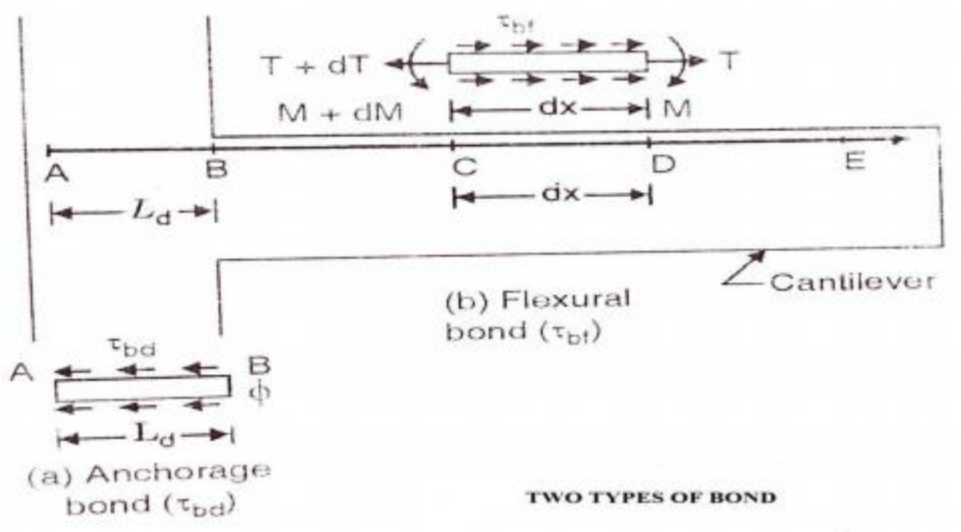
Bond Stress: It is defined as longitudinal shear acting on the surface between steel and concrete.

Types of bond:- Bond stress along the length of a reinforcing bar may be induced under two loading situations, and accordingly bond stresses are two types :

1. Flexural bond or Local bond

2. Anchorage bond or development bond

Flexural bond (τ_{bf}) is one which arises from the change in tensile force carried by the bar, along its length, due to change in bending moment along the length of the member. Evidently, flexural bond is critical at points where the shear (V=dM/dx) is significant. Since this occurs at a particular section, flexural bond stress is known as local bond stress.

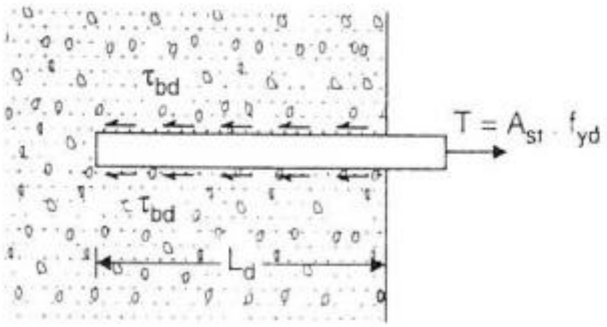


TWO TYPES OF BOND

Anchorage bond (τ_{bd}) is that which arises over the length of anchorage provided for a bar. It also arises near the end or cutoff point of reinforcing bar. The anchorage bond resists the 'pulling out' of the bar if it is in tension or 'pushing in' of the bar if it is in compression. Above figure shows the situation of anchorage bond over a length AB (=L_d). Since bond stresses are developed over specified length L_d, anchorage bond stress is also known as developed over a specified length L_d, anchorage bond stress is also known as development bond stress. Anchoring of reinforcing bars is necessary when the development length of the reinforcement is larger than the structure. Anchorage is used so that the steel's intended tension load can be reached and pop-outs will not occur. Anchorage shapes can take the form of 180^0 or 90^0 hooks.

ANCHORAGE BOND STRESS: Below shows a steel bar embedded in concrete and subjected to a tensile force T. Due to this force There will be a tendency of bar to slip out and this tendency is resisted by the bond stress developed over the perimeter of the bar, along its length of embedment.

Let us assume that average uniform bond stress is developed along the length. The required length necessary to develop full resisting force is called Anchorage length in case of axial tension or compression and development length in case of flexural tension and is denoted by L_d.



The factored to achieved increased in bond

1) Used higher grade of concrete
2) The compaction and curing should be perfect
3) Provide the adequate cover to the steel reinforcement
4) Used rough surface steel bar (i.e HYSD)
5) Used the deform or twisted bar

Development length: A **development length** can be defined as the amount of reinforcement(bar) **length** needed to be embedded or projected into the column to establish the desired bond strength between the concrete and steel Length of development of reinforcement bars. A growth length can be defined as a length of reinforcement (bar) that must be embedded or projected into the column to establish.

Reason for providing the length of development

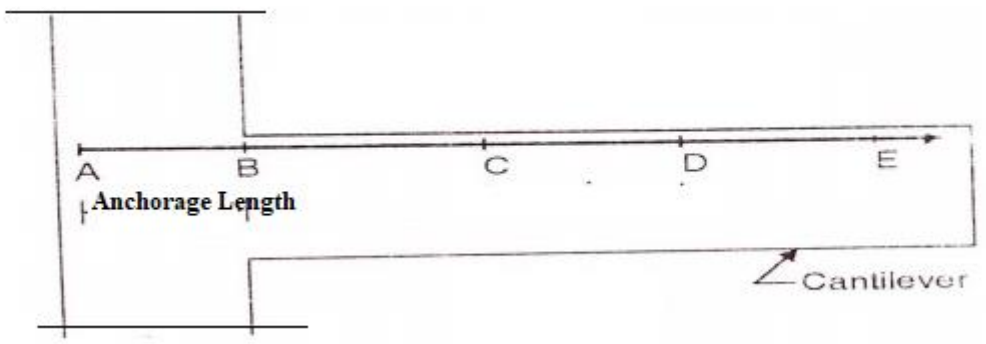
1. Develop a secure bond between the surface of the bar and the concrete so that any failure due to slippage of the bar does not occur during the final load conditions.
2. Furthermore, the additional length of the bar provided as the length of the growth is attributed to the stresses developed in any section of adjacent sections (such as the additional length of the bar provided from the beam to the column at the column beam junction).

Importance of Development Length

The provision of appropriate development is an important aspect of safe construction practices. Proper development lengths in reinforcement bars shall be provided according to the steel grade considered in the design.

Factors affecting Development Length

1) Grade of concrete: Higher the grade of concrete is used so get greater strength
2) Diameter of bar: Greater is the bar diameter less is bond resistance because length diameter having greater cracking
3) Nature of the stress: Transfer of compression from concrete increase the grip and frictional resistance.
4) The bends and hooks: The increased in bond resistance at the bend is due to increase in frictional resistance. The bend having radial components of bar tension which increased the bond and additional anchorage length.
5) Cover: If cover is not sufficient or when the horizontal distance between two parallel main reinforcement bar is less the ultimate cracking and reduction in bond strength.
6) Curtailment of bar: Curtailment of bar in tension zone create a condition of different strain adjacent bar and affecting the bond strength.
7) Grouping of the bar: The bond strength reduced for the bundles bar due to reduction in surface area.



Consider a cantilever beam of uniform c/s

Let

$\phi =$ Diameter of bar

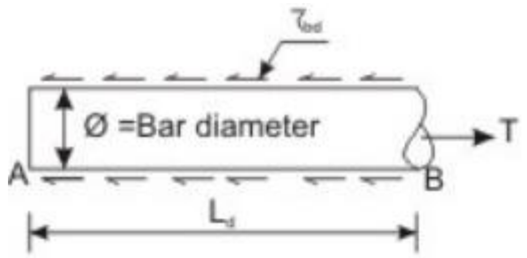
T= Maximum tension in the bar

The force acting on bar

F= 0.87 Fy A_{st}

$$F = 0.87 \text{ x Fy x } \frac{\pi}{4}\phi^2 \qquad (1)$$

This force must be transfer from steel to concrete through bond along acting over the perimeter of the bar in length AB= L_d



Free body diagram
of segment AB

τ_{bd} = bond design stress acting on the surface area

Surface Area = $\pi\ \phi\ L_d$

Force =Stress x area

The force transferred to the concrete through the bond

$$F=\tau_{bd}\ \pi\ \phi\ L_d \qquad (2)$$

For equilibrium equate equation 1 and 2

$$0.87 \text{ x Fy x } \frac{\pi}{4}\phi^2 = \tau_{bd}\ \pi\ \phi\ L_d$$

$$L_d = \frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}}$$ Page No: 42 , IS 456:2000, C.No: 26.2.1

$$L_d = k\phi$$

$$k = \frac{0.87 \text{ x Fy}}{4\tau_{bd}}$$

k = Development length factor

IS 456:2000, P. No: 43, C. No: 26.2.1.1

DESIGN BOND STRESS:-

The design bond stress in limit state method for plain bars in tension shall be as given below

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
Design bond stress τ_{bd} (N/mm^2)	1.2	1.4	1.5	1.7	1.9

Design bond stresses for deformed bars in tension : For deformed bars conforming to IS 1786. These values shall be increased by 60%.

Design bond stress for bars in compression : For bars in compression, the values of bond stress for in tension shall be increased by 25%.

1) **To calculate development length of M_{20} and Fe 250**

Solution:

F_{ck} = 20 N/mm^2

Fy= 250 N/mm^2

τ_{bd} = 1.2 N/mm^2 (P. No: 43 , T. No: 26.2.1.1, IS 456: 2000)

$$L_d = \frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}}$$ Page No: 42 , IS 456:2000, C.No: 26.2.1

$$L_d = \frac{0.87 \text{ x } 250 \text{ X } \phi}{4 \text{ x}1.2} = 45.31\phi \cong 46\phi$$

2) **To calculate development length of M_{20} and Fe 415**

Solution:

F_{ck} = 20 N/mm^2

Fy= 415 N/mm^2

τ_{bd} = 1.2 N/mm^2 (P. No: 43 , T. No: 26.2.1.1, IS 456: 2000)

τ_{bd} = 1.2 x 1.6 =1.92 N/mm^2 (P. No: 43 , T. No: 26.2.1.1, IS 456: 2000)

This value is increased by 60% by HYSD bar i.e. Fy 415 and Fy 500

$$L_d = \frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}}$$ Page No: 42 , IS 456:2000, C.No: 26.2.1

$$L_d = \frac{0.87 \text{ x } 415 \text{ X } \phi}{4 \text{ x}1.92} = 47.01\phi \cong 47\phi$$

3) **To calculate development length of M_{20} and Fe 250 for bar in compression**

Solution:

F_{ck} = 20 N/mm^2

Fy= 250 N/mm^2

τ_{bd} = 1.2 N/mm^2 (P. No: 43 , T. No: 26.2.1.1, IS 456: 2000)

τ_{bd} = 1.2 x 1.25 =1.5 N/mm^2 (P. No: 43 , T. No: 26.2.1.1, IS 456: 2000)

This value is increased by 25% when bar in compression bar

$$L_d = \frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}}$$ Page No: 42 , IS 456:2000, C.No: 26.2.1

$$L_d = \frac{0.87 \text{ x } 250 \text{ X } \phi}{4 \text{ x} 1.50} = 36.25\phi \cong 37\phi$$

4) **To calculate development length of M_{20} and Fe 415 when bar is in tension as well as compression**
Solution:
F_{ck} = 20 N/mm^2
Fy= 415 N/mm^2
τ_{bd} = 1.2 N/mm^2 (P. No: 43 , T. No: 26.2.1.1, IS 456: 2000)
τ_{bd} = 1.2 x 1.6 =1.92 N/mm^2 (Tension)
τ_{bd} = 1.92 x 1.25 = 2.4 N/mm^2 (Compression)

$$L_d = \frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}}$$ Page No: 42 , IS 456:2000, C.No: 26.2.1

$$L_d = \frac{0.87 \text{ x } 415 \text{ X } \phi}{4 \text{ x} 2.4} = 37.61\phi \cong 38\phi$$

DEVELOPMENTS LENGTH REQUIREMENTS AT SIMPLE SUPPORTS :

The code stipulates that at the simple supports (and at the point of inflection), the positive moment tension reinforcement shall be limited to a diameter such that

$$L_d = \frac{M_1}{V} + l_0$$ (IS 456:2000, P No: 44, C.No: 26.2.3.3)

Where L_d = developments length computed for design stress f_{yd} (=0.87 fy) from Eqn M_1= Moments resistance of the section assuming all reinforcement at the section to be stressed to f_{yd}(= 0.87 fy)

V= Shear force at the section due to design loads

Lo= sum of anchorage beyond the centre of supports and the equivalent anchorage value of any hook or mechanical anchorage at the simple support (At the point of inflexion, Lo is limited to d or 12ϕ whichever is greater).

In simple support or beam resting on wall or column the reaction induced compressive stress. So bond resistant increases. The IS Code allow 30% increased in the value of M_1/V

$$L_d = 1.3\frac{M_1}{V} + l_0$$ (IS 456:2000, P No: 44, C.No: 26.2.3.3)

Anchorage value of Bends and hooks - Bends and hooks shall conform to IS 2502

(IS 456:2000 , P No: 43 , C. No: 26.2.2.1)

1) Bends-The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 45^0 bend subject to a maximum of 16 times the diameter of the bar (ϕ).

Anchorage value for 45^0 bend = 4 ϕ

Anchorage value for 90^0 bend = 8 ϕ

Anchorage value for 135^0 bend =12 ϕ

Anchorage value for 180^0 bend =16 ϕ

Where ϕ =Diameter of bar

2) Hooks-The anchorage value of a standard U-type hook shall be equal to 16 times the diameter of the bar.

For mild steel i.e Fe 250 = n= 2

For HYSD bar i.e Fe 415 and Fe 500= n=4

1) **Determine the anchorage value 180^0 bend and grade of steel is Fe 250**

Solution: θ =180^0

Fy= 250 N/mm^2

Anchorage Length = L_o= X_0 + 16 ϕ (180^0 bend)

$$X_0 = \frac{b_s}{2} - X_1 - (n+1)\phi$$

$X_1 =$ Clear Cover

$b_s =$ Width of support

n=2 (Fe 250)

$$X_0 = \frac{b_s}{2} - X_1 - (2+1)\phi$$

$$X_0 = \frac{b_s}{2} - X_1 - 3\phi$$

$$L_0 = X_0 + 16\phi$$

$$L_0 = \frac{b_s}{2} - X_1 - 3\phi + 16\phi$$

$$L_0 = \frac{b_s}{2} - X_1 + 13\phi$$

2) **Determine the anchorage value 180^0 bend and grade of steel is Fe 415**

Solution: θ =180^0

Fy= 415 N/mm^2

Anchorage Length = L_o= X_0 + 16 ϕ (180^0 bend)

$$X_0 = \frac{b_s}{2} - X_1 - (n+1)\phi$$

$X_1 =$ Clear Cover

$b_s =$ Width of support

n=4 (Fe 415)

$$X_0 = \frac{b_s}{2} - X_1 - (4+1)\phi$$

$$X_0 = \frac{b_s}{2} - X_1 - 5\phi$$

$$L_0 = X_0 + 16\phi$$

$$L_0 = \frac{b_s}{2} - X_1 - 5\phi + 16\phi$$

$$L_0 = \frac{b_s}{2} - X_1 + 11\phi$$

3) Determine the anchorage value 90^0 bend and grade of steel is Fe 250

Solution: θ **=90^0**

Fy= 250 N/mm^2

Anchorage Length = L_o= X_0 + 8 ϕ (90^0 bend)

$$X_0 = \frac{b_s}{2} - X_1 - (n+1)\phi$$

$X_1 =$ Clear Cover

$b_s =$ Width of support

n=2 (*Fe* 250)

$$X_0 = \frac{b_s}{2} - X_1 - (2+1)\phi$$

$$X_0 = \frac{b_s}{2} - X_1 - 3\phi$$

$$L_0 = X_0 + 8\phi$$

$$L_0 = \frac{b_s}{2} - X_1 - 3\phi + 8\phi$$

$$L_0 = \frac{b_s}{2} - X_1 + 5\phi$$

4) Determine the anchorage value 90^0 bend and grade of steel is Fe 415

Solution: θ **=90^0**

Fy= 415 N/mm^2

Anchorage Length = L_o= X_0 + 8 ϕ (90^0 bend)

$$X_0 = \frac{b_s}{2} - X_1 - (n+1)\phi$$

$X_1 =$ Clear Cover

$b_s =$ Width of support

n=4 (*Fe* 415)

$$X_0 = \frac{b_s}{2} - X_1 - (4+1)\phi$$

$$X_0 = \frac{b_s}{2} - X_1 - 5\phi$$

$$L_0 = X_0 + 8\phi$$

$$L_0 = \frac{b_s}{2} - X_1 - 5\phi + 8\phi$$

$$L_0 = \frac{b_s}{2} - X_1 + 3\phi$$

1) **A simply supported beam is 250 mm X 500 mm overall having 2 bars of 20 mm diameter going into the support, if the shear force at the support is 135 KN as working load. Find the anchorage length, the clear cover is 25 mm. Use M_{20} and Fe 415.**

Solution:-

Given Data:- b = 250 mm

D = 500 mm

ϕ = 20 mm

No of bar = 2

Clear cover = 25 mm

d'= Effective cover = Clear cover + $\frac{\phi}{2}$

d'=Effective cover = 25 + $\frac{20}{2}$ = 35 mm

Effective depth =d = D - d' = 500 -35 = 465 mm

Ast = 2 X $\frac{\pi}{4}$ X ϕ^2 = 2 X $\frac{\pi}{4}$ X 20^2 = 628.32 mm^2

Working Shear Force= **135 KN**

Factored Shear Force=V= **135 x 1.5 = 202.5 KN**

M_{20} = Fck = 20 N/mm^2

Fe 415 = Fy = 415 N/mm^2

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$$0.36 Fck Xub = 0.87\ Fy\ Ast$$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b}$$

$$Xu = \frac{0.87 \text{ x } 415 \text{ x } 628.32}{0.36 \text{ x } 20 \text{ x } 250} = 126.03 \text{ mm}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 465 = 223.20 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu\ max$$

$$126.03 \prec 223.20$$

then section is under reinforced

STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

Mu = 0.87 x 415 x 628.32 (465- 0.42 x 126.03) = 93.48 X 10^6 Nmm = 93.48 KNm

STEP 5: To find anchorage length

$$L_d = 1.3\frac{M_1}{V} + l_0$$ (IS 456:2000, P No: 44, C.No: 26.2.3.3)

$$L_d = \frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}}$$ Page No: 42 , IS 456:2000, C.No: 26.2.1

$\tau_{bd} = 1.2\text{x } 1.6\text{x}1.25 = 2.4 \text{ N/mm}^2$

60% for HYSD bar

25% for compression zone

$$L_d = \frac{0.87 \text{ x } 415 \text{ X } 20}{4 \text{ x } 2.4} = 752.19mm$$

$$752.19 = \left(1.3x\frac{93.48 \text{ x } 10^6}{202.5 \text{ x } 10^3}\right) + l_0$$ (IS 456:2000, P No: 44, C.No: 26.2.3.3)

$$752.19 = 600.11 + l_0$$

$$l_0 = 752.19 - 600.11 = 152.07 \text{ mm}$$

2) **A simply supported beam is 300 mm X 450 mm overall having 3 bars of 14 mm diameter going into the support, if the shear force at the support is 150 KN as working load. Find the anchorage length, the clear cover is 25 mm. If the width of support is 150 mm. If it is suggested to have straight bar. How much will be diameter of bar? Use M_{20} and Fe 415.**

Solution:-

Given Data:- b = 300 mm

D = 450 mm

ϕ = 14 mm

No of bar = 3

Clear cover = 25 mm

d'= Effective cover = Clear cover + $\frac{\phi}{2}$

d'=Effective cover = 25 + $\frac{14}{2}$ = 32 mm

Effective depth =d = D - d' = 450 -32 = 418 mm

Ast = 3 X $\frac{\pi}{4}$ X ϕ^2 = 3 X $\frac{\pi}{4}$ X 14^2 = 461.81 mm^2

Working Shear Force= **150KN**

Factored Shear Force= V=**150 x 1.5 = 225 KN**

M_{20} = Fck = 20 N/mm²

Fe 415 = Fy = 415 N/mm²

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

0.36FckXub = 0.87 Fy Ast

$$\text{Xu} = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck b}}$$

$$Xu = \frac{0.87 \text{ x } 415 \text{ x } 461.81}{0.36 \text{ x } 20 \text{ x } 300} = 77.19 \text{ mm}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 418 = 200.64 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu \max$$

$$77.19 \prec 200.64$$

then section is under reinforced

STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

Mu = 0.87 x 415 x 461.81 (418- 0.42 x 77.19) = 64.29 X 10^6 Nmm = 64.29 KNm

STEP 5: To find anchorage length

Since bar is going into the support i.e. the bar are going in compression

$$L_d = 1.3\frac{M_1}{V} + l_0$$ (IS 456:2000, P No: 44, C.No: 26.2.3.3)

$$L_d = \frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}}$$ Page No: 42 , IS 456:2000, C.No: 26.2.1

$\tau_{bd} = 1.2\text{x } 1.6\text{x}1.25 = 2.4 \text{ N/mm}^2$

60% for HYSD bar

25% for compression zone

$$L_d = \frac{0.87 \text{ x } 415 \text{ X } 14}{4 \text{ x } 2.4} = 526.53$$

$$526.53 = \left(1.3x\frac{64.29 \text{ x } 10^6}{225 \text{ x } 10^3}\right) + l_0$$ (IS 456:2000, P No: 44, C.No: 26.2.3.3)

$526.53 = 371.45 + l_0$

$l_0 = 526.53 - 371.45 = 155.07 \text{ mm}$

For straight bar

$$L_0 = \frac{b_s}{2} - X_1$$

$b_s = 150$ mm

$X_1 = 25$ mm

$$L_0 = \frac{150}{2} - 25 = 50 \text{ mm}$$

$$L_d = 1.3\frac{M_1}{V} + l_0$$ (IS 456:2000, P No: 44, C.No: 26.2.3.3)

$$\frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}} = 1.3\frac{M_1}{V} + l_0$$ (IS 456:2000, P No: 44, C.No: 26.2.3.3)

$$\frac{0.87 \text{ x } 415\ \phi}{4x2.4} = 1.3\frac{64.29x10^6}{225x10^3} + 50$$

$$\phi = 11.21mm$$

Providing 10 mm diameter of bar

$$\text{Number of bar} = N = \frac{3 \text{ x } \frac{\pi}{4} x14^2}{\frac{\pi}{4} x10^2} = 5.88 \cong 6$$

3) A continuous beam 250 mm X 400 mm overall depth carries 3 bars of 16 mm diameter beyond the point of inflection in sagging moment. If factored shear force at inflection is 150 KN. Check if the bar is safe in bond. The clear cover is 25 mm. Use M_{20} and Fe 250.

Solution:-

Given Data:- b = 250 mm

D = 400 mm

ϕ = 16 mm

No of bar = 3

Clear cover = 25 mm

d'= Effective cover = Clear cover + $\frac{\phi}{2}$

d'=Effective cover = 25 + $\frac{16}{2}$ = 33 mm

Effective depth =d = D - d' = 400 -33 = 367 mm

Ast = 3 X $\frac{\pi}{4}$ X ϕ^2 = 3 X $\frac{\pi}{4}$ X 16^2 = 603.19 mm^2

Factored Shear Force=V=**150 KN**

M_{20} = Fck = 20 N/mm^2

Fe 250 = Fy = 250 N/mm^2

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$$0.36 Fck Xub = 0.87 \text{ Fy Ast}$$

$$Xu = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck b}}$$

$$Xu = \frac{0.87 \times 250 \times 603.19}{0.36 \times 20 \times 250} = 72.88 \text{ mm}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.53 x367 = 194.51 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu\max$$

$$72.99 \prec 194.51$$

then section is under reinforced

STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

$$Mu = 0.87 \text{ Fy Ast } (d - 0.42 \text{ Xu})$$

$$Mu = 0.87 \times 250 \times 603.19 \, (367 - 0.42 \times 72.99) = 44.13 \times 10^6 \text{ Nmm} = 44.13 \text{ KNm}$$

STEP 5: To find anchorage length

Since bar is not going into the support and point of inflection (Contraflexure)

IS 456:2000, P.No: 44, C. No: 26.2.3.4

$L_0 = d \ or \ 12\phi$ (Whichever is greater)

$L_0 = 367 \ or \ 12x16 = 192$ (Whichever is greater)

$L_0 = 367$ mm

$L_d = \frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}}$ Page No: 42 , IS 456:2000, C.No: 26.2.1

$\tau_{bd} = 1.2 \text{ N/mm}^2$

$L_d = \frac{0.87 \text{ x } 250 \text{ X } \phi}{4 \text{ x } 1.2} = 45.31\phi$

$L_d = \frac{M_1}{V} + l_0$

$45.31\phi = \frac{44.13 \text{ x } 10^6}{150 \text{ x } 10^3} + 367$

$\phi = 14.59 \text{ mm} < 16 \text{ mm}$

Hence 16 diameter bar is unsafe for the bond

Provide $\phi = 12mm$

$$Number \text{ of bar} = \frac{3x\frac{\pi}{4}x16^2}{\frac{\pi}{4}x12^2} = 5.3 \cong 6$$

4) **A continuous beam 300 mm X 500 mm effective depth carries 3 bars of 20 mm diameter beyond the point of inflection in sagging moment. If factored shear force at inflection is 175 KN. Check if the bar is safe in bond. Use M_{20} and Fe 250.**

Solution:-

Given Data:- b = 250 mm

d = 500 mm

ϕ = 20 mm

No of bar = 3

Ast = 3 X $\frac{\pi}{4}$ X ϕ^2 = 3 X $\frac{\pi}{4}$ X 20^2 = 942.48 mm^2

Factored Shear Force=V=**175 KN**

M_{20} = Fck = 20 N/mm^2

Fe 250 = Fy = 250 N/mm^2

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

0.36FckXub = 0.87 Fy Ast

$$Xu = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck b}}$$

$$Xu = \frac{0.87 \text{ x } 250 \text{ x } 942.48}{0.36 \text{ x } 20 \text{ x } 300} = 94.90 \text{ mm}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.53 x 500 = 265 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu \max$$

$$94.90 \prec 265$$

then section is under reinforced

STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

$$Mu = 0.87 \text{ Fy Ast } (\text{d- } 0.42 \text{ Xu })$$

$$Mu = 0.87 \text{ x } 250 \text{ x } 942.48 (500\text{- } 0.42 \text{ x } 94.90) = 94.32 \text{ X } 10^6 \text{ Nmm} = 94.32 \text{ KNm}$$

STEP 5: To find anchorage length

Since bar is not going into the support and point of inflection (Contraflexure)

IS 456:2000, P.No: 44, C, No: 26.2.3.4

$L_0 = d \ or \ 12\phi$ (Whichever is greater)

$L_0 = 500 \ or \ 12x20 = 240$ (Whichever is greater)

$L_0 = 500$ mm

$$\text{L}_\text{d} = \frac{0.87 \text{ x Fy } \phi}{4\tau_{bd}}$$ Page No: 42 , IS 456:2000, C.No: 26.2.1

$\tau_{bd} = 1.2 \text{ N/mm}^2$

$$\text{L}_\text{d} = \frac{0.87 \text{ x } 250 \text{ X } \phi}{4 \text{ x } 1.2} = 45.31\phi$$

$$\text{L}_\text{d} = \frac{M_1}{V} + l_0$$

$$45.31\phi = \frac{94.32 \text{ x } 10^6}{175 \text{ x } 10^3} + 500$$

$\phi = 22.93$ mm >20 mm

Hence 20 diameter bar is safe for the bond

UNIT 2

"DESIGN OF BEAM"

Limit state of collapse for flexure :- Page No: 69 and clause No: 38.1

Assumption made in Limit state of collapse for flexure:

1) Plane sections normal to the axis remain plane after bending.
2) The maximum strain in concrete at the outermost compression fibre is taken as 0.003 5 in bending.
3) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stressstrain curve is given in Fig. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor y, = 1.5 shall be applied in addition to this.

NOTE - For the stress-strain curve in Fig. the design stress block parameters are as follows
Area of stress block = 0.36 F_{CK} X_U
Depth of centre of compressive force = 0.425 X_U
from the extreme fibre in compression
Where
F_{CK} = characteristic compressive strength of concrete
X_U = depth of neutral axis.

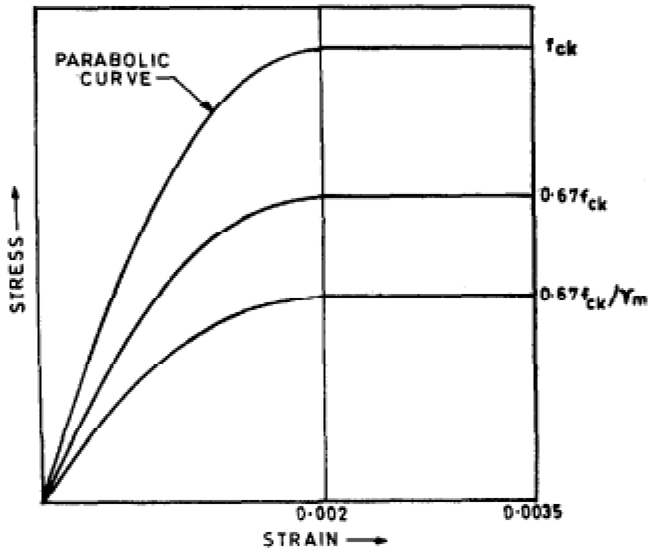
4) The tensile strength of the concrete is ignored.
5) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in Fig.. For design purposes the partial safety factor Υ_m, equal to 1.15 shall be applied.

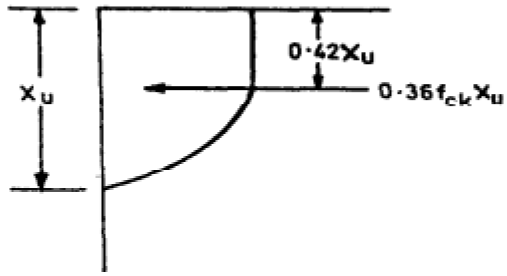
6) The maximum strain in the tension reinforcement in the section at failure shall not be less than:

$F_y/(1.15\ E_s) + 0.002$

where
F_y = characteristic strength of steel, and
E_s = modulus of elasticity of steel.

whe





Stress Strain Curve for Concrete Stress Block Parameters

For concrete stress-strain diagram shown in figure the curve from 0 to 0.002 the stain is parabolic and for 0.002 to 0.0035 strain is constant. The compressive strength in concrete is assumed to 0.67 F_{ck} and further is divided by partial safety factor 1.5.

The compressive strength in concrete for design purpose is $= 0.67\ F_{ck} /\ \Upsilon_m$

$= 0.67\ F_{ck} /\ 1.5$

$= 0.446\ F_{ck}$

For steel design purpose the yield stress in steel = $F_y /\ \Upsilon_m$

$= F_y / 1.15 = 0.87\ F_y$

Stress = Force / area = F / A_{st}

Force = Stress / area

F = Stress / A_{st}

Force in Steel = 0.87 F_y A_{st}

SECTION

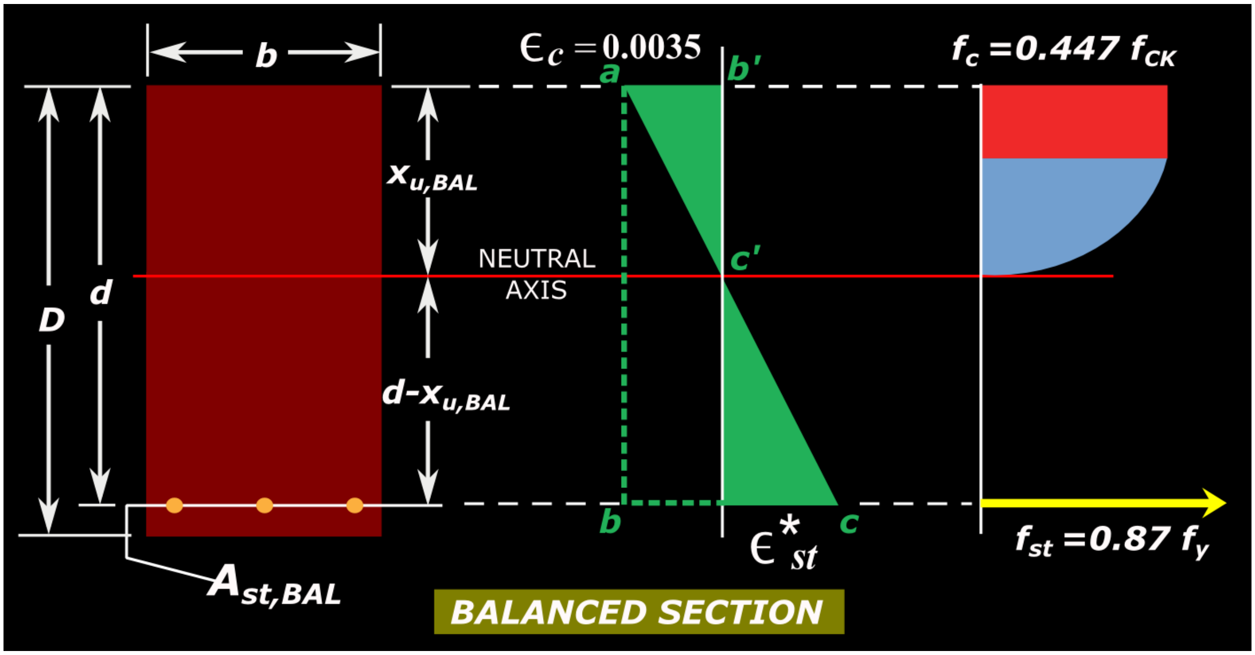
1) Balanced Section

2) Under reinforced Section

3) Over reinforced Section

1) Balanced Section

When ratio of steel at concrete in section in such that strain in steel and concrete reaches its maximum value. At the same time section is called balance section.

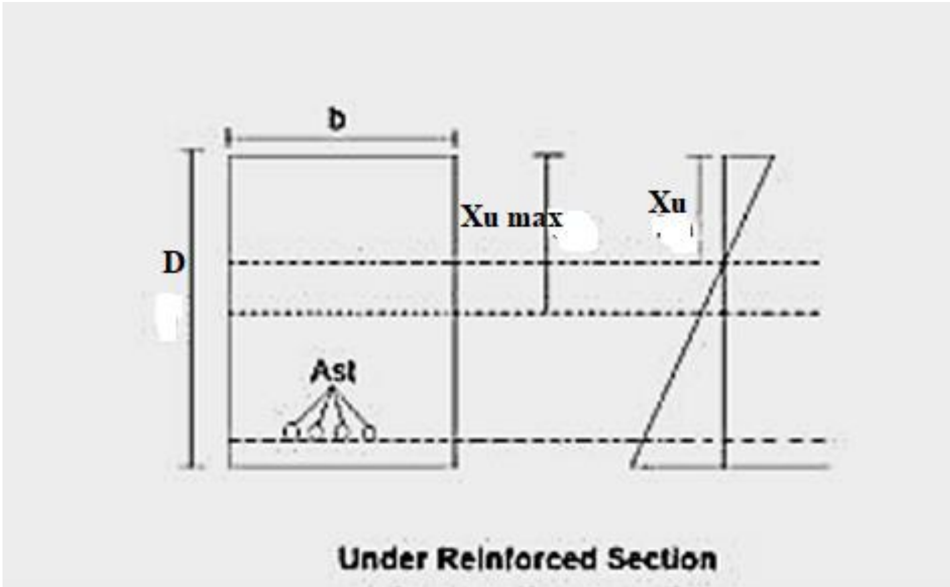


The percentage of steel in this section is caused critical percentage of steel or limiting percentage of steel and denoted by Pt limit and depth of neutral axis called depth of critical neutral axis.

2) Under reinforced Section

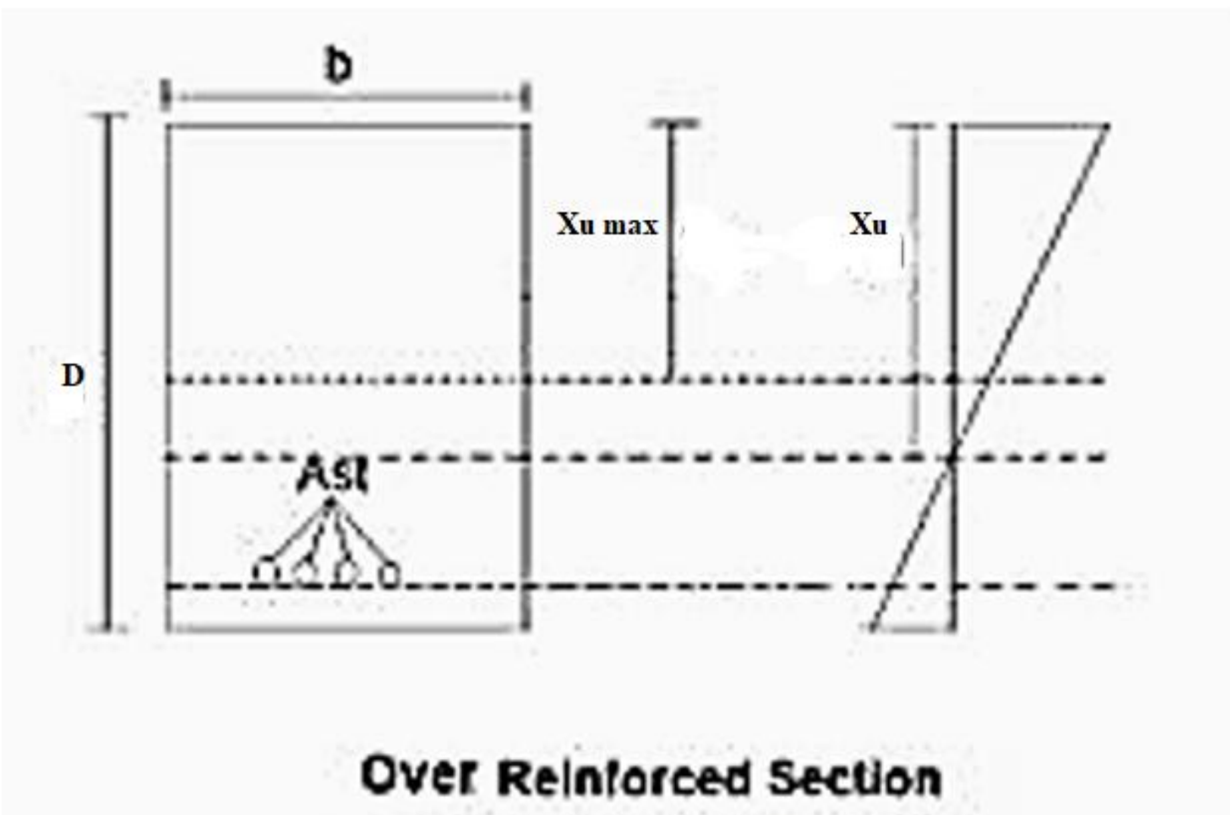
The percentage of steel in this section is less than critical percentage of steel or limiting percentage of steel, this section is called under reinforced section.

In under reinforced section steel failed first. In under reinforced section amount of steel is less than the balance section. The neutral axis more above neutral axis to satisfied equilibrium condition .



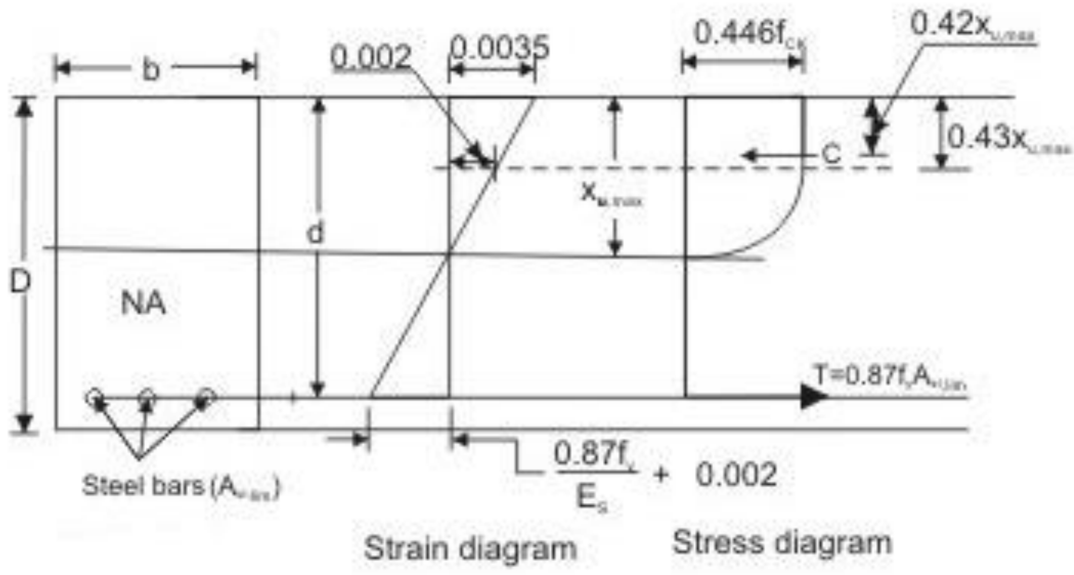
3) Over reinforced Section

The percentage of steel in this section is more than critical percentage of steel or limiting percentage of steel, this section is called over reinforced section. In over reinforced section amount of steel is more than the balance section. The neutral axis below the neutral axis to satisfied equilibrium condition .



NOTE:- Over reinforced Section is not permitted in limit state

SINGLY REINFORCED SECTION:- A section in which steel is provided on tension side only.



Where ,

D= Over all depth

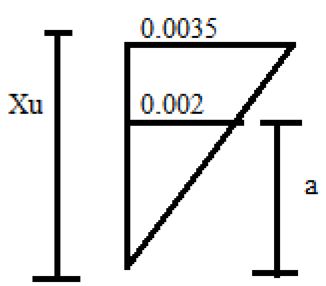
d= effective depth

b = width of section

d' = effective cover

Ast = area of steel in tension

F_{CK} = characteristic compressive strength of concrete
X_U = depth of neutral axis.
F_y = characteristic strength of steel



By similar triangle law

$$\frac{Xu}{0.0035} = \frac{a}{0.002}$$

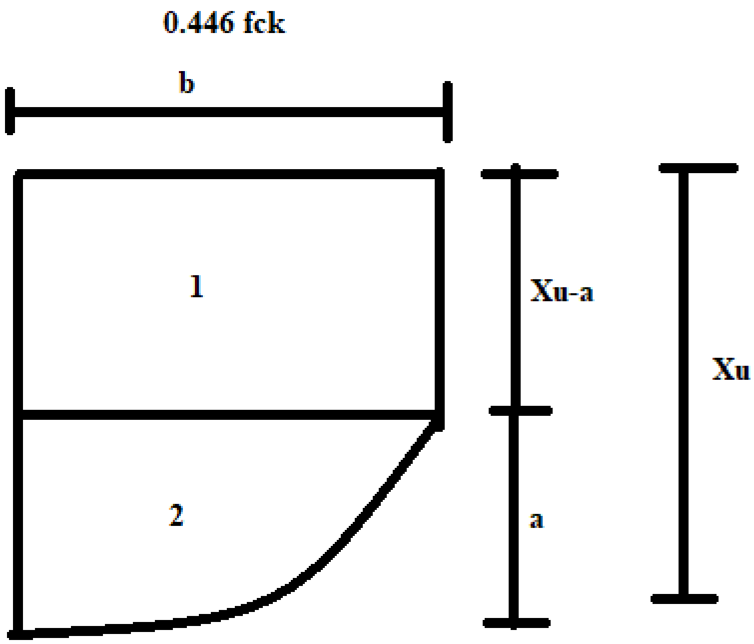
$$\frac{Xu}{0.0035}\ 0.002 = a$$

a= 0.57 Xu .... (1)

For the design purpose maximum stresses in concrete = $0.67\ F_{ck} / \gamma_m$

$= 0.67\ F_{ck} / \ 1.5$

$= 0.446\ F_{ck}$



Depth of rectangle = Xu - a

= Xu - 0.57 Xu = 0.43 Xu From (1)

Total compressive force = Area of stress block X width of section

Cu = (Area of rectangle + Area of parabola) X b

Cu = (0.43 Xu X 0.446 Fck) + (0.57 Xu X (2/3) 0.446 Fck) X b

Cu = 0.36 Fck Xu b

The position of Cu from extreme fiber

$\overline{Y}$ = 0.42 Xu

Design stress in steel = $F_y / \Upsilon_m = F_y / 1.15 = 0.87\ F_y$

Total tensile force Tu = Area of steel X Stress in steel

Tu = Ast X 0.87 F_y

Position of neutral axis

Tensile force = Compressive force

0.87 F_y Ast = 0.36 Fck Xu b

$$Xu = \frac{0.87FyAst}{0.36Fckb}$$

Dividing d on both sides

$$\frac{Xu}{d} = \frac{0.87FyAst}{0.36Fckbd}$$ Page No 96

First principle calculate the values

1) $Kumax = \frac{Xumax}{d} = \frac{700}{1100+0.87\ Fy}$

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

3) $Ptmax = \frac{0.36Fck}{0.87Fy} Ku \max X100$

1) From first principle calculate the values of Ku max, Ru max and Pt max for Fe 250 & M_{20}

Solution :- M_{20} = Fck = 20 N/mm²

Fe 250 = Fy = 250 N/mm²

1) $Kumax = \frac{Xumax}{d} = \frac{700}{1100+0.87\ Fy}$

$Kumax = \frac{Xumax}{d} = \frac{700}{1100+0.87\ x250}$ = 0.531

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

Rumax or Mu lilmit= 0.36 x 20 x 0.531 (1- 0.42 x 0.531) = 2.97

3) $Ptmax = \frac{0.36Fck}{0.87Fy} Ku \max X100$

$Ptmax = \frac{0.36\ x\ 20}{0.87\ x\ 250} x0.531X100$ = 1.757 %

2) From first principle calculate the values of Ku max, Ru max and Pt max for Fe 250 & M_{25}

Solution :- M_{25} = Fck = 25 N/mm²

Fe 250 = Fy = 250 N/mm²

1) $\text{Kumax}=\frac{\text{Xumax}}{d}=\frac{700}{1100+0.87\ \text{Fy}}$

$\text{Kumax}=\frac{\text{Xumax}}{d}=\frac{700}{1100+0.87\ \text{x}250}$ = 0.531

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

Rumax or Mu lilmit= 0.36 x 25 x 0.531 (1- 0.42 x 0.531) = 3.713

3) $\text{Ptmax}=\frac{0.36\text{Fck}}{0.87\text{Fy}}Ku\max X100$

$\text{Ptmax}=\frac{0.36\text{ x }25}{0.87\text{ x }250}x0.531X100$ = 2.197 %

3) From first principle calculate the values of Ku max, Ru max and Pt max for Fe 250 & M_{30}

Solution :- M_{30} = Fck = 30 N/mm²

Fe 250 = Fy = 250 N/mm²

1) $\text{Kumax}=\frac{\text{Xumax}}{d}=\frac{700}{1100+0.87\ \text{Fy}}$

$\text{Kumax}=\frac{\text{Xumax}}{d}=\frac{700}{1100+0.87\ \text{x}250}$ = 0.531

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

Rumax or Mu lilmit= 0.36 x 30 x 0.531 (1- 0.42 x 0.531) = 4.46

3) $\text{Ptmax}=\frac{0.36\text{Fck}}{0.87\text{Fy}}Ku\max X100$

$$\text{Ptmax}=\frac{0.36 \text{ x } 30}{0.87 \text{ x } 250} x 0.531 X 100 = 2.64\ \%$$

4) From first principle calculate the values of Ku max, Ru max and Pt max for Fe 415 & M_{20}

Solution :- M_{20} = Fck = 20 N/mm²

Fe 415 = Fy = 415 N/mm²

1) $$\text{Kumax}=\frac{\text{Xumax}}{\text{d}}=\frac{700}{1100+0.87\text{ Fy}}$$

$$\text{Kumax}=\frac{\text{Xumax}}{\text{d}}=\frac{700}{1100+0.87 \text{ x } 415} = 0.48$$

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

Rumax or Mu lilmit= 0.36 x 20 x 0.48 (1- 0.42 x 0.48) = 2.76

3) $$\text{Ptmax}=\frac{0.36\text{Fck}}{0.87\text{Fy}} Ku \max X 100$$

$$\text{Ptmax}=\frac{0.36 \text{ x } 20}{0.87 \text{ x } 415} x 0.48 X 100 = 0.96\ \%$$

5) From first principle calculate the values of Ku max, Ru max and Pt max for Fe 415 & M_{25}

Solution :- M_{25} = Fck = 25 N/mm²

Fe 415 = Fy = 415 N/mm²

1) $$\text{Kumax}=\frac{\text{Xumax}}{\text{d}}=\frac{700}{1100+0.87\text{ Fy}}$$

$$\text{Kumax}=\frac{\text{Xumax}}{\text{d}}=\frac{700}{1100+0.87 \text{ x } 415} = 0.48$$

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

Rumax or Mu lilmit= 0.36 x 25 x 0.48 (1- 0.42 x 0.48) = 3.45

3) $\text{Ptmax}=\dfrac{0.36\text{Fck}}{0.87\text{Fy}}\,Ku\,\text{max}\;X100$

$\text{Ptmax}=\dfrac{0.36 \text{ x } 25}{0.87 \text{ x } 415}\,x0.48X100$ = 1.20 %

6) From first principle calculate the values of Ku max, Ru max and Pt max for Fe 415 & M_{30}

Solution :- M_{30} = Fck = 30 N/mm^2

Fe 415 = Fy = 415 N/mm^2

1) $\text{Kumax}=\dfrac{\text{Xumax}}{\text{d}}=\dfrac{700}{1100+0.87 \text{ Fy}}$

$\text{Kumax}=\dfrac{\text{Xumax}}{\text{d}}=\dfrac{700}{1100+0.87 \text{ x } 415}$ = 0.48

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

Rumax or Mu lilmit= 0.36 x 30 x 0.48 (1- 0.42 x 0.48) = 4.14

3) $\text{Ptmax}=\dfrac{0.36\text{Fck}}{0.87\text{Fy}}\,Ku\,\text{max}\;X100$

$\text{Ptmax}=\dfrac{0.36 \text{ x } 30}{0.87 \text{ x } 415}\,x0.48X100$ = 1.44 %

7) From first principle calculate the values of Ku max, Ru max and Pt max for Fe 500 & M_{20}

Solution :- M_{20} = Fck = 20 N/mm^2

Fe 500 = Fy = 500 N/mm^2

1) $\text{Kumax}=\dfrac{\text{Xumax}}{\text{d}}=\dfrac{700}{1100+0.87 \text{ Fy}}$

$\text{Kumax}=\dfrac{\text{Xumax}}{\text{d}}=\dfrac{700}{1100+0.87 \text{ x } 500}$ = 0.46

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

Rumax or Mu lilmit= 0.36 x 20 x 0.46 (1- 0.42 x 0.46) = 2.67

3) $\text{Ptmax}=\frac{0.36\text{Fck}}{0.87\text{Fy}} Ku \max X100$

$\text{Ptmax}=\frac{0.36 \text{ x } 20}{0.87 \text{ x } 500} x0.46X100 = 0.76\ \%$

8) From first principle calculate the values of Ku max, Ru max and Pt max for Fe 500 & M_{25}

Solution :- M_{25} = Fck = 25 N/mm^2

Fe 500 = Fy = 500 N/mm^2

1) $\text{Kumax}=\frac{\text{Xumax}}{\text{d}}=\frac{700}{1100+0.87 \text{ Fy}}$

$\text{Kumax}=\frac{\text{Xumax}}{\text{d}}=\frac{700}{1100+0.87 \text{ x } 500} = 0.46$

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

Rumax or Mu lilmit= 0.36 x 25 x 0.46 (1- 0.42 x 0.46) = 3.34

3) $\text{Ptmax}=\frac{0.36\text{Fck}}{0.87\text{Fy}} Ku \max X100$

$\text{Ptmax}=\frac{0.36 \text{ x } 25}{0.87 \text{ x } 500} x0.46X100 = 0.95\ \%$

9) From first principle calculate the values of Ku max, Ru max and Pt max for Fe 500 & M_{30}

Solution :- M_{30} = Fck = 30 N/mm^2

Fe 500 = Fy = 500 N/mm^2

1) $\text{Kumax}=\frac{\text{Xumax}}{\text{d}}=\frac{700}{1100+0.87 \text{ Fy}}$

$\text{Kumax}=\frac{\text{Xumax}}{\text{d}}=\frac{700}{1100+0.87 \text{ x } 500} = 0.46$

2) Rumax or Mu lilmit= 0.36 Fck Ku max (1- 0.42 Ku max)

Rumax or Mu lilmit= 0.36 x 30 x 0.46 (1- 0.42 x 0.46) = 4.01

3) Ptmax= $\frac{0.36\text{Fck}}{0.87\text{Fy}} Ku \max X100$

Ptmax= $\frac{0.36 \text{ x } 30}{0.87 \text{ x } 500} x0.46X100$ = 1.14 %

From Page No 70 IS CODE

Sr No	Grade of Steel	$\frac{\text{Xu}}{\text{d}}\max$
1	**Fy 250**	**0.53**
2	**Fy 415**	**0.48**
3	**Fy 500**	**0.46**

Moment of resistance for balance section From Page No 96 IS CODE

1) Grade of Steel = Fy 250

$\frac{\text{Xu}}{\text{d}}\max$ = 0.53

Mu lilmit = 0.36 $\frac{Xu \max}{d}\left(1-0.42\frac{Xu \max}{d}\right)bd^2Fck$

Mu limit = 0.36 X 0.53 X (1-0.42 X 0.53) bd^2 Fck

Mu limit = 0.148 Fck bd^2

2) Grade of Steel = Fy 415

$\frac{\text{Xu}}{\text{d}}\max$ = 0.48

Mu lilmit = 0.36 $\frac{Xu \max}{d}\left(1-0.42\frac{Xu \max}{d}\right)bd^2Fck$

Mu limit = 0.36 X 0.48 X (1-0.42 X 0.48) bd^2 Fck

Mu limit = 0.138 Fck bd^2

3) Grade of Steel = Fy 500

$$\frac{Xu}{d}\max = 0.46$$

$$\text{Mu lilmit} = 0.36\ \frac{Xu\max}{d}\left(1-0.42\frac{Xu\max}{d}\right)bd^2 Fck$$

Mu limit = 0.36 X 0.46 X (1-0.42 X 0.46) bd^2 Fck

Mu limit = 0.133 Fck bd^2

Table :

Maximum depth of neutral axis (Xu max) and Moment of resistance

Sr No	Grade of Steel	$\frac{Xu}{d}\max$	Mu limit
1	**Fy 250**	**0.53**	Mu lilmit = 0.148 $Fck\ bd^2$
2	**Fy 415**	**0.48**	Mu lilmit = 0.138 $Fck\ bd^2$
3	**Fy 500**	**0.46**	Mu lilmit = 0.133 $Fck\ bd^2$

Formulae

1. Compressive Force (Cu) = 0.36FckXub
2. Tensile Force (Tu) = 0.87 Fy Ast
3. Lever Arm (Z) = d- 0.42 Xu
4. Depth of neutral axis : Cu = Tu

$$0.36FckXub = 0.87\ Fy\ Ast$$

$$Xu = \frac{0.87FyAst}{0.36Fckb}$$

5. Maximum depth of neutral axis

Sr No	Grade of Steel	Maximum depth of neutral axis (Xu max)
1	**Fy 250**	**0.53 d**

2	**Fy 415**	**0.48 d**
3	**Fy 500**	**0.46 d**

6.Moment of resistance

Sr No	Grade of Steel	Maximum depth of neutral axis (Xu max)	Mu limit
1	**Fy 250**	**0.53 d**	Mu lilmit = 0.148 $Fck\ bd^2$
2	**Fy 415**	**0.48 d**	Mu lilmit = 0.138 $Fck\ bd^2$
3	**Fy 500**	**0.46 d**	Mu lilmit = 0.133 $Fck\ bd^2$

7. Section formulae :

From Page No 96 IS CODE

1) Under reinforced Section

$Xu \prec Xu\,max$

Mu = 0.87 Fy Ast (d- 0.42 Xu max)

2) Balanced Section

$Xu = Xu\,max$

Mu = 0.36 Fck Xumax b (d- 0.42 Xu max)

3) Over reinforced Section

$Xu \succ Xu\,max$

If any numerical having over reinforced section then that numerical design for under reinforced section.

IN SINGLY REINFORCED BEAM THERE ARE THREE TYPES OF PROBLEMS

1) To find moment of resistance of section

2) To find the area of steel

3) Design the section

Type I

To find moment of resistance of section

Stepwise Procedure

:To find :- The moment of resistance of section

Given Data:- b,d,,Ast, Fy, Fck

d = D - Effective cover

d = D - d'

d'=Effective cover = Clear cover + $\frac{\phi}{2}$

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$$0.36 Fck Xu b = 0.87 \text{ Fy Ast}$$

$$Xu = \frac{0.87 Fy Ast}{0.36 Fck b}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.48 d ....For Fe 415

Xu max = 0.46 d ....For Fe 500

STEP 3: To compare Xu and Xu max

a) If $Xu \prec Xu\max$ then section is under reinforced

b) **If** $Xu = Xu\max$ then section is balance section

c) **If** $Xu \succ Xu\max$ then section is over reinforced, if section is over reinforced then consider it as balance section.

STEP 4: To find moment of resistance

a) For under reinforced section **(From page No. 96 IS CODE)**

$$Mu = 0.87\ Fy\ Ast\ (\ d- 0.42\ Xu\ )$$

b) For balance section

$$Mulimit = 0.36\ Fck\ Xumax\ b\ \ (\ d- 0.42\ Xu\ max)$$

STEP 5:

Total ultimate load = Working load X Load factor

Working load = $\dfrac{\text{Total Ultimate load}}{\text{Load factor}}$

Superimposed load = Total working load - self weight of beam

Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25

Density of Concrete = 25 KN/m^3

Examples

1) A reinforced concrete beam 250 mm X 300mm overall depth is reinforced with 3 bars of 12 mm diameter at the bottom. The clear cover of 25 mm, Calculate ultimate moment of resistance of section also find superimposed UDL over a simply supported span of 3 m. Used Fe 250 and M_{15}.

Solution:**- To find :- The moment of resistance of section**

Given Data:- b = 250 mm

D = 300 mm

ϕ = 12 mm

No of bar = 3

Clear cover = 25 mm

d'= Effective cover = Clear cover + $\dfrac{\phi}{2}$

d'=Effective cover = 25 + $\frac{12}{2}$ = 31 mm

Effective depth =d = D - d' = 300 -31 = 269 mm

Ast = 3 X $\frac{\pi}{4}$ X ϕ^2 = 3 X $\frac{\pi}{4}$ X 12^2 = 339.29 mm^2

L = 3 m

M_{15} = Fck = 15 N/mm²

Fe 250 = Fy = 250 N/mm²

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$$0.36 Fck Xub = 0.87 \text{ Fy Ast}$$

$$Xu = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck b}}$$

$$Xu = \frac{0.87 \text{ x } 250 \text{ x } 339.29}{0.36 \text{ x } 15 \text{ x } 250} = 54.66 \text{ mm}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.53 x 269 = 142.57 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu\,max$$

$$54.66 \prec 142.57$$

then section is under reinforced

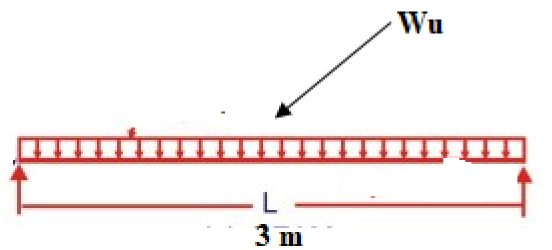
STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

$$Mu = 0.87 \text{ Fy Ast } (d - 0.42 \text{ Xu })$$

Mu = 0.87 x 250 x 339.29 (269- 0.42 x 54.66) = 18.16 X 10^6 Nmm = 18.16 KNm ....(1)

STEP 5: To find superimposed load



Maximum bending moment = $\frac{Wul^2}{8} = \frac{Wu3^2}{8}$ =1.125 Wu...(2)

Equating (1) and(2)

18.16 = 1.125 Wu

Wu = 16.14 KN/m

Total ultimate load = Working load X Load factor

Working load = $\frac{\text{Total Ultimate load}}{\text{Load factor}} = \frac{16.14}{1.5}$ = 10.76 KN/m

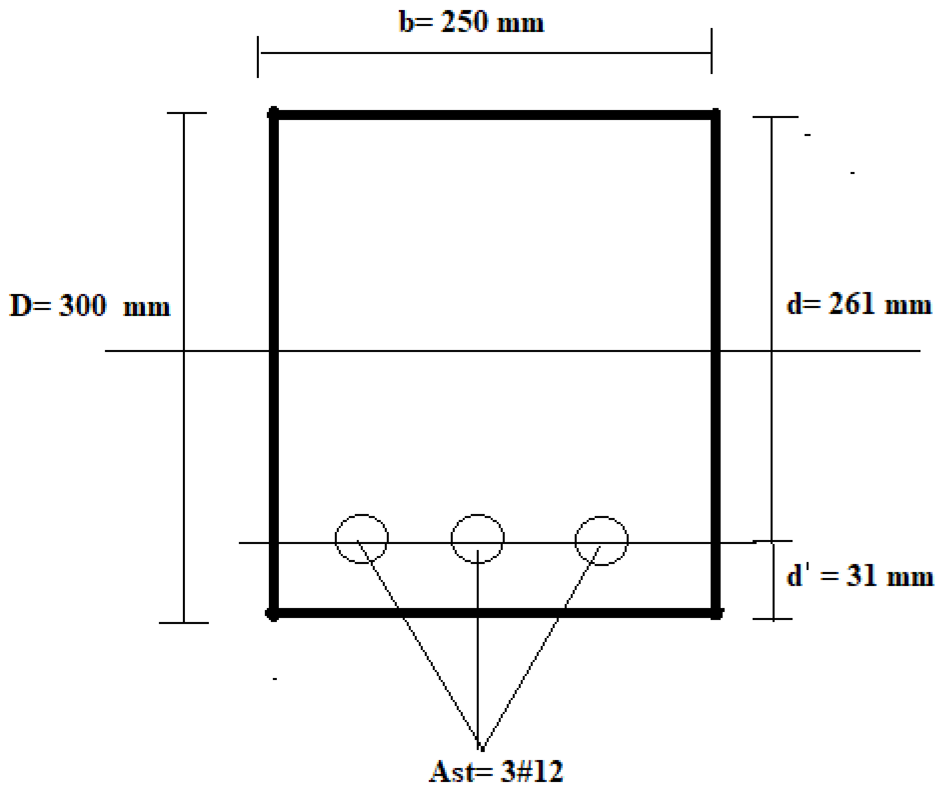
Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25= (0.25 X 0.3) X 25

Self weight of beam = 1.875 KN/m

Superimposed load = Total working load - self weight of beam

Superimposed load = 10.76 - 1.875 = 8.885 KN/m



2) A reinforced concrete beam 250 mm X 400mm overall depth is reinforced with 4 bars of 12 mm diameter at the bottom. The clear cover of 25 mm, Calculate ultimate moment of resistance of section also find superimposed UDL over a simply supported span of 4 m. Used Fe 250 and M_{15}.

Solution:- **To find :- The moment of resistance of section**

Given Data:- b = 250 mm

D = 400 mm

ϕ = 12 mm

No of bar = 4

Clear cover = 25 mm

d'= Effective cover = Clear cover + $\frac{\phi}{2}$

d'=Effective cover = 25 + $\frac{12}{2}$ = 31 mm

Effective depth =d = D - d' = 400 -31 = 369 mm

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 12^2 = 452.39 mm^2

L = 4 m

M_{15} = Fck = 15 N/mm²

Fe 250 = Fy = 250 N/mm²

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$$0.36 Fck Xub = 0.87\ Fy\ Ast$$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b}$$

$$Xu = \frac{0.87 \times 250 \times 452.39}{0.36 \times 15 \times 250} = 72.89\ mm$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.53 x 369 = 195.57 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu\ max$$

$$72.89 \prec 195.57$$

then section is under reinforced

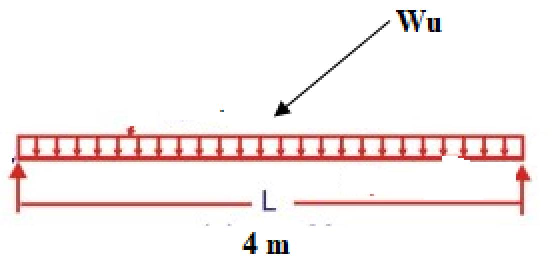
STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

Mu = 0.87 x 250 x 452.39 (369- 0.42 x 72.89) = 33.30 X 10^6 Nmm = 33.30 KNm ....(1)

STEP 5: To find superimposed load



Maximum bending moment = $\frac{Wul^2}{8} = \frac{Wu4^2}{8}$ = 2 Wu..(2)

Equating (1) and(2)

33.30 = 2 Wu

Wu = 16.65 KN/m

Total ultimate load = Working load X Load factor

Working load = $\frac{\text{Total Ultimate load}}{\text{Load factor}} = \frac{16.65}{1.5}$ = 11.10 KN/m

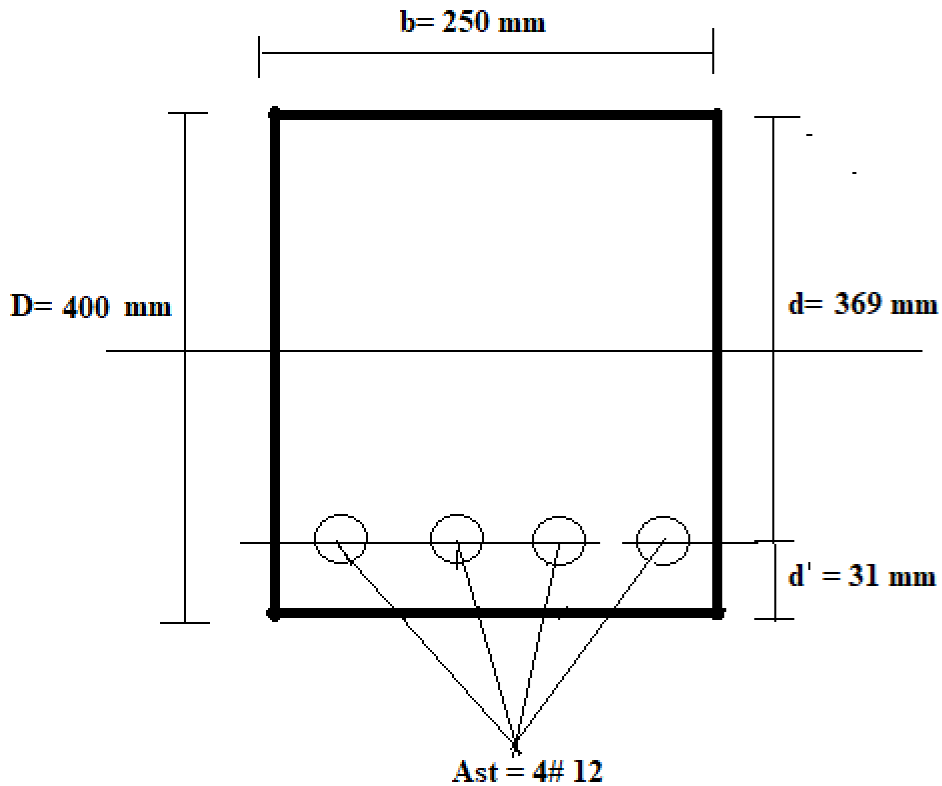
Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25= (0.25 X 0.4) X 25

Self weight of beam = 2.5 KN/m

Superimposed load = Total working load - self weight of beam

Superimposed load = 11.10 - 2.5 = 8.6 KN/m



3) A reinforced concrete beam 300 mm X 500mm overall depth is reinforced with 4 bars of 16 mm diameter on tension side with 40 mm effective cover. Calculate ultimate moment of resistance of section also find superimposed UDL over a simply supported span of 5 m. Used Fe 415 and M_{15}.

Solution:- **To find :- The moment of resistance of section**

Given Data:- b = 300 mm

D = 500 mm

ϕ = 16 mm

No of bar = 4

d'=Effective cover = 40 mm

Effective depth =d = D - d' = 500 -40 = 460 mm

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 16^2 = 804.25 mm^2

L = 5 m

M_{15} = Fck = 15 N/mm^2

Fe 415 = Fy = 415 N/mm^2

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$0.36 Fck Xub = 0.87\ Fy\ Ast$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b}$$

$$Xu = \frac{0.87 \times 415 \times 804.25}{0.36 \times 15 \times 300} = 179.24 \text{ mm}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 460 = 220.8 mm

STEP 3: To compare Xu and Xu max

$Xu \prec Xu\ max$

$179.24 \prec 220.8$

then section is under reinforced

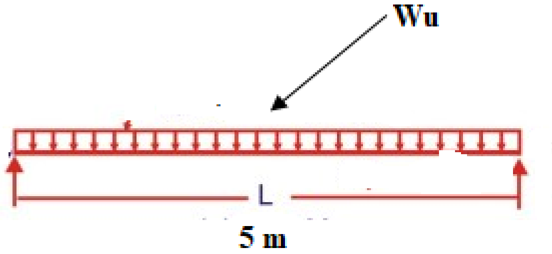
STEP 4: To find moment of resistance

a) For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

Mu = 0.87 x 415 x 804.25 (460- 0.42 x 179.24) = 111.71 X 10^6 Nmm = 111.71 KNm ....(1)

STEP 5: To find superimposed load



Maximum bending moment = $\frac{Wul^2}{8} = \frac{Wu5^2}{8}$ = 3.125 Wu...(2)

Equating (1) and(2)

111.71 = 3.125 Wu

Wu = 35.75 KN/m

Total ultimate load = Working load X Load factor

Working load = $\frac{\text{Total Ultimate load}}{\text{Load factor}} = \frac{35.75}{1.5}$ =23.83 KN/m

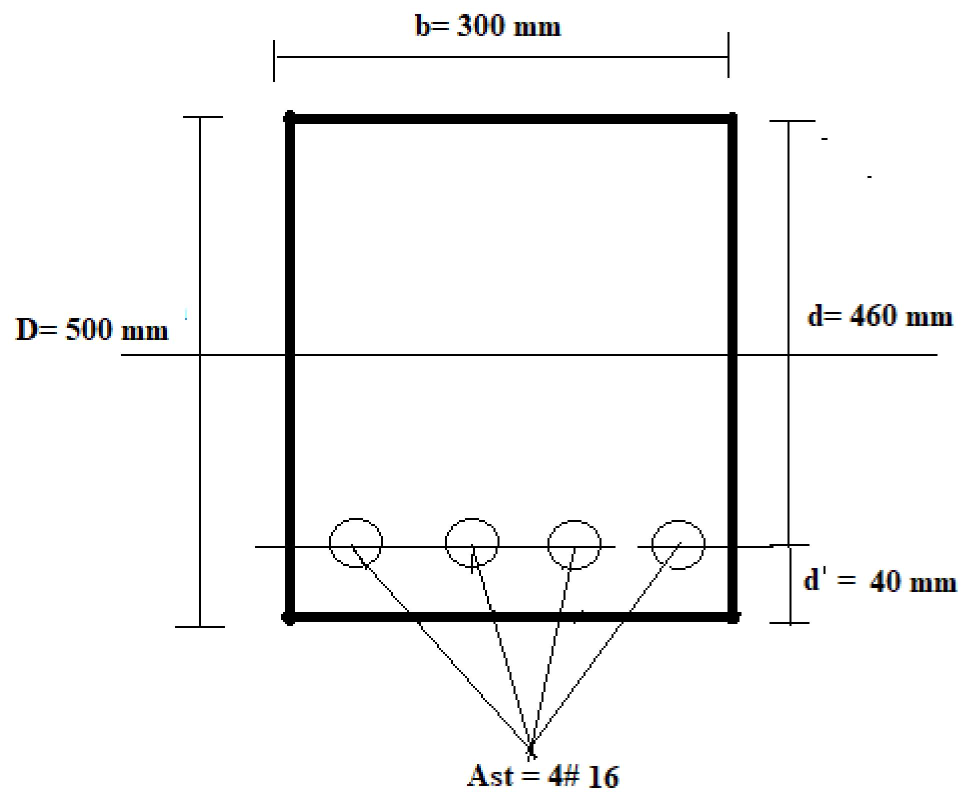
Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25= (0.3 X 0.5) X 25

Self weight of beam = 3.75 KN/m

Superimposed load = Total working load - self weight of beam

Superimposed load = 23.83 - 3.75 = 20.08 KN/m



4) A cantilever beam of span 2.5 m having width 200 mm and overall depth 400 mm provide with 4 bars of 12 mm diameter on tension side. Calculate ultimate moment of resistance of section also find superimposed UDL with 40 mm effective cover. Used Fe 415 and M_{20}.

Solution:- **To find :- The moment of resistance of section**

Given Data:- b = 200 mm

D = 400 mm

ϕ = 12 mm

No of bar = 4

d'= Effective cover = 40 mm

Effective depth =d = D - d' = 400 -40 = 360 mm

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 12^2 = 452.389 mm^2

L = 2.5 m

M_{15} = Fck = 20 N/mm²

Fe 415 = Fy = 415 N/mm²

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$$0.36 Fck Xub = 0.87 \text{ Fy Ast}$$

$$Xu = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck b}}$$

$$Xu = \frac{0.87 \times 415 \times 452.389}{0.36 \times 20 \times 200} = 113.427 \text{ mm}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 360 = 172.80 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu \max$$

$$113.427 \prec 172.80$$

then section is under reinforced

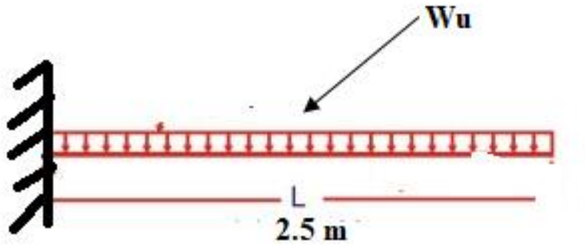
STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

Mu = 0.87 x 415 x 452.389 (360- 0.42 x 113.427) = 51.02 X 10^6 Nmm = 51.02 KNm ....(1)

STEP 5: To find superimposed load



Maximum bending moment = $\frac{Wul^2}{2} = \frac{Wu \text{ x } 2.5^2}{2}$ = 3.125 Wu...(2)

Equating (1) and(2)

51.02 = 3.125 Wu

Wu = 16.33 KN/m

Total ultimate load = Working load X Load factor

Working load = $\frac{\text{Total Ultimate load}}{\text{Load factor}} = \frac{16.33}{1.5}$ = 10.88 KN/m

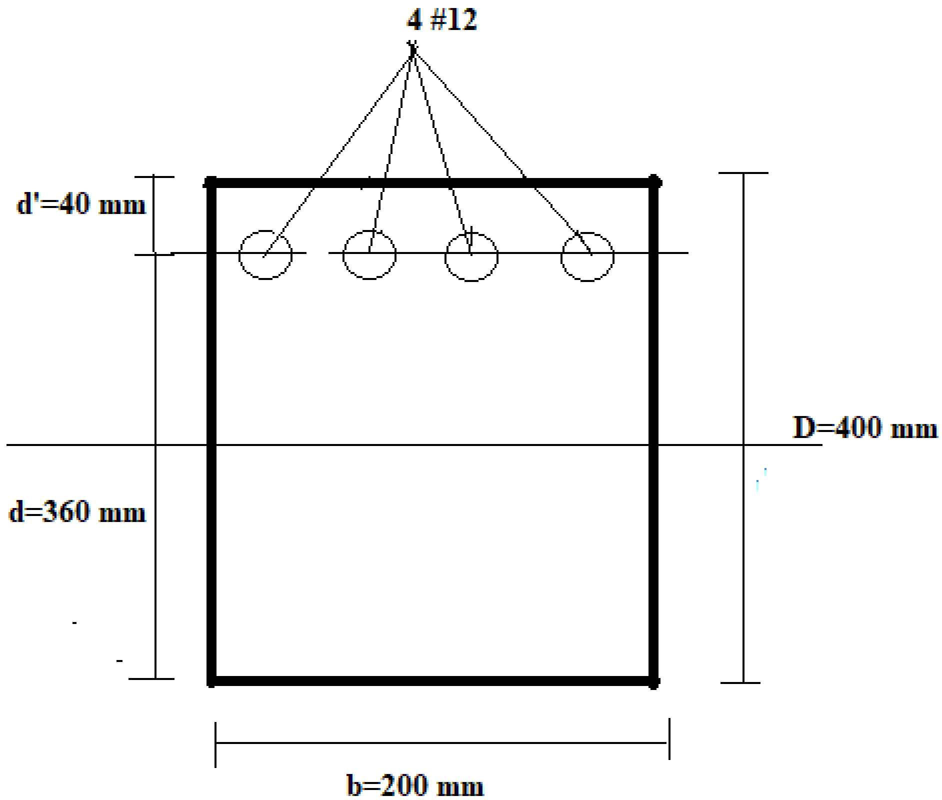
Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25= (0.2 X 0.4) X 25

Self weight of beam = 2 KN/m

Superimposed load = Total working load - self weight of beam

Superimposed load = 10.88 - 2 = 8.88 KN/m



5) A cantilever beam of span 2 m having width 250 mm and overall depth 400 mm provide with 3 bars of 12 mm diameter on tension side. Calculate ultimate moment of resistance of section also find superimposed UDL with 35 mm cover for centre of reinforcement. Used Fe 415 and M_{20}.

Solution:- **To find :- The moment of resistance of section**

Given Data:- b = 250 mm

D = 400 mm

ϕ = 12 mm

No of bar = 3

d'=Effective cover = 35 mm

Effective depth =d = D - d' = 400 -35 = 365 mm

Ast = 3 X $\frac{\pi}{4}$ X ϕ^2 = 3 X $\frac{\pi}{4}$ X 12^2 = 339.29 mm^2

L = 2 m

M_{15} = Fck = 20 N/mm^2

Fe 415 = Fy = 415 N/mm²

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$$0.36 Fck Xub = 0.87\ Fy\ Ast$$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b}$$

$$Xu = \frac{0.87 \times 415 \times 339.29}{0.36 \times 20 \times 250} = 68.09 \text{ mm}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 365 = 175.2 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu\ max$$

$$68.06 \prec 175.20$$

then section is under reinforced

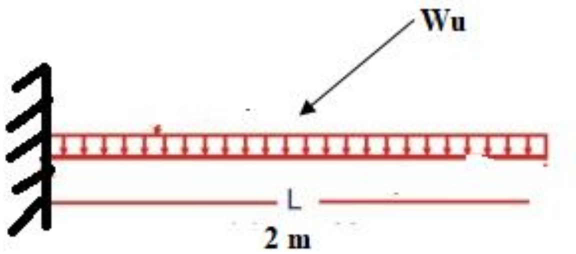
STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

Mu = 0.87 x 415 x 339.29 (360- 0.42 x 68.06) = 41.21 X 10^6 Nmm = 41.21 KNm ....(1)

STEP 5: To find superimposed load



Maximum bending moment = $\frac{Wul^2}{2} = \frac{Wu \text{ x } 2^2}{2}$ = 2 Wu..(2)

Equating (1) and(2)

41.21 = 2 Wu

Wu = 20.605 KN/m

Total ultimate load = Working load X Load factor

Working load = $\frac{\text{Total Ultimate load}}{\text{Load factor}} = \frac{20.605}{1.5}$ = 13.74 KN/m

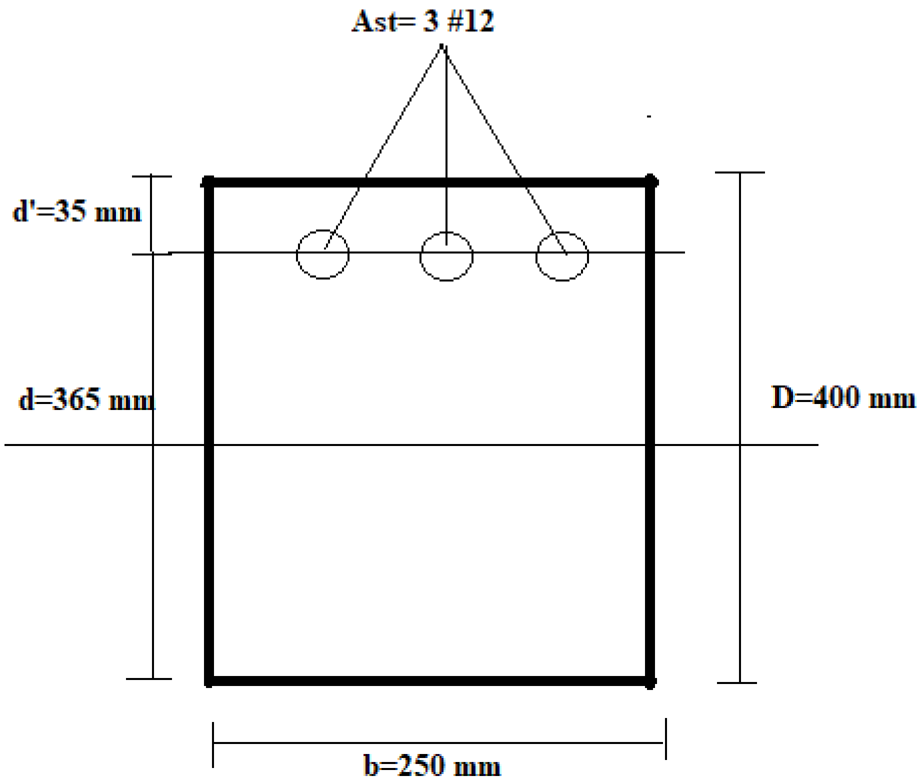
Self weight of beam = Cross sectional area X Density of Concrete

Self weight of beam = b X D X 25= (0.25 X 0.4) X 25

Self weight of beam = 2.5 KN/m

Superimposed load = Total working load - self weight of beam

Superimposed load = 13.74 - 2.5 = 11.24 KN/m



6) **A reinforced concrete beam of rectangular section 230 mm X 400 mm over all is reinforced with 4 bars of 12 mm diameter on tension side provided with clear cover 25 mm. Calculate ultimate moment of resistance of section and central point load in addition of self weight of beam, if it is simply supported over a span of 3.5 m. Use Fe 415 and M_{20}.**

Solution:- **To find :- The moment of resistance of section**

Given Data:- b = 230 mm

D = 400 mm

ϕ = 12 mm

No of bar = 4

Clear cover = 25 mm

d'=Effective cover = Clear cover + $\frac{\phi}{2}$

d'=Effective cover = 25 + $\frac{12}{2}$ = 31 mm

Effective depth =d = D - d' = 400 -31 = 369 mm

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 12^2 = 452.389 mm^2

L = 3.5 m

M_{20} = Fck = 20 N/mm^2

Fe 415 = Fy = 415 N/mm^2

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$$0.36 Fck Xub = 0.87 \text{ Fy Ast}$$

$$Xu = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck b}}$$

$$Xu = \frac{0.87 \text{ x } 415 \text{ x } 452.389}{0.36 \text{ x } 20 \text{ x } 230} = 98.63 \text{ mm}$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 369 = 177.12 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu\,max$$

$$98.63 \prec 177.120$$

then section is under reinforced

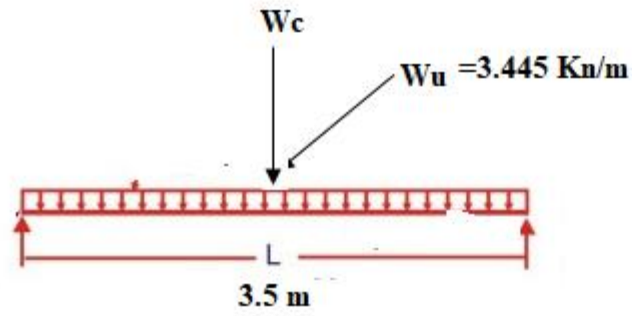
STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

$$Mu = 0.87 \text{ Fy Ast } (d - 0.42 \text{ Xu})$$

$$Mu = 0.87 \text{ x } 415 \text{ x } 452.389 \,(369 - 0.42 \text{ x } 98.63) = 53.5 \text{ X } 10^6 \text{ Nmm} = 53.50 \text{ KNm} \quad ....(1)$$

STEP 5: Central point load in addition of self weight of beam



Max BM = $\frac{Wul^2}{8} + \frac{Wcl}{4}$ ....(2)

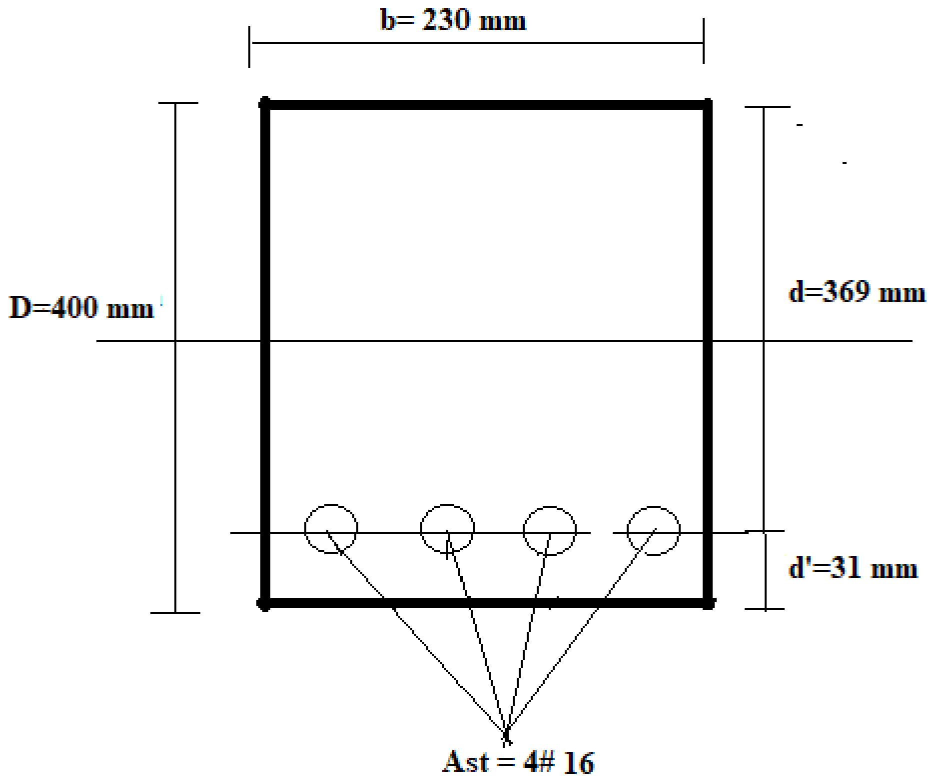
Self weight of beam = b XD X25 = 0.23 x 0.4 x25 = 2.3 KN/m

Factored self weight of beam = Wu = 1.5 x 2.3 =3.45 KN/m

Substitute all value in equation (2)

$$53.50 = \frac{3.45 \text{ x } 3.5^2}{8} + \frac{Wc \text{ x } 3.5}{4}$$

Wc = 55.109 KN



7) **A reinforced concrete beam of rectangular section 300 mm X 350 mm over all is reinforced with 3 bars of 12 mm diameter on tension side provided with effective cover 35 mm. Calculate ultimate moment of resistance of section and central point load in addition of self weight of beam, if it is simply supported over a span of 5 m. Use Fe 500 and M_{25}.**

Solution:- **To find :- The moment of resistance of section**

Given Data:- b = 300 mm

D = 350 mm

ϕ = 12 mm

No of bar = 3

d'=Effective cover = 35 mm

Effective depth =d = D - d' = 350 -35 = 315 mm

Ast = 3 X $\frac{\pi}{4}$ X ϕ^2 = 3 X $\frac{\pi}{4}$ X 12^2 = 339.29 mm^2

L = 5 m

M_{25} = Fck = 25 N/mm^2

Fe 500 = Fy = 500 N/mm^2

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

1. Cu = Tu

$$0.36FckXub = 0.87\ Fy\ Ast$$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b}$$

$$Xu = \frac{0.87 \times 500 \times 339.29}{0.36 \times 25 \times 300} = 54.66\ mm$$

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.46 d ....For Fe 500

Xu max = 0.46 x 315 = 144.9 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu\ max$$

$$54.66 \prec 144.90$$

then section is under reinforced

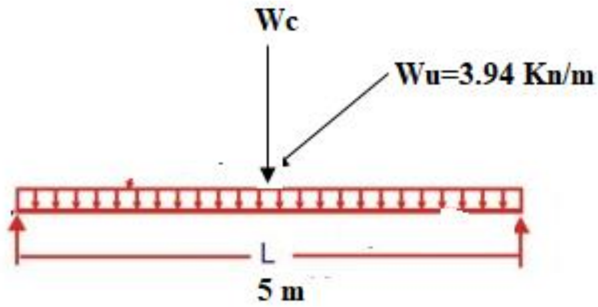
STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

Mu = 0.87 x 500 x 339.29 (315- 0.42 x 54.66) = 43.40 X 10^6 Nmm = 43.10 KNm ....(1)

STEP 5: Central point load in addition of self weight of beam



Max BM = $\frac{Wul^2}{8} + \frac{Wcl}{4}$ ....(2)

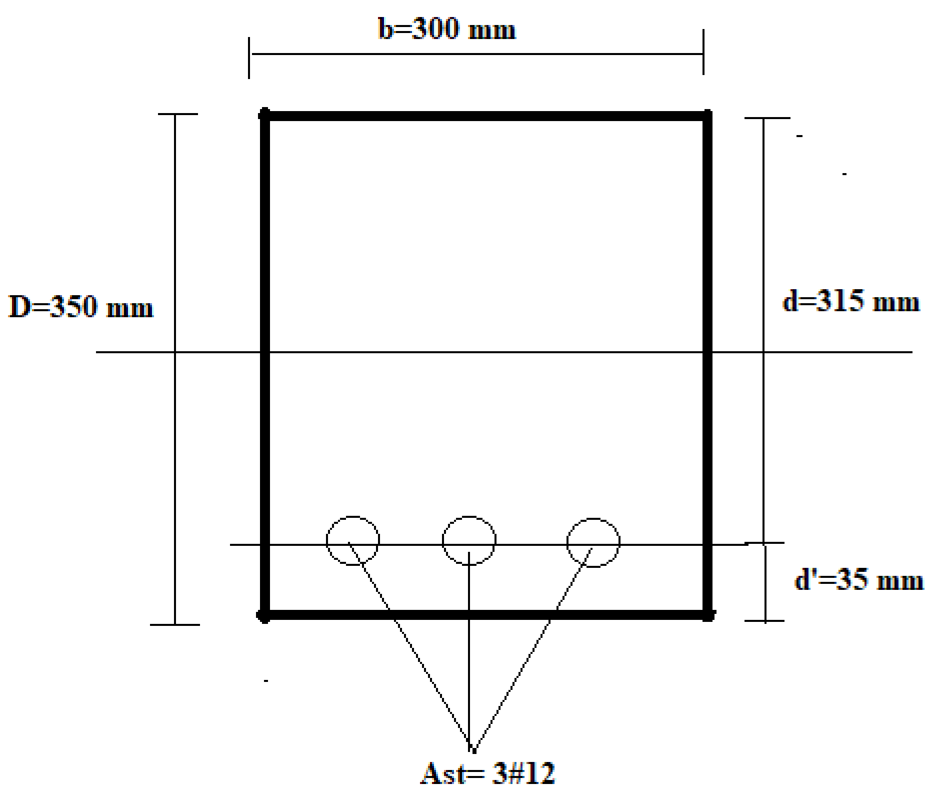
Self weight of beam = b XD X25 = 0.3 x 0.35 x25 = 2.63 KN/m

Factored self weight of beam = Wu = 1.5 x 2.63 = 3.94 KN/m

Substitute all value in equation (2)

$$4310 = \frac{3.94 \text{ x } 5^2}{8} + \frac{Wc \text{ x } 5}{4}$$

Wc = 24.63 KN



Type II :- To find area of steel (Ast)

Stepwise Procedure

To find :- The area of steel (Ast)

Given Data:- b,d, Fy, Fck, Design Moment (M_d)

Effective depth =d = D - Effective cover

Effective depth =d = D - d'

Effective cover = Clear cover + $\frac{\phi}{2}$

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.148 $Fck\ bd^2$ .... Fe 250

Mu lilmit = 0.138 $Fck\ bd^2$ .... Fe 415

Mu lilmit = 0.133 $Fck\ bd^2$ .... Fe 500

STEP 2: To compare M_d and $M_{U\ limit}$

$M_d \prec M_{U\ limit}$ section is Under reinforced Section

$M_d = M_{U\ limit}$ section is Balanced Section

$M_d \succ M_{U\ limit}$ section is Over reinforced Section

STEP 3: Equating M_d and $M_{U\ limit}$ and calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckbd^2}}\right]bd$$

EXAMPLES:

1) Calculate the area of steel required for the singly reinforced beam 230 mm wide and 400 mm deep overall to resist ultimate moment of 60 KNm, the effective cover is 40 mm. M_{15} and Fe 250.

Solution:- Given data

b= 230 mm

D =400 mm

d' = 40 mm

Effective depth =d= D-d' = 400- 40 =360 mm

M_d=60 KNm = 60 X 10^6 Nmm

Fck = 15 N/mm^2

Fy= 250 N/mm^2

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.148 $Fck\ bd^2$ .... Fe 250

Mu lilmit = 0.148 X 15 X 230 X 360^2 = 66.17 X 10^6 N.mm = 66.17 KNm

STEP 2: To compare M_d and $M_{U\ limit}$

60 $\prec$ 66.17

$M_d \prec M_{U\ limit}$ section is Under reinforced Section

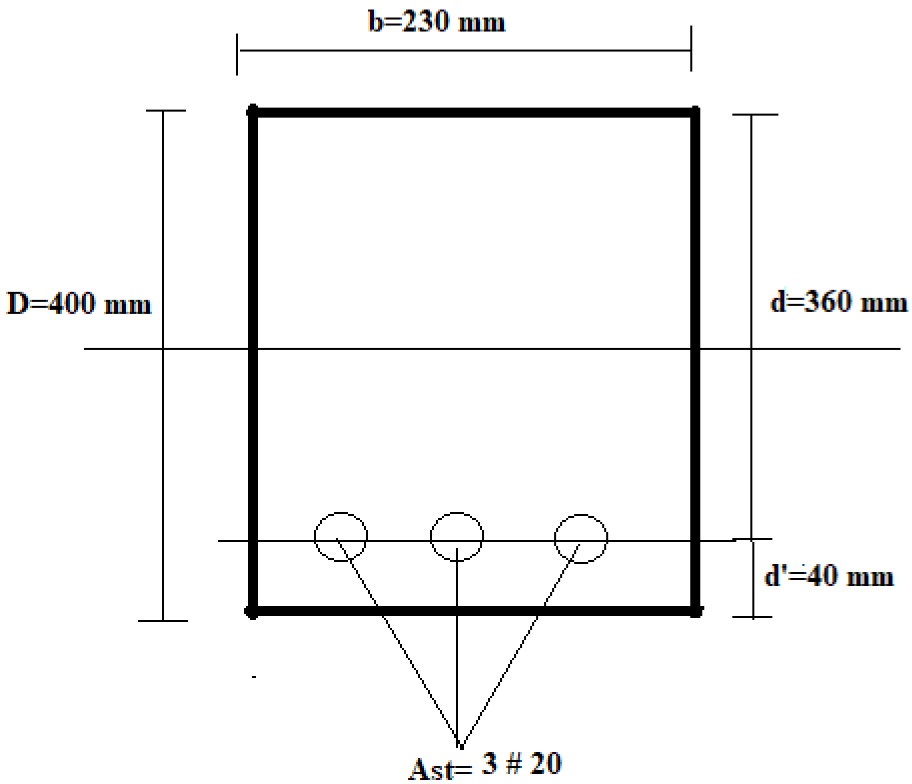
STEP 3: Equating M_d and $M_{U\ limit}$ and calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckbd^2}}\right]bd$$

$$A_{st} = \frac{0.5 \text{ x } 15}{250}\left[1-\sqrt{1-\frac{4.6 \text{ x } 60 \text{ x } 10^6}{15 \text{ x } 230 \text{ x } 360^2}}\right]230 \text{ x } 360 = 947.30 \text{ mm}^2$$

Assume diameter of bar = Φ = 20 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2} = \frac{947.30}{(\pi/4) \text{ x } 20^2} = 3.01 \cong 3$



2) Calculate the area of steel required for the singly reinforced beam 230 mm wide and 650 mm effective carrying <u>working moment</u> of 130 KNm. Use M $_{15}$ and Fe 415.

Solution:- Given data

b= 230 mm

Effective depth =d= 650 mm

Working Moment=130 KNm

Design Moment = M_d =130 X 1.5 = 195 KNm = 195 X 10^6 Nmm

Fck = 15 N/mm^2

Fy= 415 N/mm^2

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.138 $Fck\ bd^2$ .... Fe 415

Mu lilmit = 0.138 X 15 X 230 X 650^2 = 201.15 X 10^6 N.mm = 201.15 KNm

STEP 2: To compare M_d and $M_{U\ limit}$

195 $\prec$ 201.15

$M_d \prec M_{U\ limit}$ section is Under reinforced Section

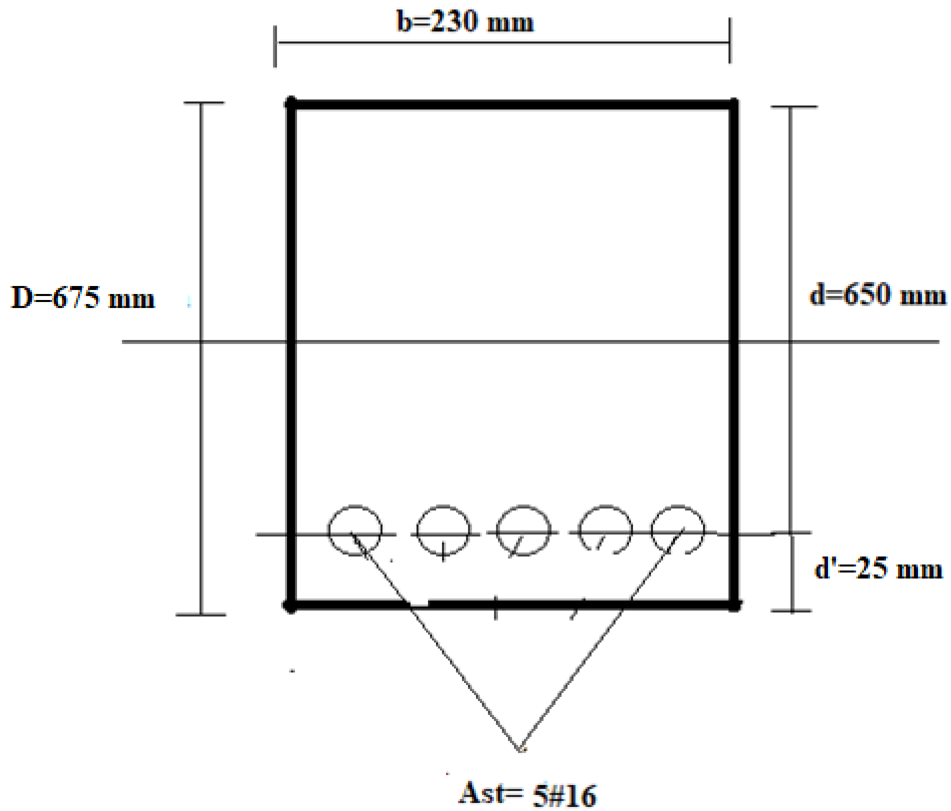
STEP 3: Equating M_d and $M_{U\ limit}$ and calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckbd^2}}\right]bd$$

$$A_{st} = \frac{0.5 \text{ x } 15}{415}\left[1-\sqrt{1-\frac{4.6 \text{ x } 195 \text{ x } 10^6}{15 \text{ x } 230 \text{ x } 650^2}}\right]230 \text{ x } 650 = 1026.22 \text{ mm}^2$$

Assume diameter of bar = Φ = 16 mm

Number of bars = $\dfrac{Ast}{(\pi/4) \text{ x } \phi^2} = \dfrac{1026.22}{(\pi/4) \text{ x } 16^2} = 5.10 \cong 5$



3) **A reinforced concrete beam of rectangular section 230 mm X 400 mm overall ,the clear cover is 25 mm. The beam is carrying UDL of 24 KN/m including self weight over a span of 3.5 m. Find the area of steel required, Use M_{20} and Fe 415.**

Solution:- Given data

b= 230 mm

D =400 mm

Assuming ϕ = 20 mm

Clear cover = 25 mm

Effective cover = d'=Clear cover + $\frac{\phi}{2}$

Effective cover = d'=25 + $\frac{20}{2}$ = 35 mm

d = D - d' = 400 -35 = 365 mm

Fck = 20 N/mm^2

Fy= 415 N/mm^2

UDL W= 24 KN/m

Factored UDL = Wu = 1.5 X 24 = 36 KN/m

Assuming simply supported beam

Maximum bending moment =M$_d$= $\frac{Wul^2}{8} = \frac{36 \text{ x } 3.5^2}{8}$ = 55.125 KNm

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.138 $Fck\ bd^2$ .... Fe 415

Mu lilmit = 0.138 X 20 X 230 X 365^2 = 84.57 X 10^6 N.mm = 84.57 KNm

STEP 2: To compare M$_d$ and M$_{U\ limit}$

55.125 $\prec$ 84.57

M$_d$ $\prec$ **M$_{U\ limit}$ section is Under reinforced Section**

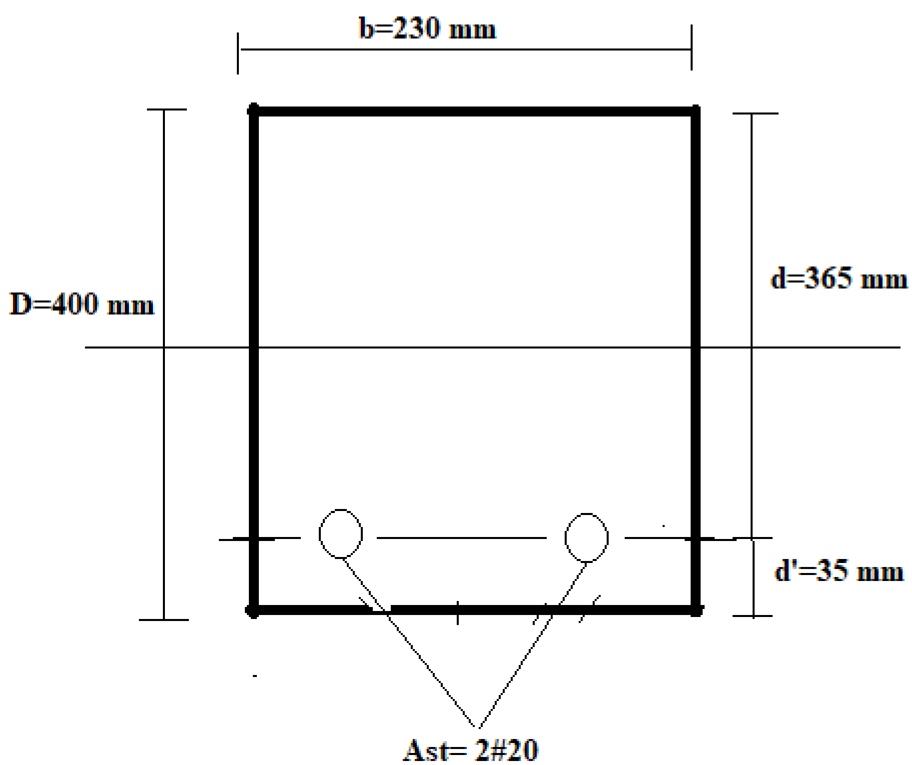
STEP 3: Equating M$_D$ and M$_{U\ limit}$ and calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckbd^2}}\right]bd$$

$$A_{st} = \frac{0.5 \text{ x } 20}{415}\left[1-\sqrt{1-\frac{4.6 \text{ x } 55.125 \text{ x } 10^6}{20 \text{ x } 230 \text{ x } 365^2}}\right]230 \text{ x } 365 = 474.06 \text{ mm}^2$$

Assume diameter of bar = Φ = 20 mm (Allready assumed)

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2} = \frac{474.06}{(\pi/4) \text{ x } 20^2} = 1.51 \cong 2$



Type III :- To design of beam

Stepwise Procedure

To design of beam

Given Data:- M_d**, Fy, Fck**

To find :- b,d,Ast

d = D - Effective cover

Effective depth =d = D - d'

Effective cover = Clear cover + $\frac{\phi}{2}$

STEP 1: Calculate ultimate moment of resistance

$\text{Mu lilmit} = 0.148\, Fck\ bd^2$ .... Fe 250

$\text{Mu lilmit} = 0.138\, Fck\ bd^2$ .... Fe 415

$\text{Mu lilmit} = 0.133\, Fck\ bd^2$ .... Fe 500

STEP 2: Equating Mulimit to M_d

Assuming the value of " b" and find "d" or take minimum value of "b " = 230 mm or "b" = d/2

STEP 3: To find area of steel

a) For balance section

$$M_d = 0.87\ \text{Fy Ast} \left(\text{d- } 0.42\ \text{Xu max}\right)$$

To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.48 d ....For Fe 415

Xu max = 0.46 d ....For Fe 500

<u>or</u>

$$A_{st} = \frac{0.5\text{Fck}}{\text{Fy}}\left[1-\sqrt{1-\frac{4.6M_d}{\text{Fckbd}^2}}\right]\text{bd}$$

To find minimum area of steel Ast (From page No. 47 and cl no 26.5.1.1 IS CODE)

Compare Ast with Ast minimum

$$A_{st\ min} = \frac{0.85\ \text{b d}}{\text{Fy}}$$

STEP 4 :Number of bars

$$\textbf{Number of bars} = \frac{Ast}{(\pi / 4)\ \text{x}\ \phi^2}$$

1) **Design a rectangular beam to resist a ultimate moment of 100 KNm, usin**g M_{20} and Fe 415.

Solution:-

To design of beam

Given Data:- M_d= 100 KNm= 100 X 10^6 Nmm

Fy= 415 N/mm^2

Fck= 20 N/mm^2

Assuming "b" = 230 mm

To find :- d,Ast

STEP 1: Calculate ultimate moment of resistance

$\text{Mu lilmit} = 0.138\ Fck\ bd^2$ .... Fe 415

$\text{Mu lilmit} = 0.138 \text{ x } 20\text{x } 230\ d^2$

$\text{Mu lilmit} = 634.8\ d^2$

STEP 2: Equating Mulimit to M_d

b= 230 mm

M_d = Mulimit

100 X 10^6 = 634.8 d^2

d= 396.90 mm

Effective depth =d $\cong$ 400 mm

STEP 3: To find area of steel

a) For balance section

$M_d = 0.87 \text{ Fy Ast (d- 0.42 Xu max)}$

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 400 = 192 mm

$100 \text{ x } 10^6 = 0.87 \text{ x } 415 \text{ x Ast (} 400 - 0.42 \text{ x } 192)$

Ast = 867.27 mm^2

Compare Ast with Ast minimum

$$A_{st\ min} = \frac{0.85 \text{ b d}}{\text{Fy}}$$ .... IS CODE PAGE NO 47

$$A_{st\ min} = \frac{0.85 \text{ x } 230 \text{ x } 400}{415} = 188.43 \text{ mm}^2$$

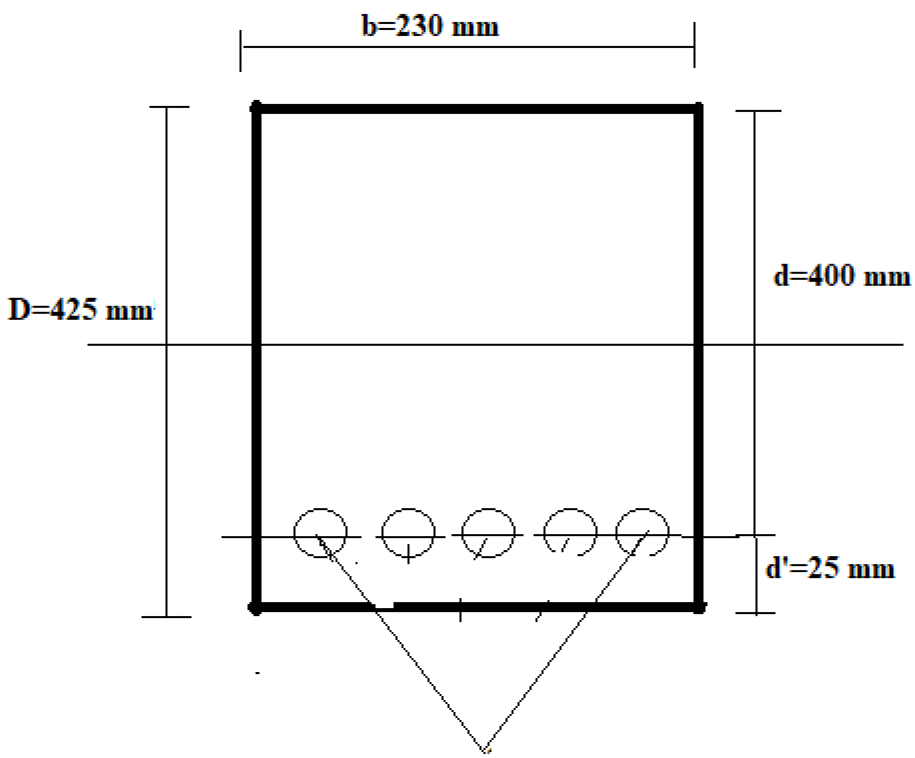
Ast $\succ$ Ast Min (ok)

STEP 4:Number of bars

Assuming diameter of bar = ϕ = 16 mm

$$\textbf{Number of bars} = \frac{Ast}{(\pi / 4) \text{ x } \phi^2}$$

$$\textbf{Number of bars} = \frac{867.27}{(\pi / 4) \text{ x } 16^2} = 4.31 \cong \mathbf{5}$$



2) A simply supported beam 4 m span subjected to central point load of 60 KN. Design the beam section using M_{20} and Fe 415.

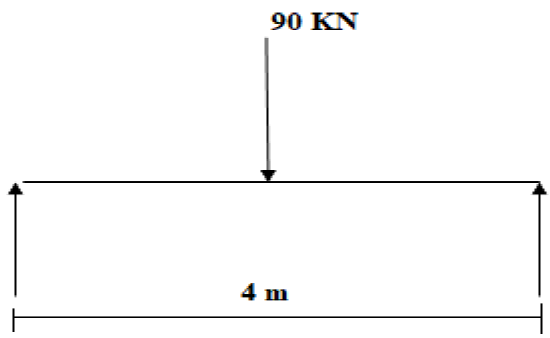
To design of beam

Given Data:- Fy= 415 N/mm^2

Fck= 20 N/mm^2, L = 4 m

Working load =W= 60 KN

Factored load = Wu= 60 x1.5 = 90KN



Max BM = (WL/4) = (90 X 4)/ 4 = 90 KNm

M_d= 90 X 10^6 Nmm

Assuming "b" = 230 mm

To find :- b,d,,Ast

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.138 $Fck\ bd^2$ .... Fe 415

Mu lilmit = 0.138 x 20x 230 d^2

Mu lilmit = 634.8 d^2

STEP 2: Equating Mulimit to M_d

b = 230 mm

M_d = Mulimit

90 X 10^6 = 634.8 d^2

d= 376.532 mm

Effective depth =d $\cong$ 380 mm

STEP 3: To find area of steel

a) For balance section

$M_d = 0.87$ Fy Ast (d- 0.42 Xu max)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 380 = 182.4 mm

$90 \text{ x } 10^6 = 0.87 \text{ x } 415 \text{ x Ast } (380- 0.42 \text{ x } 182.4)$

Ast = 821.62 mm^2

Compare Ast with Ast minimum

$A_{st\ min} = \frac{0.85 \text{ b d}}{Fy}$ .... IS CODE PAGE NO 47

$A_{st\ min} = \frac{0.85 \text{ x } 230 \text{ x } 380}{415}$ = 179.01 **mm^2**

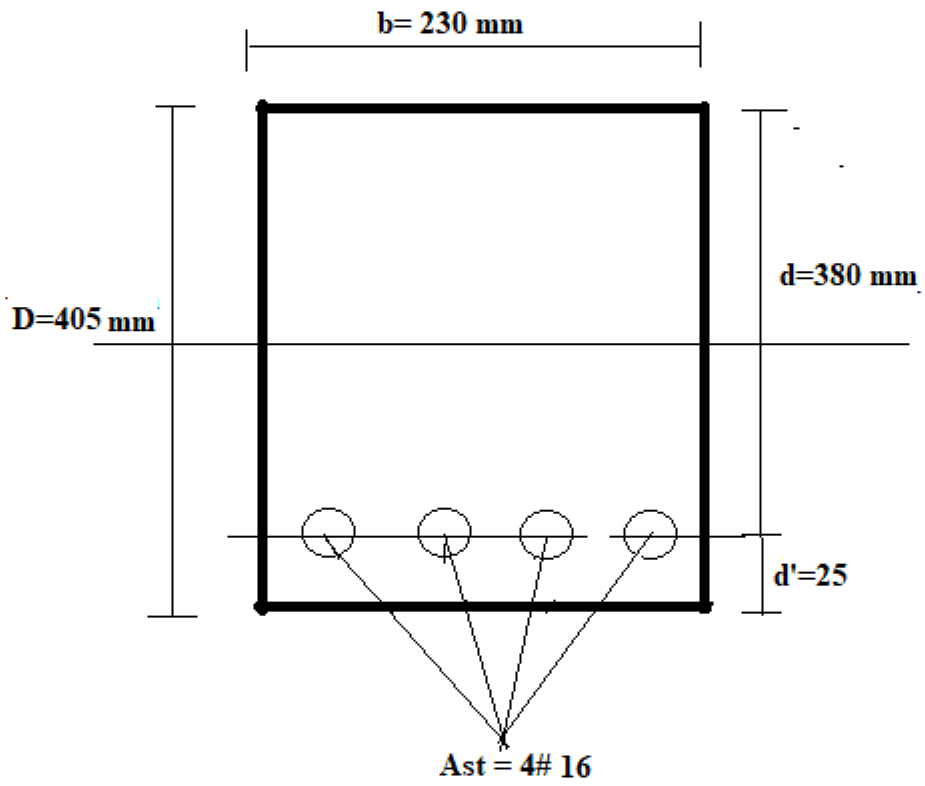
Ast $\succ$ Ast Min (ok)

STEP 4:Number of bars

Assuming diameter of bar = ϕ = 16 mm

Number of bars = $\frac{Ast}{(\pi / 4) \text{ x } \phi^2}$

Number of bars = $\dfrac{821.62}{(\pi / 4) \times 16^2}$ = 4.08 ≅ **4**



3) A simply supported beam 5 m span subjected to <u>working</u> UDL of 30 KN/m. Design the beam section using M_{20} and Fe 415.

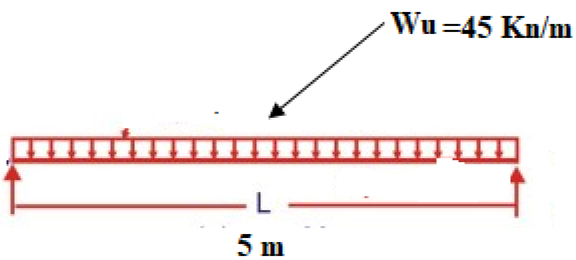
To design of beam

Given Data:- Fy= 415 N/mm^2

Fck= 20 N/mm^2 , L = 5 m

Working UDL =W= 30 KN/m

Factored UDL = Wu= 30 x1.5 = 45 KN/m



Max BM = $(WL^2/8)$ = $(45 \times 5^2)/8$ = 140.625 KNm

M_d= 140.625 X 10^6 Nmm

Assuming "b" = 230 mm

To find :- b,d,,Ast

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = $0.138\ Fck\ bd^2$ .... Fe 415

Mu lilmit = $0.138 \times 20 \times 230\ d^2$

Mu lilmit = $634.8\ d^2$

STEP 2: Equating Mulimit to M_d

b = 230 mm

M_d = Mulimit

$140.625 \times 10^6 = 634.8\ d^2$

d= 470.665 mm

Effective depth =d $\cong$ 480 mm

STEP 3: To find area of steel

a) For balance section

M_d = 0.87 Fy Ast (d- 0.42 Xu max)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 480 = 230.4 mm

$140.625 \times 10^6 = 0.87 \times 415 \times Ast\ (480 - 0.42 \times 230.4)$

A_{st} = 1016.32 mm^2

Compare Ast with Ast minimum

$A_{st\ min} = \dfrac{0.85\ b\ d}{Fy}$ .... IS CODE PAGE NO 47

$$A_{st\,min} = \frac{0.85 \text{ x } 230 \text{ x } 480}{415} = 226.120 \text{ mm}^2$$

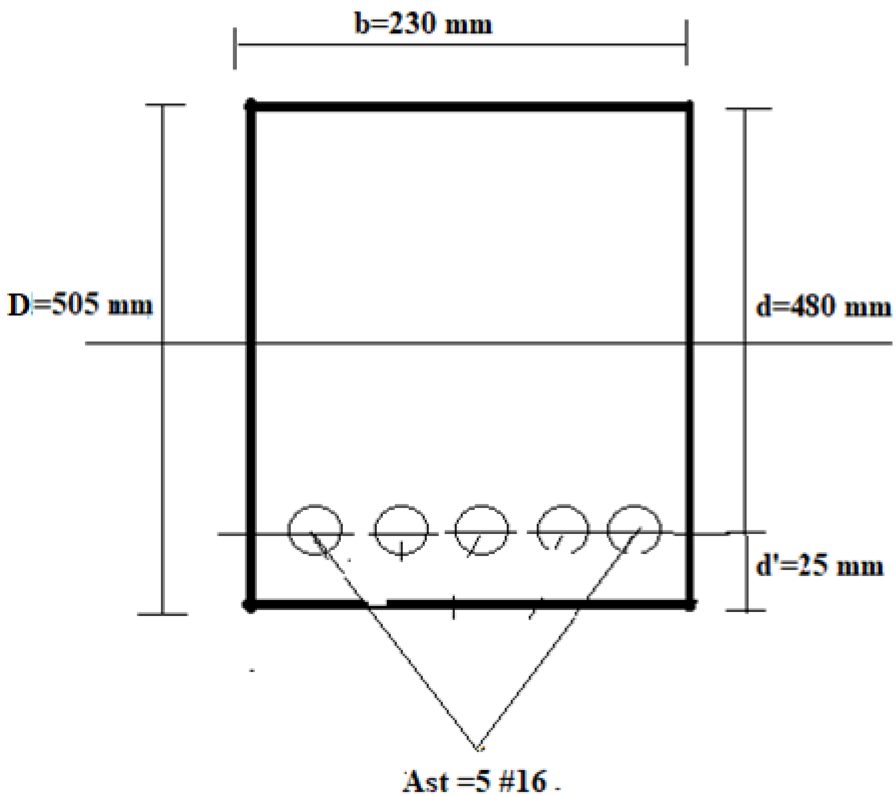
Ast $\succ$ Ast Min (ok)

STEP 4:Number of bars

Assuming diameter of bar = ϕ = 16 mm

Number of bars = $\dfrac{Ast}{(\pi/4) \text{ x } \phi^2}$

Number of bars = $\dfrac{1016.32}{(\pi/4) \text{ x } 16^2}$ = 5.05 $\cong$ **5**



4) Design under reinforced section to resist factored BM of 125 KNm having width 250 mm using M_{20} and Fe 250.

To design of beam

Given Data:- Fy= 250 N/mm²

Fck= 20 N/mm² ,

Max BM = 125 KNm

M_d= 125 X 10^6 Nmm

"b" = 250 mm

To find :- d, Ast

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.148 $Fck\ bd^2$ .... Fe 250

Mu lilmit = 0.148 x 20x 250 d^2

Mu lilmit = 740 d^2

STEP 2: Equating Mulimit to M_d

M_d = Mulimit

125 X 10^6 = 740 d^2

d= 410.99 mm

Effective depth =d $\cong$ 420 mm

STEP 3: To find area of steel

a) For balance section

$$M_d = 0.87 \text{ Fy Ast (d- 0.42 Xu max)}$$

Xu max = 0.53 d ....For Fe 250

Xu max = 0.53 x 420 = 222.6 mm

$$125 \times 10^6 = 0.87 \times 250 \times \text{Ast} (420- 0.42 \times 222.6)$$

A_{st} = 1760.18 mm^2

Compare Ast with Ast minimum

$$A_{st\ min} = \frac{0.85\ b\ d}{Fy}$$ .... IS CODE PAGE NO 47

$$A_{st\ min} = \frac{0.85 \times 250 \times 420}{250}$$ = 357 **mm^2**

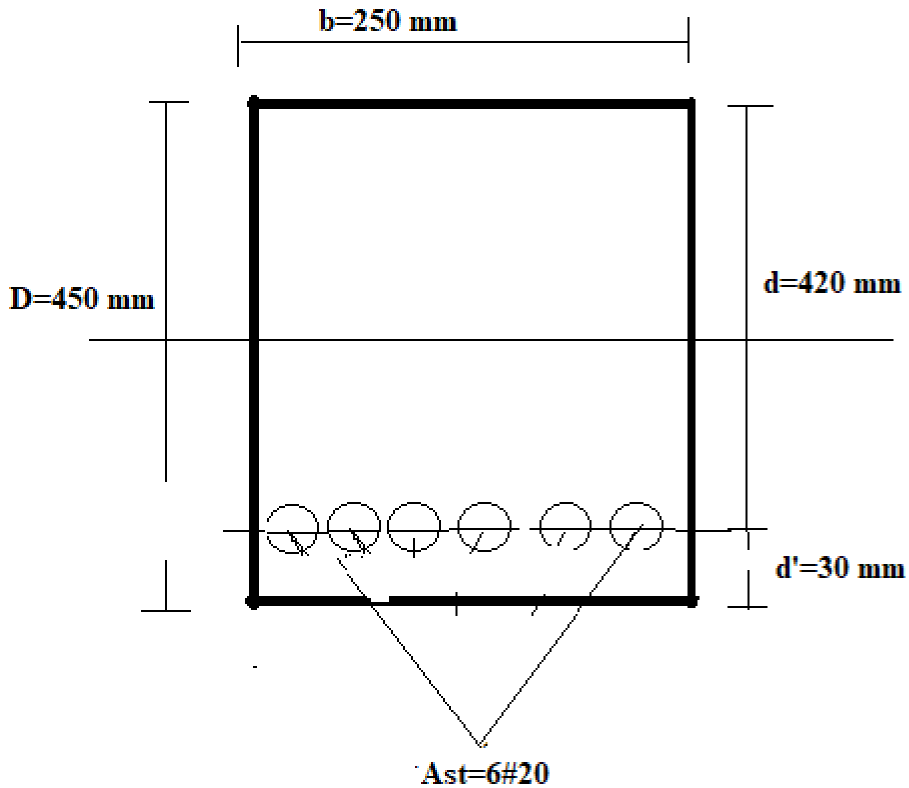
Ast $\succ$ Ast Min (ok)

STEP 4:Number of bars

Assuming diameter of bar = ϕ = 20 mm

Number of bars = $\dfrac{Ast}{(\pi/4) \text{ x } \phi^2}$

Number of bars = $\dfrac{1760.18}{(\pi/4) \text{ x } 20^2}$ = 5.60 $\cong$ **6**



5) Design under reinforced section to resist factored BM of 75 KNm having width 230 mm using M_{15} and Fe 500.

To design of beam

Given Data:- Fy= 500 N/mm^2

Fck= 15 N/mm^2 ,

Max BM = 75 KNm

M_d= 75 X 10^6 Nmm

"b" = 230 mm

To find :- d, Ast

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.133 $Fck\ bd^2$ .... Fe 500

Mu lilmit = 0.133 x 15 x 230 d^2

Mu lilmit = 458.85 d^2

STEP 2: Equating Mulimit to M_d

M_d = Mulimit

75 X 10^6 = 458.85 d^2

d= 404.29 mm

Effective depth =d $\cong$ 410 mm

STEP 3: To find area of steel

a) For balance section

M_d = 0.87 Fy Ast (d- 0.42 Xu max)

Xu max = 0.46 d ....For Fe 500

Xu max = 0.46 x 410 = 188.6 mm

75 x 10^6 = 0.87 x 500 x Ast (41- 0.42 x 188.6)

A_{st} = 521.22 mm^2

Compare Ast with Ast minimum

$$A_{st\,min} = \frac{0.85\ b\ d}{Fy}$$ .... IS CODE PAGE NO 47

$$A_{st\,min} = \frac{0.85 \times 230 \times 410}{500}$$ = 160.31 **mm^2**

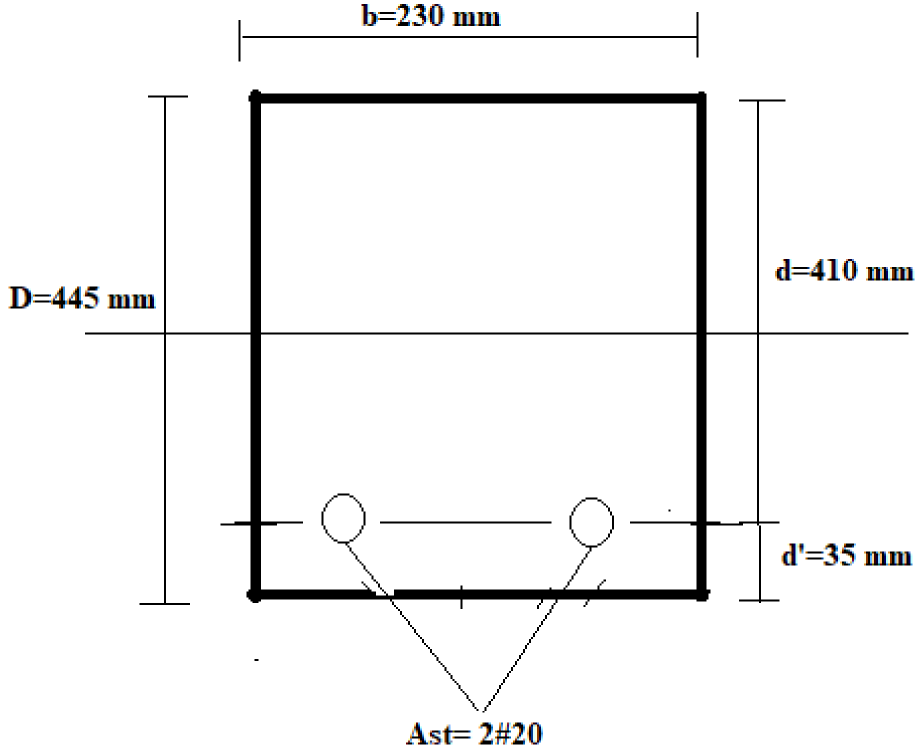
Ast $\succ$ Ast Min (ok)

STEP 4:Number of bars

Assuming diameter of bar = ϕ = 20 mm

Number of bars = $\dfrac{Ast}{(\pi/4) \text{ x } \phi^2}$

Number of bars = $\dfrac{521.22}{(\pi/4) \text{ x } 20^2}$ = 1.66 ≅ **2**



6) To design a rectangular section to resist a factored moment of 90 KNm

a) The Xu should not be greater than 0.3 d

b) Mu = 1.785 b d^2

assuming b = 300 mm use M_{15} and Fe 415.

Solution :- b = 300 mm

M_d = 90 KNm = 90 x 10^6 Nmm

M_{15} = Fck = 15 N/mm^2

Fe 415 = Fy = 415 N/mm^2

CASE I:- The Xu should not be greater than 0.3 d

STEP 1: To find depth of neutral axis (From page No. 96 IS CODE)

1. **The Xu should not be greater than 0.3 d**
   Assuming Xu = 0.3 d
2. **Xu max = 0.48 d ....For Fe 415**
3. Comparing Xu and Xu max
   Xu < Xu max
   0.3 d < 0.48 d
   The section is under reinforced

4. Cu = Tu

$$0.36 Fck Xub = 0.87\ Fy\ Ast$$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b}$$

$$0.3d = \frac{0.87 \times 415 \times Ast}{0.36 \times 15 \times 300}$$

Ast = 1.346 d

5: To find moment of resistance

a) For under reinforced section **(From page No. 96 IS CODE)**

$$Mu = 0.87\ Fy\ Ast\ (d - 0.42\ Xu)$$

$$90 \times 10^6 = 0.87 \times 415 \times 1.346\ d\ (d - 0.42 \times 0.3d)$$

Effective depth = d = 460.32 mm ≅ 470 mm

Ast = 1.346 x 470 = 632.62 mm^2

Number of bars

Assuming diameter of bar = ϕ = 20 mm

$$\textbf{Number of bars} = \frac{Ast}{(\pi/4) \times \phi^2}$$

$$\textbf{Number of bars} = \frac{632.62}{(\pi/4) \times 20^2} = 2.01 \cong \mathbf{2}$$

CASE II:- Mu = 1.785 b d^2

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.138 $Fck\ bd^2$ .... Fe 415

Mu lilmit = 0.138 X 15 X b X d^2 = 2.07 bd^2

STEP 2: To compare M_d and $M_{U\ limit}$

1.785 b d^2 $\prec$ 2.07 bd^2

$M_d \prec M_{U\ limit}$ section is Under reinforced Section

Mu = 1.785 b d^2

90 x 10^6 = 1.785 x 300 x d^2

d= 409.96 mm $\cong$ 410 mm

STEP 3: Equating M_d and $M_{U\ limit}$ and calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckbd^2}}\right]bd$$

$$A_{st} = \frac{0.5 \text{ x } 15}{415}\left[1-\sqrt{1-\frac{4.6 \text{ x } 90 \text{ x } 10^6}{15 \text{ x } 300 \text{ x } 410^2}}\right]300 \text{ x } 410 = 727.25 \text{ mm}^2$$

STEP 4: Number of bars

Assuming diameter of bar = ϕ = 20 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2}$

Number of bars = $\frac{727.25}{(\pi/4) \text{ x } 20^2}$ = 2.31 $\cong$ **3**

7) A fixed beam of a span 5m carries superimposed load of 19 KN/m over a whole span assuming rectangular cross section 230 mm X 450 mm. Find the area of steel required at mid span and at the support. Take effective cover 50 mm. use M_{20} and Fe 250.

Solution :- b = 230 mm

D = 450 mm

d' = 50 mm

Effective depth =d = D- d' = 450 -50 =400 mm

superimposed load = 19 KN/m

Span = L = 5 m

M_{20} = Fck = 20 N/mm^2

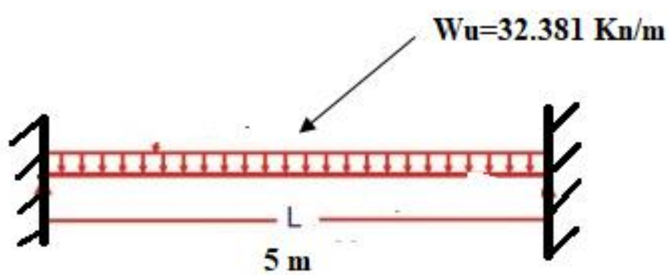
Fe250 = Fy = 250 N/mm^2

Self weight of beam = b x D x 25 = 0.23 x .45 x 25 = 2.5875 KN/ m

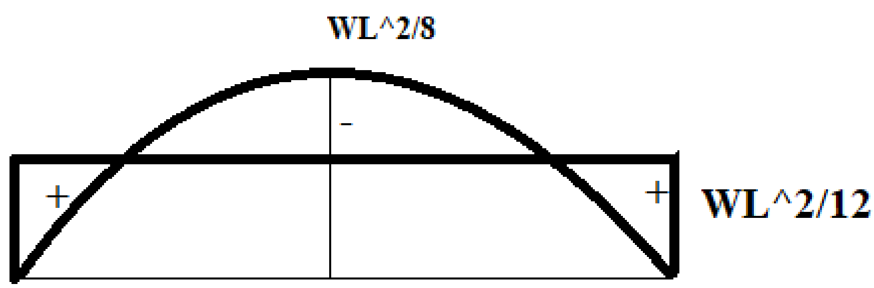
Total load on beam = Superimposed load + Self weight of beam

Total load on beam = 19 +2.5875 = 21.5875 KN/ m

Total factored load on beam=Wu = 1.5 x 21.5875 = 32.381 KN/ m



BM at support = $\frac{Wul^2}{12} = \frac{32.381 \text{ x } 5^2}{12}$ = 67.44 KN m



BM at mid span = $\frac{Wul^2}{8} - \frac{Wul^2}{12} = \frac{32.381\text{x } 5^2}{8} - \frac{32.381 \text{ x } 5^2}{12}$ = 33.75 KN m

BM at support M_d = 67.44 KN m

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.148 $Fck\ bd^2$ .... Fe 250

Mu lilmit = 0.148 X 20 X 230 X 400^2 = 108.93 X 10^6 KNm=108.93 KN m

STEP 2: To compare M_d and $M_{U\ limit}$

67.44 $\prec$ 108.93

$M_d \prec M_{U\ limit}$ section is Under reinforced Section

STEP 3: Equating M_d and $M_{U\ limit}$ and calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1\text{-}\sqrt{1\text{-}\frac{4.6M_d}{Fckbd^2}}\right]bd$$

$$A_{st} = \frac{0.5 \text{ x } 20}{250}\left[1\text{-}\sqrt{1\text{-}\frac{4.6 \text{ x } 67.46 \text{ x } 10^6}{20 \text{ x } 230 \text{ x } 400^2}}\right]230 \text{ x } 400 = 881.32 \text{ mm}^2$$

STEP 4:Number of bars

Assuming diameter of bar = ϕ = 20 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2}$

Number of bars = $\frac{881.32}{(\pi/4) \text{ x } 20^2}$ = 2.81 $\cong$ **3**

BM at mid span design B M = M_d = 33.75 KN m

STEP 1: To compare M_d and $M_{U\ limit}$

33.75 $\prec$ 108.93

$M_d \prec M_{U\ limit}$ section is Under reinforced Section

STEP 2: Equating M_D and $M_{U\ limit}$ and calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1\text{-}\sqrt{1\text{-}\frac{4.6M_d}{Fckbd^2}}\right]bd$$

$$A_{st} = \frac{0.5 \text{ x } 20}{250}\left[1\text{-}\sqrt{1\text{-}\frac{4.6 \text{ x } 33.75 \text{ x } 10^6}{20 \text{ x } 230 \text{ x } 400^2}}\right]230 \text{ x } 400 = 411.09 \text{ mm}^2$$

STEP 3:Number of bars

Assuming diameter of bar = ϕ = 20 mm

Number of bars = $\dfrac{Ast}{(\pi/4) \text{ x } \phi^2}$

Number of bars = $\dfrac{411.09}{(\pi/4) \text{ x } 20^2}$ = 1.31 $\cong$ **2**

8) Design a RC singly reinforced rectangular section for the span of 5 m with both ends fixed carries udl of 15 KN/m acting on whole span inclusive self weight. Use M_{20} and Fe 415.

Solution :-

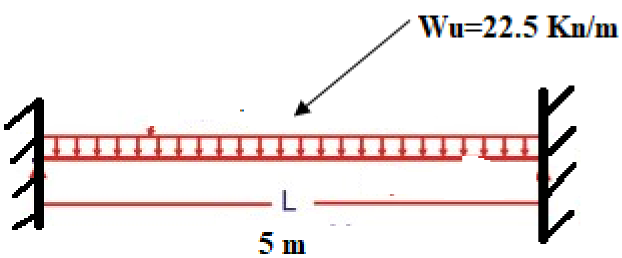
udl = 15 KN/m

Span = L = 5 m

M_{20} = Fck = 20 N/mm^2

Fe415 = Fy = 415 N/mm^2

Total factored udl on beam = 1.5 x 15 = 22.5 KN/ m



BM at support = $\dfrac{Wul^2}{12} = \dfrac{22.5 \text{ x } 5^2}{12}$ = 46.88 KN m

Maximum BM M_d = 46.88 KN m

STEP 1: Calculate ultimate moment of resistance

Mu lilmit = 0.138 $Fck\ bd^2$ .... Fe 415

Assuming b = 230 mm

0.138 X 20 X 230 X d^2

STEP 1: To compare M_D and $M_{U\,limit}$

$46.88 \times 10^6 = 0.138 \times 20 \times 230 \times d^2$

Effective depth = d = 271.74 mm ≅ 280 mm

$M_d \prec M_{U\,limit}$ section is Under reinforced Section

STEP 3: Equating M_d and $M_{U\,limit}$ and calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckbd^2}}\right]bd$$

$$A_{st} = \frac{0.5 \text{ x } 20}{415}\left[1-\sqrt{1-\frac{4.6 \text{ x } 46.8745 \text{ x } 10^6}{20 \text{ x } 230 \text{ x } 280^2}}\right]230 \text{ x } 280 = 567.78 \text{ mm}^2$$

STEP 4:Number of bars

Assuming diameter of bar = ϕ = 20 mm

Number of bars = $\dfrac{Ast}{(\pi / 4) \text{ x } \phi^2}$

Number of bars = $\dfrac{567.78}{(\pi / 4) \text{ x } 20^2}$ = 1.81 ≅ **2**

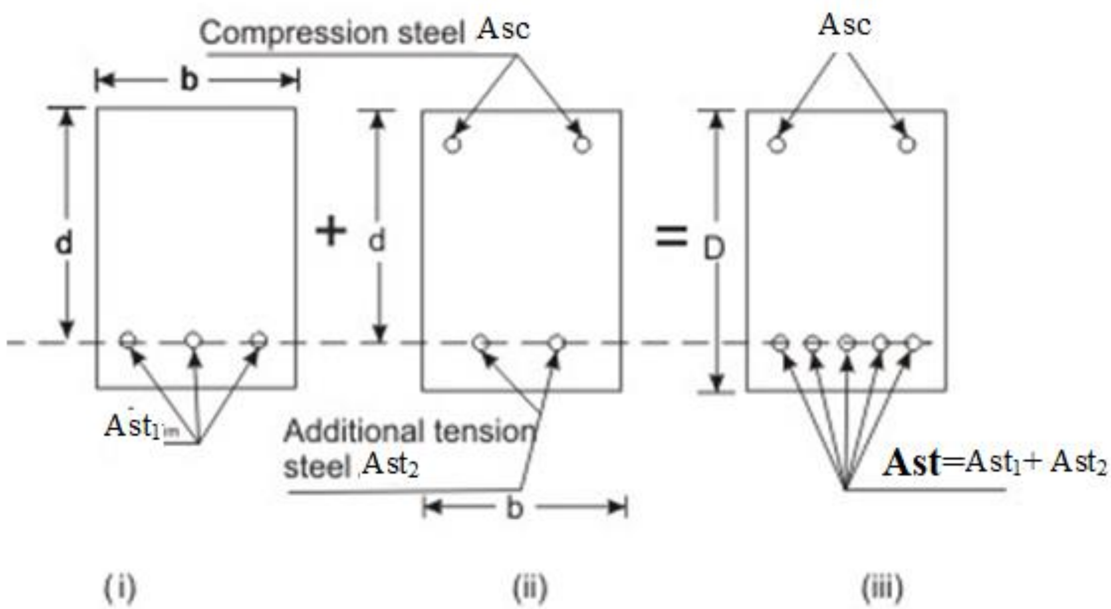
Doubly Reinforced Beams

Doubly Reinforced Beams :- These are those section in which R/F is provided on tension as well as compression side.

Condition for doubly reinforced section:-

1. When section are restricted due to architectural consideration.
2. To reduce deflection of beam
3. In the continuous beam at intermediate support, the beam is design doubly reinforced beam.
4. The member which are subjected to reversal of stress.
5. In case of vibration in structures.

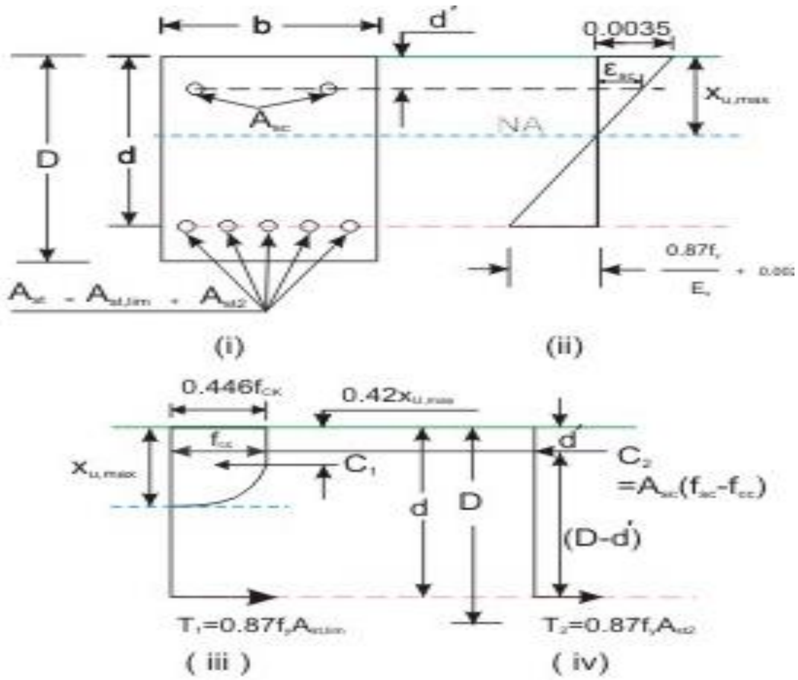
DESIGN PROCEDURE FOR DOUBLY REINFORCED BEAM



DOUBLY REINFORCED BEAM

A doubly reinforced beam section is normally provided when a given concrete structure with maximum steel Ast on tension is in adequate resist to given design moment. The inadequacy is made up by adding the steel on tension side equal to Ast_2 but also adding steel on compression Asc.

Balance section with concrete for resisting compression and balance tension steel Ast_1 resisting tension.



STRESS STRAIN DIAGRAM & FORCE DIAGRAM FOR DOUBLY REINFORCED BEAM

Cu_1 = Compressive force in concrete

Cu_1 = 0.36 Fck Xu b

Cu_2 = Compressive force in compressive steel

Cu_2 = Asc (Fsc - Fcc)

Fsc = Stress in compressive steel

Fsc = 0.85 Fy...Fe 250

Fcc = Stress in concrete at level of d'

Fcc = 0.45 Fck

$Mu = Cu_1 \; Z_1 + Cu_2 \; Z_2$

where,

Z_1 = (d- 0.42 Xu)

Z_2 = (d-d')

$Mu = Cu_1 \; Z_1 + Cu_2 \; Z_2$

Mu = 0.36 Fck Xu b_1 (d- 0.42 Xu) + Asc (Fsc - Fcc) (d-d')

IN DOUBLY REINFORCED BEAM THERE ARE THREE TYPES OF PROBLEMS

1) To find moment of resistance of section

2) To find the area of steel i.e (Asc and Ast)

3) Design the section

TYPE I

1) To find moment of resistance of section

CASE I : - If Fe 250 is given in numerical

STEPWISE PROCESURE

STEP 1: Given Data

b, d, d', Asc ,Ast, Fck, Fy

STEP 2: To find depth of neutral axis (Xu)

Compressive force = Tensile force

Cu =Tu

$Cu_1 + Cu_2 = Tu$

0.36 Fck Xu b + Asc (Fsc -Fcc) = 0.87 Fy Ast

Xu = ?

Fsc = Stress in compressive steel
Fsc = 0.85 Fy..Fe 250
Fcc = Stress in concrete at level of d'
Fcc = 0.45 Fck

STEP 3: To find Moment of resistance of section

$Mu = Cu_1 \; Z_1 + Cu_2 \; Z_2$
Mu = 0.36 Fck Xu b (d- 0.42 Xu) + Asc (Fsc - Fcc) (d-d')

1) Find moment of resistance of rectangular section 300 mm X 380 mm effective with 6 bars of 20 mm diameter on tension side and 2 bars of 20 mm diameter on compressive side and effective cover is 40 mm. Use M_{15} and Fe 250.

Solution:-

STEP 1: -Given Data

Width of rectangular section = b = 300 mm

Effective Depth of rectangular section = d = 380 mm

Effective cover = d' = 40 mm

Number of bar on tension side = 6

Area of steel on tension side = Ast = 6 X $\frac{\pi}{4} X \;\; \phi^2 = 6X \; \frac{\pi}{4} X \; 20^2 = 1884.95 \text{ mm}^2$

Number of bar on compressive side = 2

Area of steel on compressive side = Asc =2 X $\frac{\pi}{4} X \ \phi^2 = 2 \text{ X } \frac{\pi}{4} X \ 20^2$ =628.32 mm^2

M_{15} = Fck = 15 N/ mm^2

Fe 250 =Fy = 250 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 15 = 6.75 N/ mm^2

Fsc = 0.87 Fy = 0.87 X 250 = 217.5 N/ mm^2

STEP 2: To find depth of neutral axis (Xu)

Compressive force = Tensile force

Cu =Tu

$Cu_1 + Cu_2$ = Tu

0.36 Fck Xu b + Asc (Fsc -Fcc) = 0.87 Fy Ast

0.36 x 15x Xu x 300 + 628.32 x(217.5 - 6.75) = 0.87 x 250x 1884.95

Xu = 171.33 mm

STEP 3: To find Moment of resistance of section

Mu = $Cu_1 \ Z_1 + Cu_2 \ Z_2$

Mu = 0.36 Fck Xu b (d- 0.42 Xu) + Asc (Fsc - Fcc) (d-d')

Mu = 0.36 x 15 x 171.33 x300 (380 - 0.42 x 171.33) + 628.32 x(217.5 - 6.75) (380-40)

Mu = 130.52 X 10^6 Nmm

Mu = 130.52 KNm

2) Find moment of resistance of rectangular section 230 mm X 460 mm effective with 4 bars of 20 mm diameter on tension side and 2 bars of 16 mm diameter on compressive side and effective cover is 40 mm. Use M_{15} and Fe 250.

Solution:-

STEP 1: -Given Data

Width of rectangular section = b = 230 mm

Effective Depth of rectangular section = d = 460 mm

Effective cover = d' = 40 mm

Number of bar on tension side = 4

Area of steel on tension side = Ast =4X $\frac{\pi}{4} X \ \phi^2 = 4\text{X } \frac{\pi}{4} X \ 20^2$ = 1256.63 mm^2

Number of bar on compressive side = 2

Area of steel on compressive side = Asc =2 X $\frac{\pi}{4} X \ \phi^2 = 2 \text{ X } \frac{\pi}{4} X \ 16^2$ = 402.12 mm^2

M_{15} = Fck = 15 N/ mm^2

Fe 250 =Fy = 250 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 15 = 6.75 N/ mm^2

Fsc = 0.87 Fy = 0.87 X 250 = 217.5 N/ mm^2

STEP 2: To find depth of neutral axis (Xu)

Compressive force = Tensile force

Cu =Tu

$Cu_1 + Cu_2 = Tu$

0.36 Fck Xu b + Asc (Fsc -Fcc) = 0.87 Fy Ast

0.36 x 15x Xu x 230 + 402.12 x(217.5 - 6.75) = 0.87 x 250x 1256.63

Xu = 151.83 mm

STEP 3: To find Moment of resistance of section

$Mu = Cu_1 \; Z_1 + Cu_2 \; Z_2$

Mu = 0.36 Fck Xu b (d- 0.42 Xu) + Asc (Fsc - Fcc) (d-d')

Mu = 0.36 x 15 x 151.33 x 230 (460 - 0.42 x 151.83) + 402.12 x (217.5 - 6.75) (460-40)

Mu = 110.31 X 10^6 Nmm

Mu = 110.31 KNm

3) Find moment of resistance of doubly R/F rectangular section 250 mm X 450 mm effective with 2 bars of 25 mm diameter on tension side and 2 bars of 16 mm diameter on compressive side and effective cover is 50 mm. Use M_{15} and Fe 250.

Solution:-

STEP 1: -Given Data

Width of rectangular section = b = 250 mm

Effective Depth of rectangular section = d = 450 mm

Effective cover = d' = 50 mm

Number of bar on tension side = 2

Area of steel on tension side = Ast = 2 X $\frac{\pi}{4} X \; \phi^2 = 2 \text{ X } \frac{\pi}{4} X \; 20^2 =$ 981.74 mm^2

Number of bar on compressive side = 2

Area of steel on compressive side = Asc =2 X $\frac{\pi}{4} X \; \phi^2 = 2 \text{ X } \frac{\pi}{4} X \; 16^2 =$ 402.12 mm^2

M_{15} = Fck = 15 N/ mm^2

Fe 250 =Fy = 250 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 15 = 6.75 N/ mm^2

Fsc = 0.87 Fy = 0.87 X 250 = 217.5 N/ mm^2

STEP 2: To find depth of neutral axis (Xu)

Compressive force = Tensile force

Cu =Tu

$Cu_1 + Cu_2 = Tu$

0.36 Fck Xu b + Asc (Fsc -Fcc) = 0.87 Fy Ast

0.36 x 15x Xu x 250 + 402.12 x(217.5 - 6.75) = 0.87 x 250x 981.74

Xu = 95.39 mm

STEP 3: To find Moment of resistance of section

$Mu = Cu_1 \; Z_1 + Cu_2 \; Z_2$

Mu = 0.36 Fck Xu b (d- 0.42 Xu) + Asc (Fsc - Fcc) (d-d')

Mu = 0.36 x 15 x 95.39 x 250 (450 - 0.42 x 95.39) + 402.12 x(217.5 - 6.75) (450-50)

$Mu = 86.68 \times 10^6$ Nmm

Mu = 86.68 KNm

<u>CASE II : - If Fe 415 or Fe 500 is given in numerical</u>

If Fe 415 or Fe500 is given in numerical then use stress-strain relationship

Sr No	Strain	Stress (N/mm^2)
1	144×10^{-5}	288.00
2	145×10^{-5}	289.65
3	150×10^{-5}	294.88
4	160×10^{-5}	303.86
5	170×10^{-5}	311.07
6	180×10^{-5}	317.07
7	190×10^{-5}	323.55
8	200×10^{-5}	327.14
9	210×10^{-5}	331.41
10	220×10^{-5}	335.15
11	230×10^{-5}	338.76
12	240×10^{-5}	342.43
13	250×10^{-5}	345.11
14	260×10^{-5}	347.67
15	270×10^{-5}	350.26
16	280×10^{-5}	352.14
17	290×10^{-5}	353.02
18	300×10^{-5}	353.90
19	310×10^{-5}	354.77
20	320×10^{-5}	355.65
21	330×10^{-5}	356.52
22	340×10^{-5}	357.40
23	350×10^{-5}	358.27
24	360×10^{-5}	359.15
25	370×10^{-5}	360.02
26	380×10^{-5}	360.09

NOTE: - **This table is prepared from IS Code having page number 70 from figure number 23 A (Cold Worked Deformed Bar).**
If table is not given in exam then use IS Code having page number 70 from figure number 23 A (Cold Worked Deformed Bar).

1) Find moment of resistance of rectangular beam section 300 mm X 500 mm effective with 4 bars of 22 mm diameter on tension side and 4 bars of 16 mm on compressive side and effective cover is 50 mm. Use M_{15} and Fe 415.

STEP 1: -Given Data

Width of rectangular section = b = 300 mm

Effective Depth of rectangular section = d = 500 mm

Effective cover = d' = 50 mm

Number of bar on tension side = 4

Area of steel on tension side = Ast = 4 X $\frac{\pi}{4} X \ \phi^2 = 4 X \ \frac{\pi}{4} X \ 22^2 = 1520.53 \text{ mm}^2$

Number of bar on compressive side = 4

Area of steel on compressive side = Asc = 4 X $\frac{\pi}{4} X \ \phi^2 = 4 X \ \frac{\pi}{4} X \ 16^2 = 804.25 \text{ mm}^2$

M_{15} = Fck = 15 N/ mm^2

Fe 415 =Fy = 415 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 15 = 6.75 N/ mm^2

STEP 2: To find depth of neutral axis (Xu)

Compressive force = Tensile force

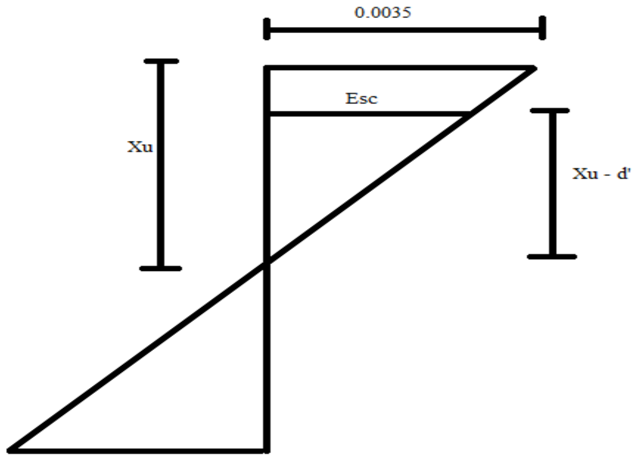
Cu =Tu

$Cu_1 + Cu_2$ = Tu

0.36 Fck Xu b + Asc (Fsc -Fcc) = 0.87 Fy Ast

0.36 x 15x Xu x 300 + 804.25 x(Fsc - 6.75) = 0.87 x 415 x 1520.53

1620 Xu + 804.25 Fsc - 804.25 x 6.75 = 548.98 X 10^3
1620 Xu + 804.25 Fsc = 554.415 X 10^3
Dividing above equation by 1620
Xu + 0.4967 Fsc = 342.23...(1)
Xu max = 0.48 d
Xu max = 0.48 x 500 = 240 mm
Trial 1 :- Assuming Xu = 230 mm (IS 456:2000, P. No:96, C No: G.1.2



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad \frac{230}{0.0035} = \frac{230 - 50}{\xi sc}$$

$\xi sc = 274 \text{ X } 10^{-5}$

From table

ξsc	**Fsc**
270 X 10^{-5}	**350.26**
274 X 10^{-5}	**?**
280 X 10^{-5}	**352.14**

Fsc = 351.01 N/ mm^2 substituting in equation (1)

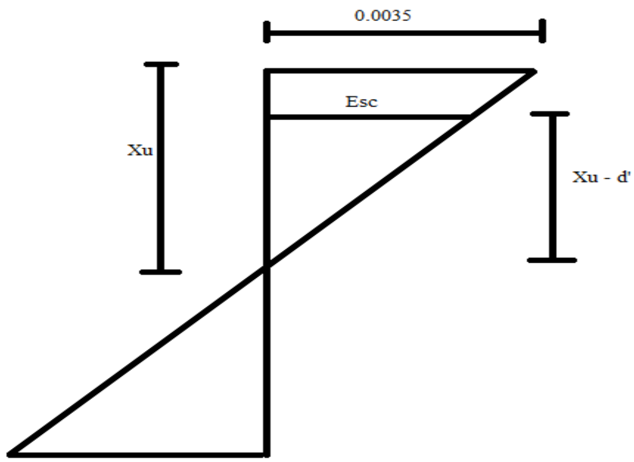
Xu + 0.4967 Fsc = 342.23

Xu + 0.4967 X 351.01 = 342.23

Xu = 167.88 mm

Assume value of Xu = 230 mm and current value of Xu = 167.88 mm, this two value not equal to each other. Therefore above assumption for Xu = 230 mm is wrong.

Trial 2 :- Assuming Xu = 170 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad \frac{170}{0.0035} = \frac{170 - 50}{\xi sc}$$

$\xi sc = 247 \text{ X } 10^{-5}$

From table

ξsc	**Fsc**
240 X 10^{-5}	**342.43**
247 X 10^{-5}	**?**
250 X 10^{-5}	**345.11**

Fsc = 351.01 N/ mm^2 substituting in equation (1)

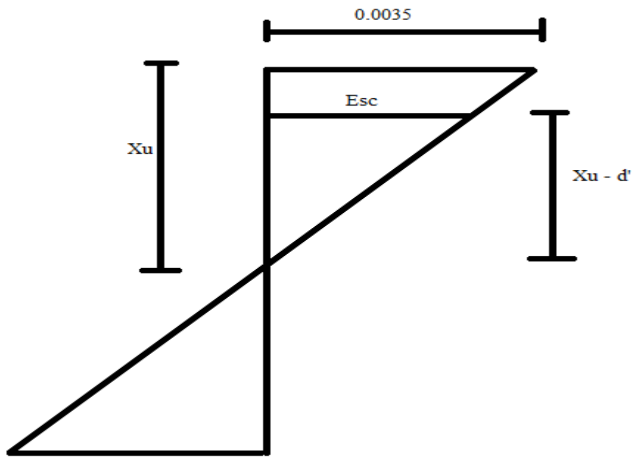
Xu + 0.4967 Fsc = 342.23

Xu + 0.4967 X 351.11 = 342.23

Xu = 171.21 mm

Assume value of Xu = 170 mm and current value of Xu = 171.21 mm, this two value not equal to each other. Therefore above assumption for Xu = 170 mm is wrong.

Trial 3 :- Assuming Xu = 171 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad \frac{171}{0.0035} = \frac{171 - 50}{\xi sc}$$

$\xi sc = 247.66 \text{ X } 10^{-5}$

From table

ξsc	**Fsc**
240 X 10^{-5}	**342.43**
247.66 X 10^{-5}	**?**
250 X 10^{-5}	**345.11**

Fsc = 344.48 N/ mm^2 substituting in equation (1)

Xu + 0.4967 Fsc = 342.23

Xu + 0.4967 X 344.48 = 342.23

Xu = 171.13 mm

Assume value of Xu = 171 mm and current value of Xu = 171.13 mm, this two value nearly equal to each other. Therefore above assumption for Xu = 171 mm is correct.

Xu $\cong$ 171 mm

Xu max = 0.48 d = 0.48 X 500 = 240 mm

Xu < Xumax

The section under reinforced section

STEP 3: To find Moment of resistance of section

$Mu = Cu_1 \ Z_1 + Cu_2 \ Z_2$

Mu = 0.36 Fck Xu b (d- 0.42 Xu) + Asc (Fsc - Fcc) (d-d')

Mu = 0.36 x 15 x 171 x 300 (500 - 0.42 x 171) + 804.25 x(344.48 - 6.75) (500-50)

$Mu = 240.77 \ X \ 10^6$ Nmm

Mu = 240.77 KNm

2) Find moment of resistance of rectangular beam section 250 mm X 450 mm effective with 4 bars of 18 mm diameter on tension side and 2 bars of 14 mm diameter on compressive side and effective cover is 35 mm. Use M_{20} and Fe 415.

STEP 1: -Given Data

Width of rectangular section = b = 250 mm

Effective Depth of rectangular section = d = 450 mm

Effective cover = d' = 35 mm

Number of bar on tension side = 4

Area of steel on tension side = Ast = 4 X $\frac{\pi}{4} X \ \phi^2 = 4 \ X \ \frac{\pi}{4} X \ 18^2 = 1017.88 \ mm^2$

Number of bar on compressive side = 2

Area of steel on compressive side = Asc = 2 X $\frac{\pi}{4} X \ \phi^2 = 2 \ X \ \frac{\pi}{4} X \ 14^2 = 307.88 \ mm^2$

M_{20} = Fck = 20 N/ mm^2

Fe 415 =Fy = 415 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 20 = 9 N/ mm^2

STEP 2: To find depth of neutral axis (Xu)

Compressive force = Tensile force

Cu =Tu

$Cu_1 + Cu_2 = Tu$

0.36 Fck Xu b + Asc (Fsc -Fcc) = 0.87 Fy Ast

0.36 x 20 x Xu x 250 + 307.88 x(Fsc - 9) = 0.87 x 415 x 1017.88

1800 Xu + 307.88 Fsc - 2770.92 = 367.51 X 10^3

1800 Xu + 307.88 Fsc = 370.28 X 10^3

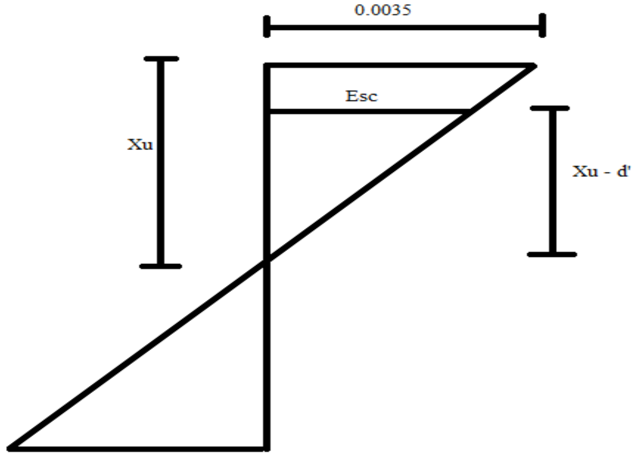
Dividing above equation by 1800

Xu + 0.1710 Fsc = 205.71...(1)

Xu max = 0.48 d

Xu max = 0.48 x 450 = 216 mm

Trial 1 :- Assuming Xu = 200 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad \frac{200}{0.0035} = \frac{200 - 35}{\xi sc}$$

$\xi sc = 288.75 \text{ X } 10^{-5}$

From table

ξsc	**Fsc**
280 X 10^{-5}	**352.12**
288.75 X 10^{-5}	**?**
290 X 10^{-5}	**353.02**

Fsc = 352.91 N/ mm^2 substituting in equation (1)

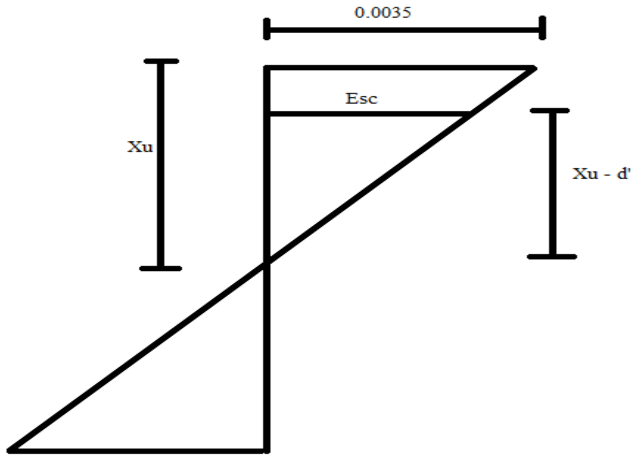
Xu + 0.1710 Fsc = 205.71

Xu + 0.1710 x 352.91 = 205.71

Xu = 145.36 mm

Assume value of Xu = 200 mm and current value of Xu = 145.36 mm, this two value not equal to each other. Therefore above assumption for Xu = 200 mm is wrong.

Trial 2 :- Assuming Xu = 145 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad\qquad \frac{145}{0.0035} = \frac{145 - 35}{\xi sc}$$

$\xi sc = 265.52 \text{ X } 10^{-5}$

From table

ξsc	**Fsc**
260 X 10^{-5}	**347.67**
265.52X 10^{-5}	**?**
270 X 10^{-5}	**350.26**

Fsc = 349.10 N/ mm^2 substituting in equation (1)

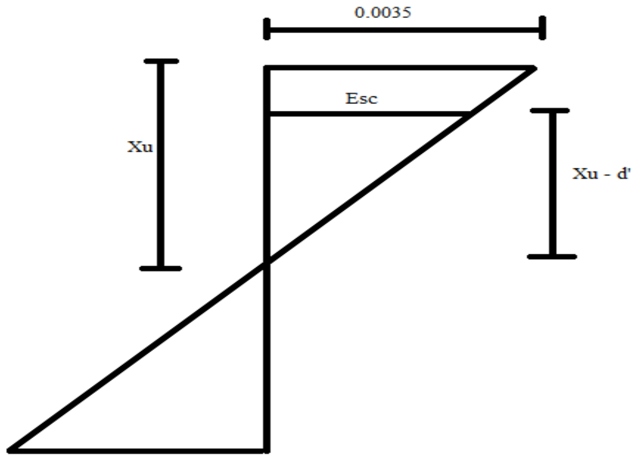
Xu + 0.1710 Fsc = 205.71

Xu + 0.1710 x 349.10 = 205.71

Xu = 146.01 mm

Assume value of Xu = 145 mm and current value of Xu = 146.01 mm, this two value not equal to each other. Therefore above assumption for Xu = 145 mm is wrong.

Trial 3 :- Assuming Xu = 146 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad \frac{146}{0.0035} = \frac{146 - 35}{\xi sc}$$

$\xi sc =$ 266.10 X 10^{-5}

From table

ξsc	**Fsc**
260 X 10^{-5}	**347.67**
266.10 X 10^{-5}	**?**
270 X 10^{-5}	**350.26**

Fsc = 349.25 N/ mm^2 substituting in equation (1)

Xu + 0.1710 Fsc = 205.71

Xu + 0.1710 x 349.25 = 205.71

Xu = 145.99 mm

Assume value of Xu = 146 mm and current value of Xu = 145.99 mm, this two value nearly equal to each other. Therefore above assumption for Xu = 146 mm is correct.

Xu $\cong$ 146 mm

Xu max = 0.48 d = 0.48 X 450 = 216 mm

Xu < Xumax

The section is under reinforcement

STEP 3: To find Moment of resistance of section

Mu = Cu_1 Z_1 + Cu_2 Z_2

Mu = 0.36 Fck Xu b (d- 0.42 Xu) + Asc (Fsc - Fcc) (d-d')

Mu = 0.36 x 20 x 146 x 250 (450 - 0.42 x 146) + 307.88 x(349.25 - 9) (450-35)

Mu = 145.62 X 10^6 Nmm

Mu = 145.62 KNm

3) A doubly reinforced rectangular beam section is 250 mm wide , the effective depth of section 650 mm. The effective cover to both compressive and tensile reinforcements is 50 mm. The tension reinforcement with 4 bars of 16 mm diameter of tension side and 4 bars of 10 mm diameter on compressive Use M_{20} and Fe 415.

STEP 1: -Given Data

Width of rectangular section = b = 250 mm

Effective Depth of rectangular section = d = 650 mm

Effective cover = d' = 50 mm

Number of bar on tension side = 4

Area of steel on tension side = Ast = 4 X $\frac{\pi}{4} X\ \phi^2$ = 4 X $\frac{\pi}{4} X\ 16^2$ = 804.23 mm^2

Number of bar on compressive side = 4

Area of steel on compressive side = Asc = 4 X $\frac{\pi}{4} X \ \phi^2 = 4 \text{ X } \frac{\pi}{4} X \ 10^2 = 314.16 \text{ mm}^2$

M_{20} = Fck = 20 N/ mm^2

Fe 415 =Fy = 415 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 20 = 9 N/ mm^2

STEP 2: To find depth of neutral axis (Xu)

Compressive force = Tensile force

Cu =Tu

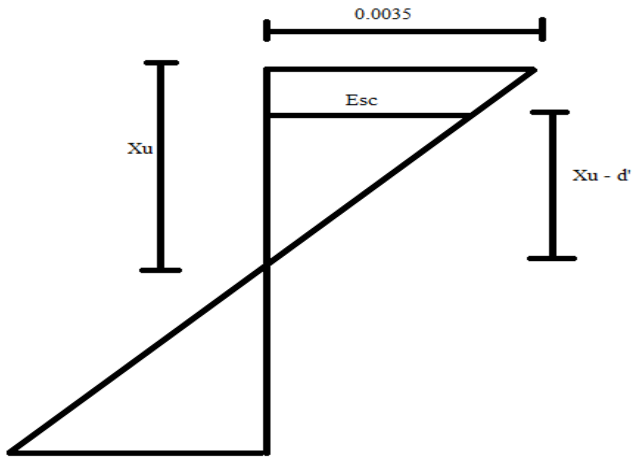
$Cu_1 + Cu_2$ = Tu

0.36 Fck Xu b + Asc (Fsc -Fcc) = 0.87 Fy Ast

0.36 x 20 x Xu x 250 + 314.16 x(Fsc - 9) = 0.87 x 415 x 804.23

1800 Xu + 314.16 Fsc - 2827.44 = 290.37 X 10^3
1800 Xu + 314.16 Fsc = 293.197 X 10^3

Dividing above equation by 1800
Xu + 0.1745 Fsc = 162.887..(1)
Xu max = 0.48 d
Xu max = 0.48 x 650 = 312 mm
Trial 1 :- Assuming Xu = 310 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu \text{ - d'}}{\xi sc} \qquad\qquad \frac{310}{0.0035} = \frac{310 \text{ - } 50}{\xi sc}$$

$\xi sc =$ 293.55 X 10^{-5}

From table

ξsc	Fsc
290 X 10^{-5}	**353.02**
293.55X 10^{-5}	**?**

300 X 10^{-5}	**353.90**

Fsc = 353.33 N/ mm^2 substituting in equation (1)

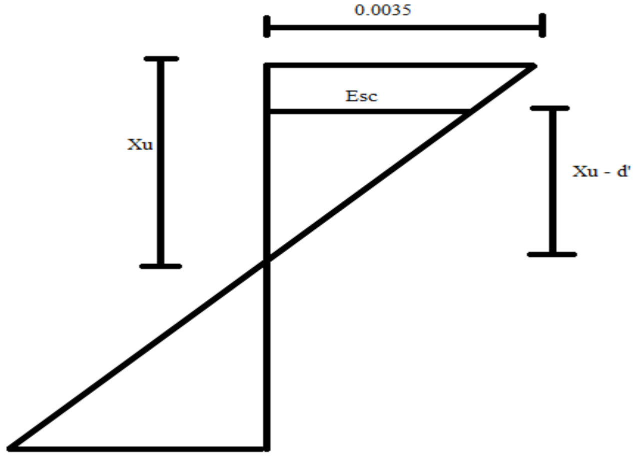
Xu + 0.1745 Fsc = 162.887

Xu + 0.1745 x 353.33 = 162.887

Xu = 101.23 mm

Assume value of Xu = 310 mm and current value of Xu = 101.23 mm, this two value not equal to each other. Therefore above assumption for Xu = 310 mm is wrong.

Trial 2 :- Assuming Xu = 100 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad \frac{100}{0.0035} = \frac{100 - 50}{\xi sc}$$

$\xi sc = 175 \text{ X } 10^{-5}$

From table

ξsc	Fsc
170 X 10^{-5}	**311.07**
175X 10^{-5}	**?**
180 X 10^{-5}	**317.31**

Fsc = 314.19 N/ mm^2 substituting in equation (1)

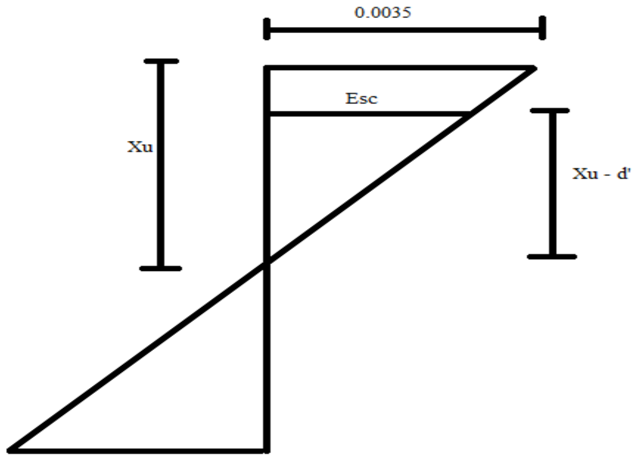
Xu + 0.1745 Fsc = 162.887

Xu + 0.1745 x 314.19 = 162.887

Xu = 108.06 mm

Assume value of Xu = 100 mm and current value of Xu = 108.06 mm, this two value not equal to each other. Therefore above assumption for Xu = 100 mm is wrong.

Trial 3 :- Assuming Xu = 107 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad\qquad \frac{107}{0.0035} = \frac{107 - 50}{\xi sc}$$

$\xi sc = 186.45 \times 10^{-5}$

From table

ξsc	**Fsc**
180×10^{-5}	**317.31**
186.45×10^{-5}	**?**
190×10^{-5}	**323.55**

Fsc = 321.33 N/ mm^2 substituting in equation (1)

Xu + 0.1745 Fsc = 162.887

Xu + 0.1745 x 321.33 = 162.887

Xu = 106.81 mm

Assume value of Xu = 107 mm and current value of Xu = 106.81 mm, this two value nearly equal to each other. Therefore above assumption for Xu = 107 mm is correct.

Xu $\cong$ 107 mm

Xu max = 0.48 d = 0.48 X 650 = 312 mm

Xu $<$ Xumax

The section is under reinforcement

STEP 3: To find Moment of resistance of section

$Mu = Cu_1 \; Z_1 + Cu_2 \; Z_2$

Mu = 0.36 Fck Xu b (d- 0.42 Xu) + Asc (Fsc - Fcc) (d-d')

Mu = 0.36 x 20 x 107 x 250 (650 - 0.42 x 107) + 314.16 x(321.33 - 9) (650-50)

$Mu = 175.41 \times 10^{6}$ Nmm

Mu = 175.41 KNm

4) A doubly reinforced rectangular beam section is 250 mm wide , the effective depth of section 450 mm. The effective cover to both compressive and tensile reinforcements is 35 mm. The tension reinforcement with 4 bars of 22 mm diameter of tension side and 2 bars of 14 mm diameter on compressive Use M_{20} and Fe 415.

STEP 1: -Given Data

Width of rectangular section = b = 250 mm

Effective Depth of rectangular section = d = 450 mm

Effective cover = d' = 35 mm

Number of bar on tension side = 4

Area of steel on tension side = Ast = 4 X $\frac{\pi}{4} X\ \phi^2 = 4\ X\ \frac{\pi}{4} X\ 22^2 = 1520.53\ mm^2$

Number of bar on compressive side = 2

Area of steel on compressive side = Asc = 2 X $\left(\frac{Xu\max\ - d'}{Xu\max}\right)$ x $0.0035 = 2\ X\ \frac{\pi}{4} X\ 14^2 = 307.08$ mm^2

M_{20} = Fck = 20 N/ mm^2

Fe 415 =Fy = 415 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 20 = 9 N/ mm^2

STEP 2: To find depth of neutral axis (Xu)

Compressive force = Tensile force

Cu =Tu

$Cu_1 + Cu_2 = Tu$

0.36 Fck Xu b + Asc (Fsc -Fcc) = 0.87 Fy Ast

0.36 x 20 x Xu x 250 + 307.08 x(Fsc - 9) = 0.87 x 415 x 1520.53

1800 Xu + 307.88 Fsc - 2770.92 = 548.99 X 10^3

1800 Xu + 307.88 Fsc = 551.76 X 10^3

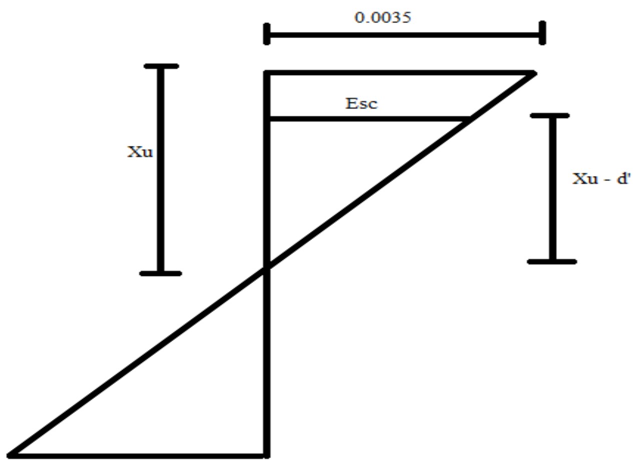
Dividing above equation by 1800

Xu + 0.1710 Fsc = 306.53...(1)

Xu max = 0.48 d

Xu max = 0.48 x 450 = 216 mm

Trial 1 :- Assuming Xu = 210 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad\qquad \frac{210}{0.0035} = \frac{2100 - 35}{\xi sc}$$

$\xi sc = 291.67 \text{ X } 10^{-5}$

From table

ξsc	**Fsc**
290 X 10^{-5}	**353.02**
291.67X 10^{-5}	**?**
300 X 10^{-5}	**353.90**

Fsc = 353.17 N/ mm^2 substituting in equation (1)

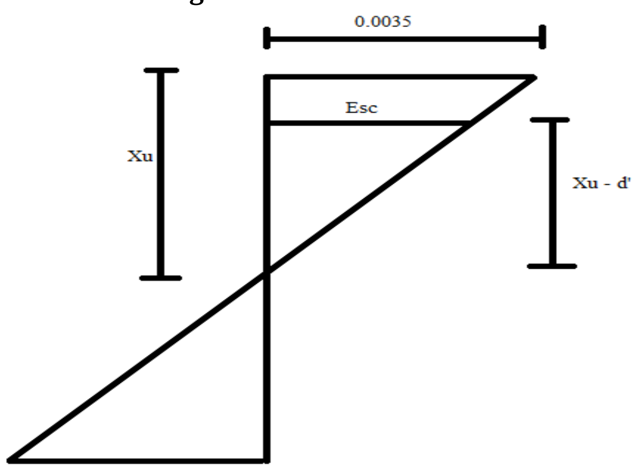
Xu + 0.1710 Fsc = 306.53

Xu + 0.1710 x 353.17 = 306.53

Xu = 246.14 mm

Assume value of Xu = 210 mm and current value of Xu = 246.14 mm, this two value not equal to each other. Therefore above assumption for Xu = 210 mm is wrong.

Trial 2 :- Assuming Xu = 246 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu - d'}{\xi sc} \qquad \frac{246}{0.0035} = \frac{246 - 35}{\xi sc}$$

$\xi sc = 300.30 \times 10^{-5}$

From table

ξsc	**Fsc**
300×10^{-5}	**353.90**
300.3×10^{-5}	**?**
310×10^{-5}	**354.77**

Fsc = 353.92 N/ mm^2 substituting in equation (1)

Xu + 0.1710 Fsc = 306.53

Xu + 0.1710 x 353.92 = 306.53

Xu = 246.01 mm

Assume value of Xu = 246 mm and current value of Xu = 246.01 mm, this two value nearly equal to each other. Therefore above assumption for Xu = 246 mm is correct.

Xu $\cong$ 246 mm

Xu max = 0.48 d = 0.48 X 450 = 216 mm

Xu $>$ Xumax

The section over reinforced section

Note: - If Xu $>$ Xumax then consider the value of Xumax in the calculation of moment of resistance.

STEP 3: To find Moment of resistance of section

$Mu = Cu_1 \; Z_1 + Cu_2 \; Z_2$

Mu = 0.36 Fck Xu max b (d- 0.42 Xumax) + Asc (Fsc - Fcc) (d-d')

Mu = 0.36 x 20 x 246 x 250 (450 - 0.42 x 246) + 307.886 x(353.92 - 9) (450-35)

Mu = 183.76 X 10^6 Nmm

Mu = 183.76 KNm

5) A doubly reinforced rectangular beam section is 250 mm wide , the effective depth of section 450 mm. The effective cover to both compressive and tensile reinforcements is 30 mm. The tension reinforcement with 4 bars of 20 mm diameter of tension side and 2 bars of 10 mm diameter on compressive Use M_{15} and Fe 415.

STEP 1: -Given Data

Width of rectangular section = b = 250 mm

Effective Depth of rectangular section = d = 450 mm

Effective cover = d' = 30 mm

Number of bar on tension side = 4

Area of steel on tension side = Ast = 4 X $\frac{\pi}{4} X\ \phi^2 = 4\text{ X } \frac{\pi}{4} X\ 20^2 = 1256.64\text{ mm}^2$

Number of bar on compressive side = 2

Area of steel on compressive side = Asc = 2 X $\frac{\pi}{4} X\ \phi^2 = 2\text{ X } \frac{\pi}{4} X\ 10^2 = 157.08\text{ mm}^2$

M_{15} = Fck = 15 N/ mm^2

Fe 415 =Fy = 415 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 15 = 6.75 N/ mm^2

STEP 2: To find depth of neutral axis (Xu)

Compressive force = Tensile force

Cu =Tu

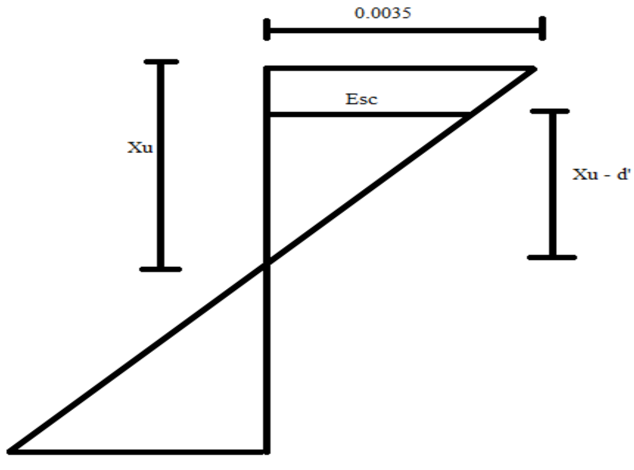
$Cu_1 + Cu_2$ = Tu

0.36 Fck Xu b + Asc (Fsc -Fcc) = 0.87 Fy Ast

0.36 x 15 x Xu x 250 + 157.08 x(Fsc - 6.75) = 0.87 x 415 x 1256.64

1350 Xu + 157.08 Fsc - 1060.29 = 453.71X 10^3
1350Xu + 157.08 Fsc = 454.697 X 10^3

Dividing above equation by 1350
Xu + 0.1164 Fsc = 336.87..(1)
Xu max = 0.48 d
Xu max = 0.48 x 450 = 216 mm
Trial 1 :- Assuming Xu = 210 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu\text{ - d'}}{\xi sc} \qquad\qquad \frac{210}{0.0035} = \frac{2100 - 30}{\xi sc}$$

$\xi sc = 300\text{ X }10^{-5}$

From table

ξsc	Fsc
-	-
-	-
300 X 10^{-5}	**353.90**

Fsc = 353.90 N/ mm^2 substituting in equation (1)

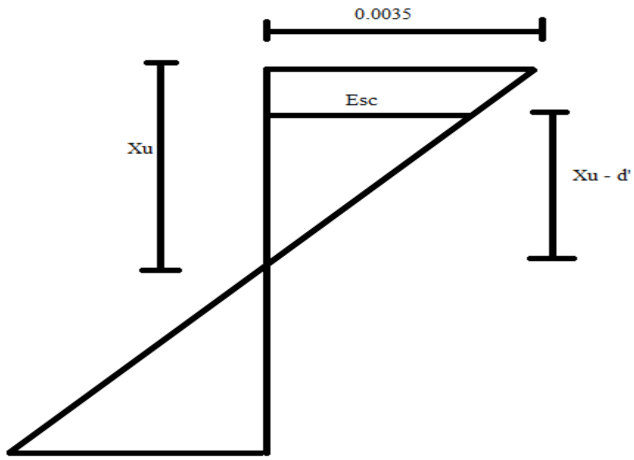
Xu + 0.1164 Fsc = 336.87

Xu + 0.1164 x 353.90 = 336.87

Xu = 295.78 mm

Assume value of Xu = 210 mm and current value of Xu = 295.78 mm, this two value not equal to each other. Therefore above assumption for Xu = 210 mm is wrong.

Trial 2 :- Assuming Xu = 295 mm



By using similar triangle law

$$\frac{Xu}{0.0035} = \frac{Xu \text{ - d'}}{\xi sc} \qquad \frac{295}{0.0035} = \frac{295 - 30}{\xi sc}$$

$\xi sc = 314.41 \text{ X } 10^{-5}$

From table

ξsc	Fsc
310 X 10^{-5}	**354.71**
314.19X 10^{-5}	**?**
320X 10^{-5}	**355.65**

Fsc = 355.16 N/ mm^2 substituting in equation (1)

Xu + 0.1710 Fsc = 306.53

Xu + 0.1710 x 355.16 = 306.53

Xu = 295.53 mm

Assume value of Xu = 295 mm and current value of Xu = 295.53 mm, this two value nearly equal to each other. Therefore above assumption for Xu = 295 mm is correct.

Xu $\cong$ 295 mm
Xu max = 0.48 d = 0.48 X 450 = 216 mm
Xu $>$ Xumax
The section over reinforced section
Note: - If Xu $>$ Xumax then consider the value of Xumax in the calculation of moment of resistance.

STEP 3: To find Moment of resistance of section

$Mu = Cu_1 Z_1 + Cu_2 Z_2$
Mu = 0.36 Fck Xu max b (d- 0.42 Xumax) + Asc (Fsc - Fcc) (d-d')
Mu = 0.36 x 15 x 295 x 250 (450 - 0.42 x 295) + 157.08 x(355.16 - 6.75) (450-30)
$Mu = 127.75 \times 10^6$ Nmm
Mu = 127.75 KNm

Type II :- To find area of steel (Ast and Asc)
Stepwise Procedure
STEP 1:To find :- The area of steel (Ast)
Given Data:- b,d, Fy, Fck, Design Moment (M_d)
d = D - Effective cover
d = D - d'
Effective cover = d'=Clear cover + $\frac{\phi}{2}$
STEP 2: Calculate ultimate moment of resistance

Mu lilmit = $0.148\ Fck\ bd^2$ .... Fe 250
Mu lilmit = $0.138\ Fck\ bd^2$ .... Fe 415
Mu lilmit = $0.133\ Fck\ bd^2$ .... Fe 500

STEP 3: To compare M_d and $M_{U\ limit}$
$M_d \prec M_{U\ limit}$ section is Under reinforced Section
$M_d = M_{U\ limit}$ section is Balanced Section
$M_d \succ M_{U\ limit}$ section is Over reinforced Section
Unbalance moment
$Mu_2 = M_d - Mu_1$
Mu_1 = Mulimit

STEP 4: Calculate area of steel
To find Ast_1
Mulimit = $Mu_1 = 0.87\ Fy\ Ast_1$ (d-0.42 Xu max)
Ast_1 = ?

To find Ast_2

$Mu_2 = 0.87$ Fy Ast_2 (d-d')

Ast_2 = ?

Total Ast = $Ast_1 + Ast_2$

Assume diameter of bar = Φ

Number of bars = $\dfrac{Ast}{(\pi / 4) \text{ x } \phi^2}$

To find Asc

Mu_2 = Asc (Fsc - Fcc) (d-d')

Asc=?

Fsc= 0.87 x Fy………………………………………. Fe 250

Fsc is calculated from P No :- 70 (IS code) And Figure No:- 23 (a) For Fe 415 and Fe 500 Grade of steel only

Assume diameter of bar = Φ

Number of bars = $\dfrac{Ast}{(\pi / 4) \text{ x } \phi^2}$

1) A rectangular beam 230 mm wide and 460 mm overall depth has to resist factored Bending moment of 160 KNm. Design reinforcement assuming effective cover of 35 mm on both side. Use M_{15} and Fe 415.

STEP 1: -Given Data

Width of rectangular section = b = 230 mm

Depth of rectangular section = D = 460 mm

Effective cover = d' = 35 mm

Effective depth of section = d = 460 - 35 = 425 mm

Factored moment = M_d = 160 KNm

M_{15} = Fck = 15 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 15 = 6.75 N/ mm^2

Fe 415 =Fy = 415 N/ mm^2

STEP 2: Calculate ultimate moment of resistance

Mu lilmit = 0.138 $Fck\ bd^2$ .... Fe 415

Mu lilmit = 0.138 x 15 x 230 x 425^2

Mu limit = 85.99 X 10^6 Nmm

Mu limit = Mu_1 = 85.99 KNm

STEP 3: To compare M_d and $M_{U\ limit}$

M_d = 160 KNm

Mu limit = Mu_1 = 85.99 KNm

$M_d \succ M_{U\ limit}$

The section is doubly reinforced Section

Unbalance moment

$\mathbf{Mu_2 = M_d - Mu_1}$

$\mathbf{Mu_1 = Mulimit}$

Mu_2 = 160 - 85.99 = 74.01 KNm

STEP 4: Calculate area of steel

To find Ast_1

Mulimit = Mu_1 = 0.87 Fy Ast_1 (d-0.42 Xu max)

Xu max = 0.48 d = 0.48 x 425 = 204 mm

85.99 X 10^6 = 0.87 x 415 x **Ast_1 x (425 - 0.42 x 204)**

Ast_1 = 701.89 mm^2

To find Ast_2

Mu_2 = 0.87 Fy Ast_2 (d-d')

74.01 X 10^6 = 0.87 x 415 x Ast_2 x (425 - 35)

Ast_2 = 525.60 mm^2

Total Ast = Ast_1 + Ast_2

Total Ast = 701.89 + 525.60 = 1227.49 mm^2

Assume diameter of bar = Φ = 20 mm

Number of bars = $\frac{Ast}{(\pi / 4) \text{ x } \phi^2} = \frac{1227.49}{(\pi / 4) \text{ x } 20^2} = 3.90 \cong 4$

To find Asc

Mu_2 = Asc (Fsc - Fcc) (d-d')

Fsc is calculated from P No :- 70 (IS code) And Figure No:- 23 (a)

$\varepsilon sc = 0.0035 \text{ x } \left(\frac{Xu_{max} - d'}{Xu_{max}}\right)$ from P No :- 96 (IS code)

$\varepsilon sc = 0.0035 \text{ x} \left(\frac{204 - 35}{204}\right)$

$\varepsilon sc = 2.8995 \text{ x } 10^{-3} = 289.95 \text{ x } 10^{-5}$

Fsc = 0.84 Fy = 0.84 x 415 = 348.60 N/ mm^2 P No :- 70 (IS code) And Figure No:- 23 (a)

74.01 X 10^6 = Asc (348.60 - 6.75) x (425- 35)

Asc = 553.12 **mm^2**

Assume diameter of bar = Φ = 20 mm

Number of bars = $\frac{Ast}{(\pi / 4) \text{ x } \phi^2} = \frac{553.12}{(\pi / 4) \text{ x } 20^2} = 1.76 \cong 2$

2) A reinforced concrete beam 230 mm wide and 600 mm overall depth carries live load of 38 KN/m over a simply supported span of 5 m. Design reinforcement for the beam having effective cover of 40 mm on both side. Use M_{15} and Fe 415.

STEP 1: -Given Data

Width of rectangular section = b = 230 mm

Depth of rectangular section = D = 600 mm

Effective cover = d' = 40 mm

Effective depth of section = d =D-d'= 600 - 40 = 560 mm

Span of beam = L=5m

beam is simply supported

Live load = 38 KN/m

Self weight of beam = b x D x 25 = 0.23 x 0.6 x 25 = 3.45 KN/m

Total working load = Live load + Self weight of beam = 38 + 3.45 = 41.45 KN/m

Total factored load (Wu) = 1.5 x 41.45 = 62.175 KN/m

Factored moment = $M_d = \frac{W_u \text{ x } L^2}{8} = \frac{62.175 \text{ x } 5^2}{8}$ = 194.30 KN m

Factored moment = M_d **= 194.30 KNm**

M_{15} = Fck = 15 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 15 = 6.75 N/ mm^2

Fe 415 =Fy = 415 N/ mm^2

STEP 2: Calculate ultimate moment of resistance

Mu lilmit = 0.138 $Fck\ bd^2$ .... Fe 415

Mu lilmit = 0.138 x 15 x 230 x 560^2

Mu limit = 149.30 X 10^6 Nmm

Mu limit = Mu_1 = 149.30 KNm

STEP 3: To compare M_d **and** $M_{U\ limit}$

M_d **= 194.30 KNm**

Mu limit = Mu_1 =149.30 KNm

$M_d \succ M_{U\ limit}$

The section is doubly reinforced Section

Unbalance moment

$Mu_2 = M_d - Mu_1$

Mu_1 **= Mulimit**

Mu_2 **= 194.30 - 149.30 = 45** KNm

STEP 4: Calculate area of steel

To find Ast_1

Mulimit = Mu_1 **= 0.87 Fy** Ast_1 **(d-0.42 Xu max)**

Xu max = 0.48 d = 0.48 x 560 = 268.8 mm

149.30 X 10^6 = 0.87 x 415 x Ast_1 **x (560 - 0.42 x268.8)**

Ast_1 **= 924.88 mm**^{2}

To find Ast_2

Mu_2 **= 0.87 Fy** Ast_2 **(d-d')**

45 X 10^6 **= 0.87 x 415 x** Ast_2 **x (560 - 40)**

Ast_2 **= 239.69 mm**^{2}

Total Ast = Ast_1 + Ast_2

Total Ast = 924.88 + 239.69 = 1164.57 mm^2

Assume diameter of bar = Φ = 20 mm

Number of bars = $\dfrac{Ast}{(\pi/4) \text{ x } \phi^2} = \dfrac{1164.57}{(\pi/4) \text{ x } 20^2} = 3.70 \cong 4$

To find Asc

Mu_2 = Asc (Fsc - Fcc) (d-d')

Fsc is calculated from P No :- 70 (IS code) And Figure No:- 23 (a)

$\varepsilon sc = 0.0035 \text{ x } \left(\dfrac{Xu_{max} - d'}{Xu_{max}}\right)$ from P No :- 96 (IS code)

$\varepsilon sc = 0.0035 \text{ x} \left(\dfrac{268.8 - 40}{268.8}\right) = 2.9792 \text{ x } 10^{-3} = 297.92 \text{ x } 10^{-5}$

Fsc = 0.84 Fy = 0.84 x 415 = 348.60 N/ mm^2 P No :- 70 (IS code) And Figure No:- 23 (a)

45 X 10^6 = Asc (348.60 - 6.75) x (560- 40)

Asc = 253.15 **mm^2**

Assume diameter of bar = Φ = 12 mm

Number of bars = $\dfrac{Ast}{(\pi/4) \text{ x } \phi^2} = \dfrac{253.147}{(\pi/4) \text{ x } 12^2} = 2.23 \cong 3$

3)) A rectangular beam 230 mm wide and 460 mm overall depth has to resist factored moment of 150 KNm. Design reinforcement assuming effective cover of 35 mm on both side. Use M_{15} and Fe 250.

STEP 1: -Given Data

Width of rectangular section = b = 230 mm

Depth of rectangular section = D = 460 mm

Effective cover = d' = 35 mm

Effective depth of section = d = 460 - 35 = 425 mm

Factored moment = M_d = 150 KNm

M_{15} = Fck = 15 N/ mm^2

Fcc = 0.45 Fck = 0.45 X 15 = 6.75 N/ mm^2

Fe 250 =Fy = 250 N/ mm^2

Fsc = 0.87 Fy = 0.87 X 250 = 217.5 N/ mm^2

STEP 2: Calculate ultimate moment of resistance

Mu lilmit = 0.148 $Fck\ bd^2$ .... Fe 250

Mu lilmit = 0.148 x 15 x 230 x 425^2

Mu limit = 92.27 X 10^6 Nmm

Mu limit = Mu_1 = 92.27 KNm

STEP 3: To compare M_d and $M_{U\ limit}$

M_d = 150 KNm

Mu limit = Mu_1 = 92.27 KNm

$M_d \succ M_{U\ limit}$

The section is doubly reinforced Section

Unbalance moment

$Mu_2 = M_d - Mu_1$

Mu_1 = Mulimit

Mu_2 = 150 – 92.27 = 57.77 KNm

STEP 4: Calculate area of steel

To find Ast_1

Mulimit = Mu_1 = 0.87 Fy Ast_1 (d-0.42 Xu max)

Xu max = 0.53 d = 0.53x 425 = 225.25 mm

92.27 X 10^6 = 0.87 x 250x **Ast_1 x (425 - 0.42 x 225.25)**

Ast_1 = 1284.00 mm^2

Assume diameter of bar = Φ = 20 mm

Number of bars = $\dfrac{Ast}{(\pi / 4) \text{ x } \phi^2} = \dfrac{1284}{(\pi / 4) \text{ x } 20^2}$ = 4.087 ≅ 5

To find Ast_2

Mu_2 = 0.87 Fy Ast_2 (d-d')

92.27 X 10^6 = 0.87 x 250x Ast_2 x (425 - 35)

Ast_2 = 1087.7 mm^2

Total Ast = Ast_1 + Ast_2

Total Ast = 1284 + 1087.7 = 2371.76 mm^2

To find Asc

Mu_2 = Asc (Fsc - Fcc) (d-d')

92.27 X 10^6 = Asc (217.5- 6.75) x (425- 35)

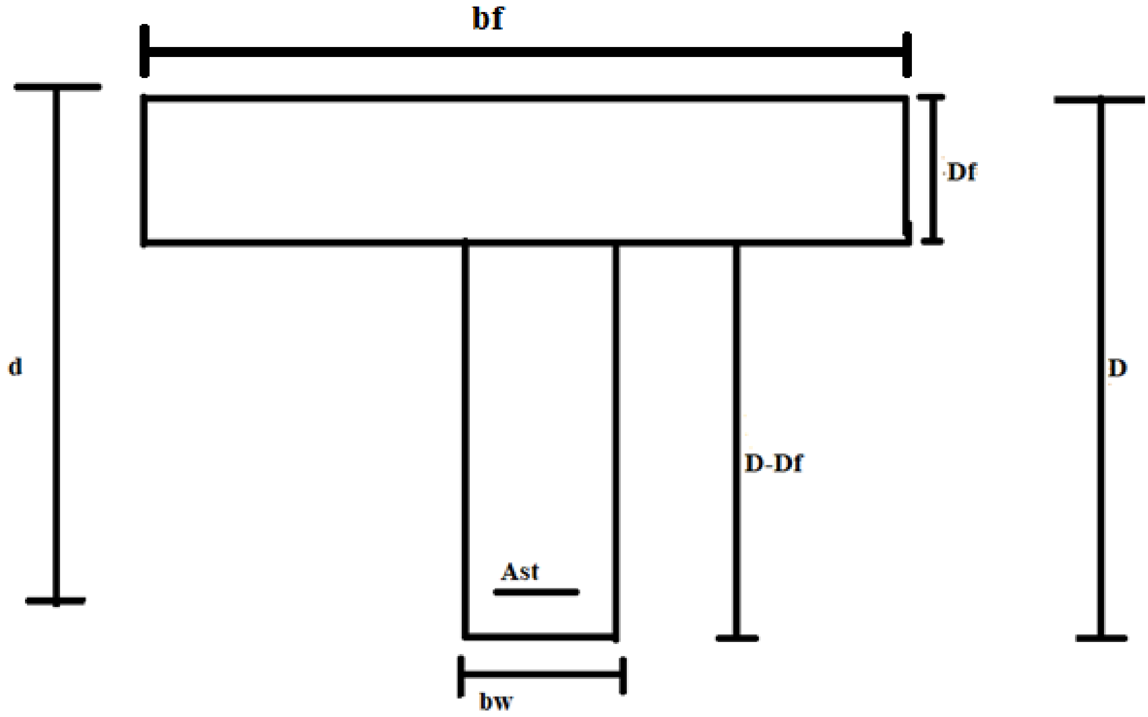
Asc = 1122.60 **mm^2**

Assume diameter of bar = Φ = 20 mm

Number of bars = $\dfrac{Ast}{(\pi / 4) \text{ x } \phi^2} = \dfrac{1122.60}{(\pi / 4) \text{ x } 20^2}$ = 3.57 ≅ 4

Limit State For shear (T and L Beam)

When beam and slab cast monolithically then it is called ‘T’ Beam.



Where b_f = Width of flange

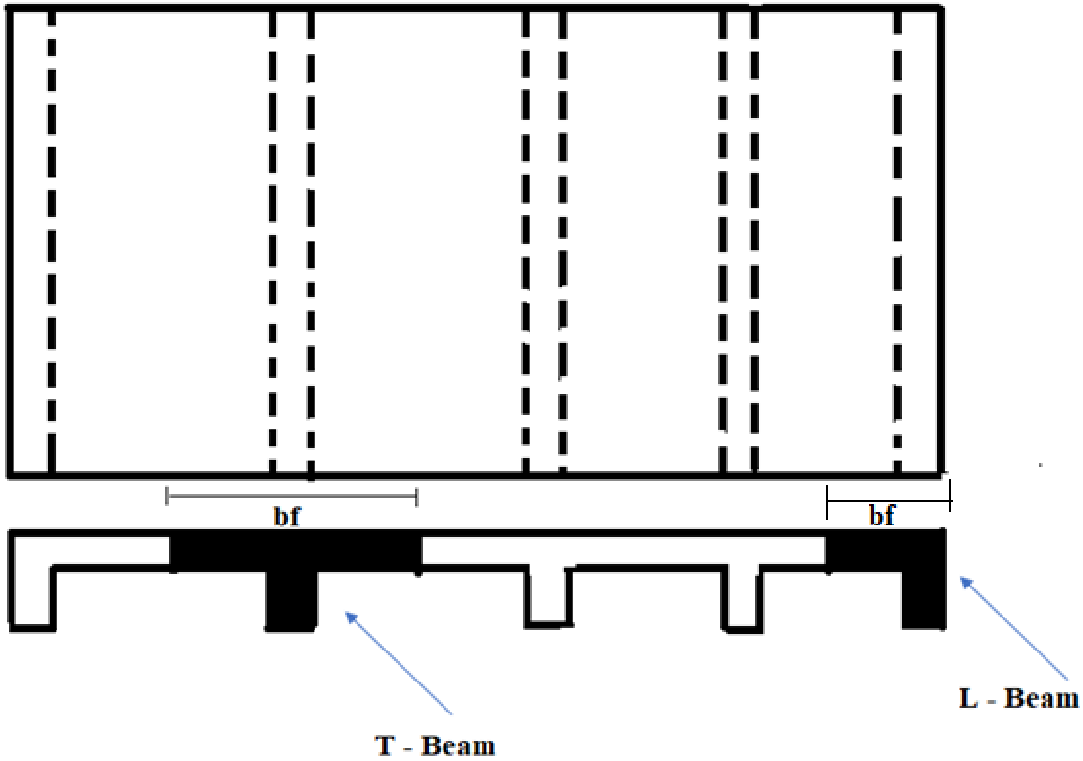
D_f = Depth of flange

D= Overall depth

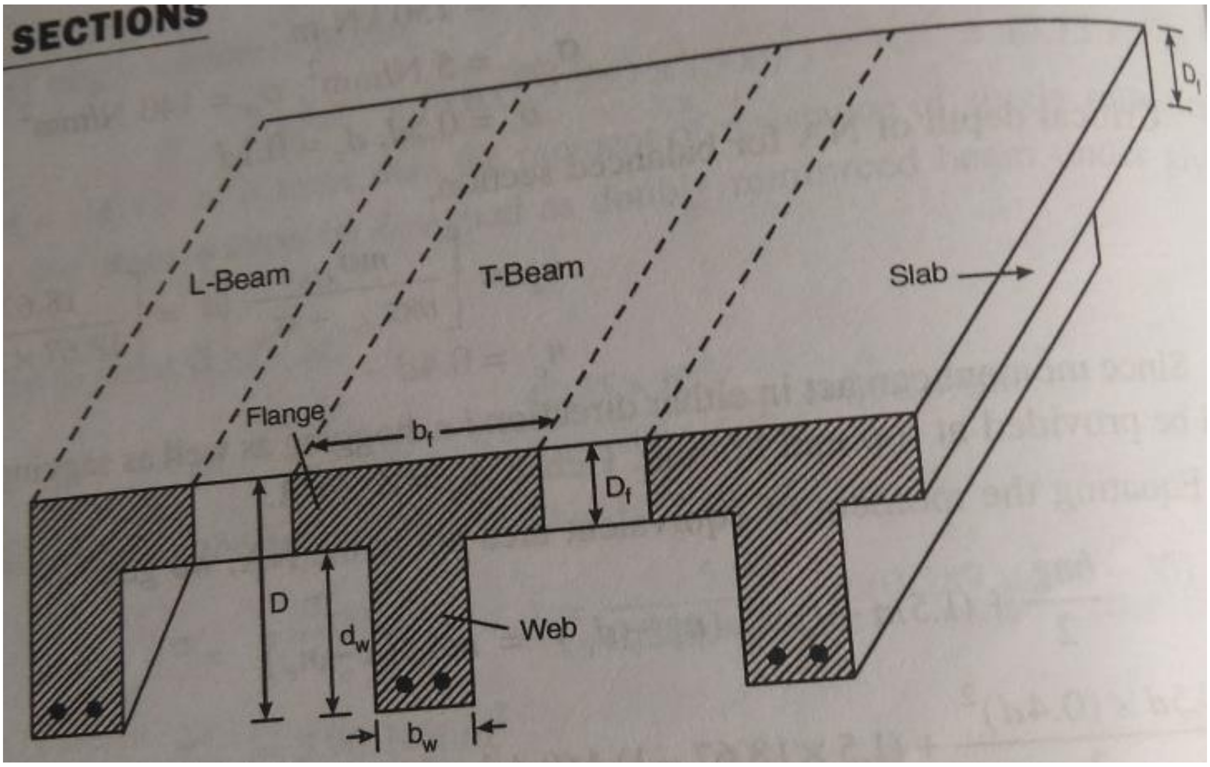
d= Effective depth

b_w =Width of beam

A_{st} =Area of tensile steel



T and L Beam



Conditions:

1) Construction should be monolithically
2) The sufficient amount of R/F should be transparently place to the beam

   Depth of neutral axis
   Case I: $Xu < D_f$ (Neutral axis lies in flange)
   Case II: $Xu > D_f$ (Neutral axis lies in web)

   Types of problems

1) To find moment of resistance of section

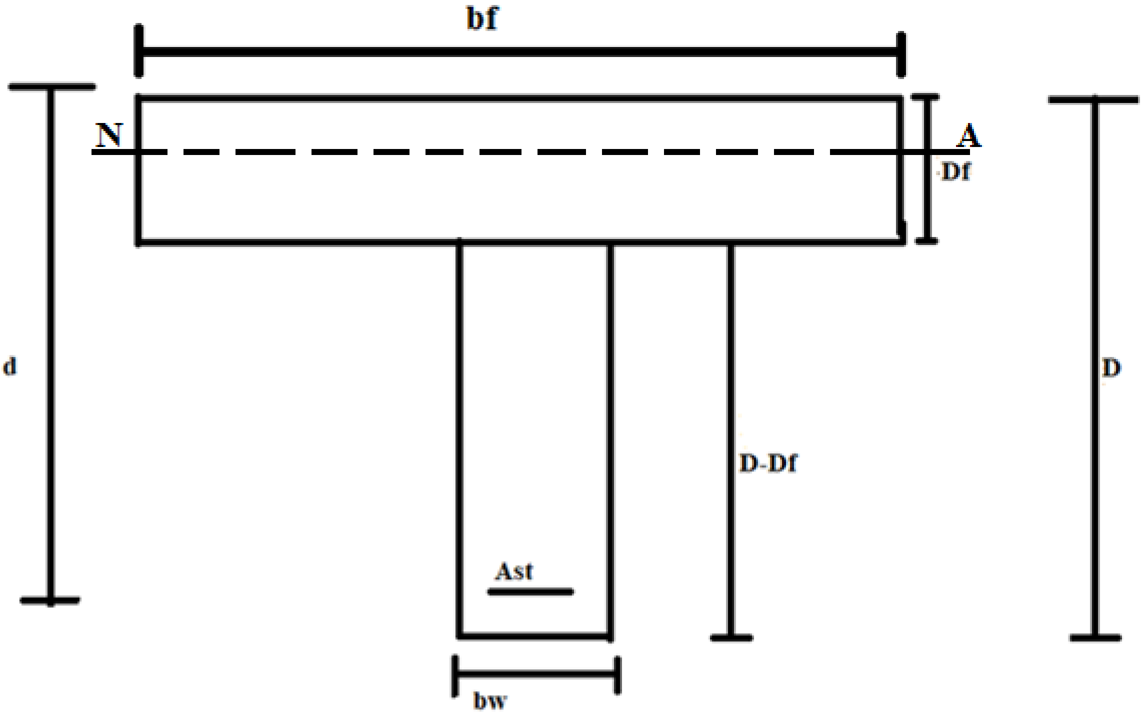
2) To find the area of steel

3) To Design the section

Type I

To find moment of resistance of section

Case I: $Xu < D_f$ (Neutral axis lies in flange)



Stepwise Procedure

Case I: $Xu < D_f$ (Neutral axis lies in flange)

Given Data:- b_w,b_f, d,Ast, Fy, Fck

d = D - Effective cover

d = D - d'

d'=Effective cover = Clear cover + $\frac{\phi}{2}$

STEP 1: To find depth of neutral axis: $Xu < D_f$ (Neutral axis lies in flange)

(From page No. 96 IS CODE)

Cu = Tu

$$0.36FckXub_f = 0.87 \text{ Fy Ast}$$

$$Xu = \frac{0.87FyAst}{0.36Fckb_f}$$

$xu < D_f$ The assumption is correct

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.48 d ....For Fe 415

Xu max = 0.46 d ....For Fe 500

STEP 3: To compare Xu and Xu max

a) If $Xu \prec Xu\max$ then section is under reinforced

b) **If** $Xu = Xu\max$ then section is balance section

c) **If** $Xu \succ Xu\max$ then section is over reinforced, if section is over reinforced then consider it as balance section.

STEP 4: To find moment of resistance

a) For under reinforced section **(From page No. 96 IS CODE)**

$$Mu = 0.87\ Fy\ Ast\ (d - 0.42\ Xu)$$

b) For balance section

$$M_{ulimit} = 0.36\ Fck\ Xu_{max}\ b\ (d - 0.42\ Xu_{max})$$

1) A 'T' section having flange width 1000 mm and thickness of flange is 100 mm, width of web is 250 mm has effective depth of 500 mm. The beam is reinforced with 4 bars of 22 mm diameter. Find moment of resistance of section . Used Fe 250 and M_{15}.

Solution:- **To find :- The moment of resistance of section**

Given Data:- Width of Flange =b_f = 1000 mm

Depth of flange= D_f = 100 mm

Width of web =b_w =250 mm

Effective depth =d= 500 mm

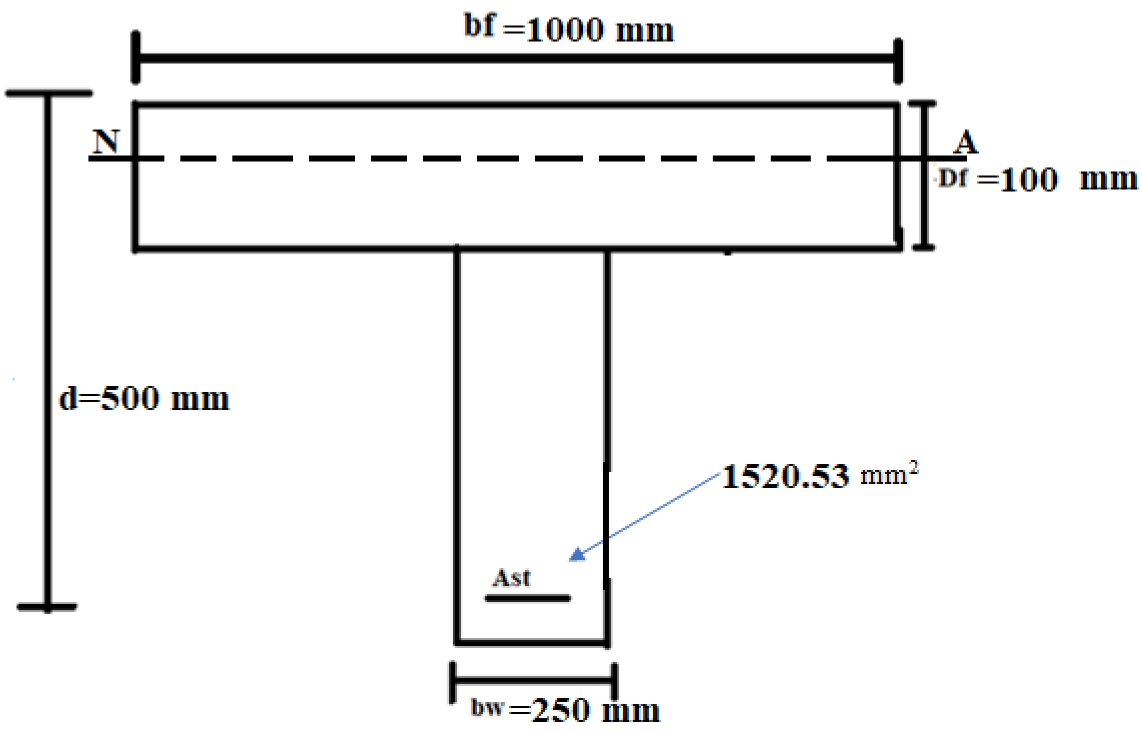
ϕ =Diameter of bar= 22 mm

No of bar = 4

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 22^2 = 1520.53 mm^2

M_{15} = Fck = 15 N/mm^2

Fe 250 = Fy = 250 N/mm^2



STEP 1: To find depth of neutral axis : Xu < D_f (Neutral axis lies in flange)

(From page No. 96 IS CODE)

Cu = Tu

$0.36 Fck Xu b_f = 0.87\ Fy\ Ast$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b_f}$$

$$Xu = \frac{0.87 \times 250 \times 1520.53}{0.36 \times 15 \times 1000} = 61.24 \text{ mm}$$

xu < D_f

61.24 < 100 ,The assumption is correct

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.53 x 500 = 265 mm

STEP 3: To compare Xu and Xu max

$Xu \prec Xu\ max$

$61.24 \prec 265$

then section is under reinforced

STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

Mu = 0.87 x 250 x 1520.53 (500- 0.42 x 61.24) = 156.85 X 10^6 Nmm = 156.85 KNm

2) A 'T' section having flange width 1400 mm and thickness of flange is 100 mm, width of web is 250 mm has effective depth of 500 mm. The beam is reinforced with 4 bars of 20 mm diameter. Find moment of resistance of section . Used Fe 415 and M_{15}.

Solution:- **To find :- The moment of resistance of section**

Given Data:- Width of Flange =b_f = 1400 mm

Depth of flange= D_f = 100 mm

Width of web =b_w =250 mm

Effective depth =d= 500 mm

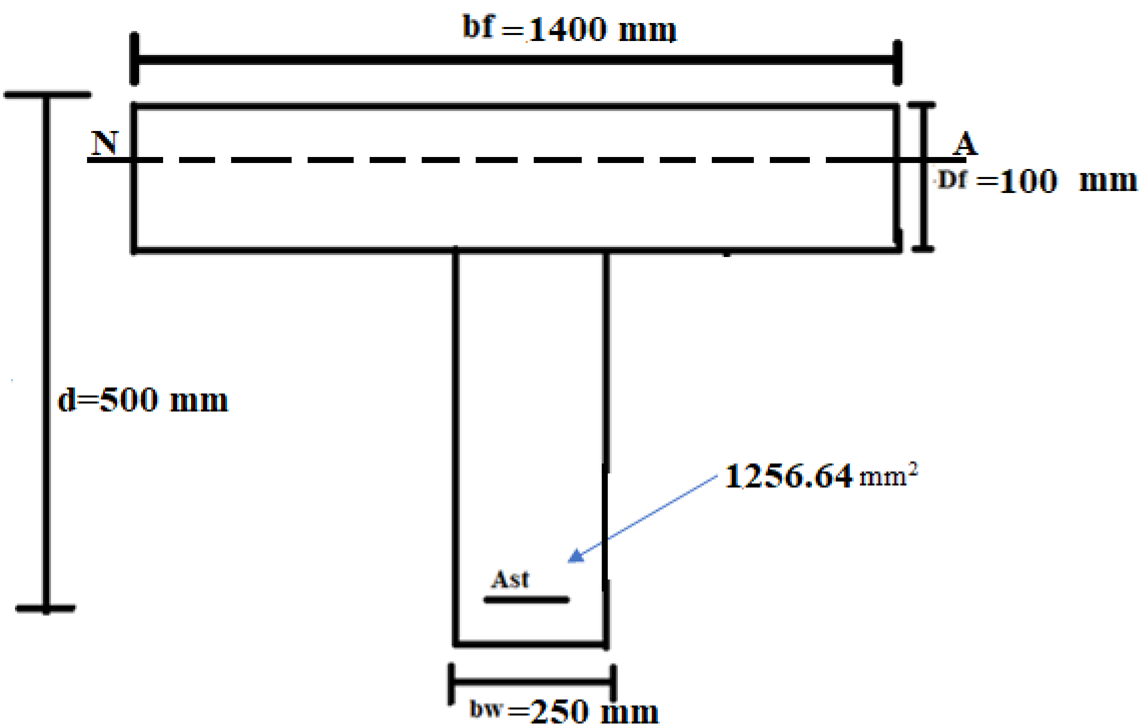
ϕ =Diameter of bar= 20 mm

No of bar = 4

Ast = 4 X $\frac{\pi}{4}$ X ϕ^2 = 4 X $\frac{\pi}{4}$ X 20^2 = 1256.64 mm^2

M_{15} = Fck = 15 N/mm^2

Fe 415 = Fy = 415N/mm^2



STEP 1: To find depth of neutral axis : Xu < D_f (Neutral axis lies in flange)

(From page No. 96 IS CODE)

Cu = Tu

$0.36FckXub_f = 0.87$ Fy Ast

$$Xu = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck } b_f}$$

$$Xu = \frac{0.87 \text{ x } 415 \text{ x } 1256.64}{0.36 \text{ x } 15 \text{ x } 1400} = 60.01 \text{ mm}$$

$xu < D_f$

60.01 < 100 ,The assumption is correct

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 500 = 240 mm

STEP 3: To compare Xu and Xu max

$Xu \prec Xu \max$

$60.01 \prec 240$

then section is under reinforced

STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

Mu = 0.87 x 415 x 1256.64 (500- 0.42 x 60.01) = 215.41 X 10^6 Nmm = 215.41 KNm

Effective width of flange (IS: 456: 2000, Page Number :37, C. No :23.1.2)

For T beam

$b_f = \frac{l_0}{6} + b_w + 6D_f <$ C/C distance

For L beam

$b_f = \frac{l_0}{12} + b_w + 3D_f <$ C/C distance

l_0 = Distance between point of zero moment i.e. Effective span

$l_0 = l$ (For simply supported beam)

$l_0 = 0.7l$ (For continuous beam)

3) Find ultimate moment of resistance of T beam for the following particulars
   1) Simply supported span = 4.5 m

2) Slab thickness = 115 mm
3) Width of web = 230 mm
4) Effective depth = 450 mm
5) Tensile steel = 8 bars of 16 mm diameter in 2 rows
6) Spacing of beam = 3.5 m c/c
   Used Fe 415 and M_{20}.

Solution:- **To find :- The moment of resistance of section**

Given Data:- T beam

Simply supported beam

l= 4.5m

l_o = l =4.5 m= 4500 mm

Depth of flange= D_f = 115 mm

Width of web =b_w =230 mm

Effective depth =d= 450 mm

C/C distance= 3.5 m = 3500 mm

For T beam $(\mathbf{IS:\ 456:\ 2000,\ Page\ Number\ :37,\ C.\ No\ :23.1.2})$

$b_f = \frac{l_0}{6} + b_w + 6D_f <$ C/C distance

$b_f = \frac{4500}{6} + 230 + 6x115 < 3500mm$

$b_f = 1670$ mm $< 3500mm$ (ok)

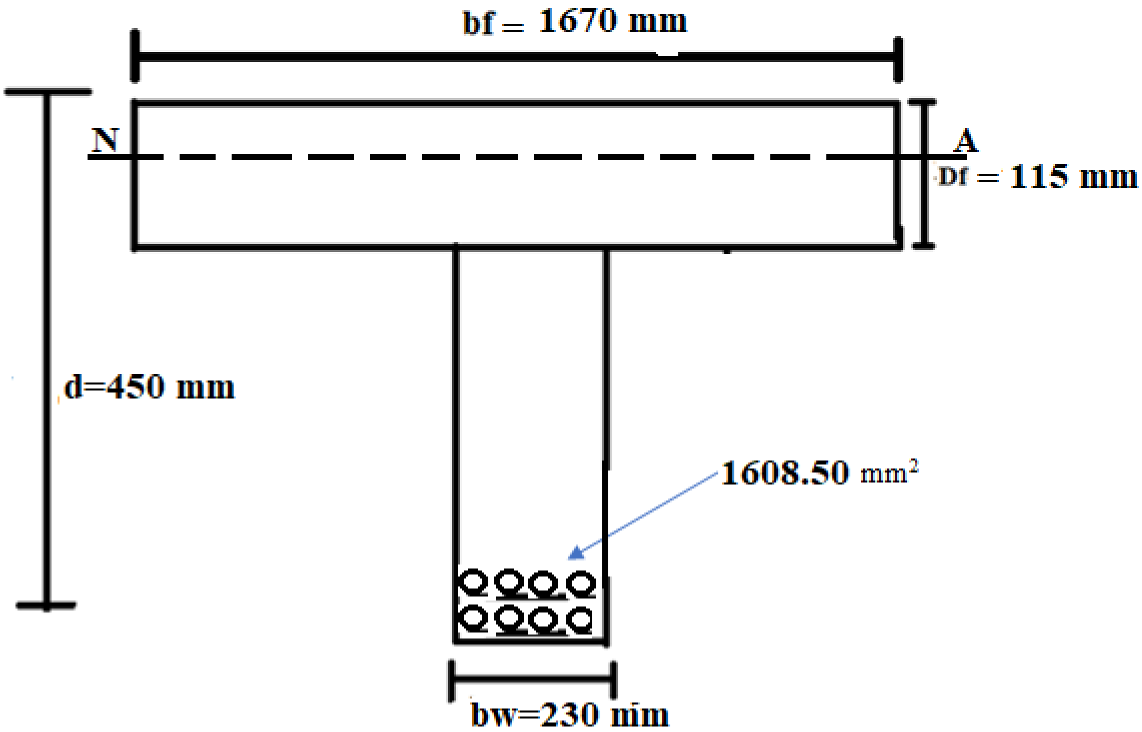
ϕ =Diameter of bar= 16 mm

No of bar = 8

Ast = 8X $\frac{\pi}{4}$ X ϕ^2 = 8 X $\frac{\pi}{4}$ X 16^2 = 1608.50 mm^2

M_{20} = Fck = 20 N/mm^2

Fe 415 = Fy = 415N/mm^2



STEP 1: To find depth of neutral axis: $Xu < D_f$ (Neutral axis lies in flange)

(From page No. 96 IS CODE)

$$Cu = Tu$$

$$0.36 Fck Xu b_f = 0.87 \, Fy \, Ast$$

$$Xu = \frac{0.87 \, Fy \, Ast}{0.36 \, Fck \, b_f}$$

$$Xu = \frac{0.87 \times 415 \times 1608.50}{0.36 \times 20 \times 1670} = 48.30 \text{ mm}$$

$xu < D_f$

48.30 < 115 ,The assumption is correct

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 450 = 216 mm

STEP 3: To compare Xu and Xu max

$Xu \prec Xu \max$

$48.30 \prec 115$

then section is under reinforced

STEP 4: To find moment of resistance

For under reinforced section **(From page No. 96 IS CODE)**

$Mu = 0.87\ Fy\ Ast\ (d - 0.42\ Xu)$

$Mu = 0.87 \times 415 \times 1608.50\ (415 - 0.42 \times 48.03) = 249.55 \times 10^6\ Nmm = 249.55\ KNm$

4) A 'T' beam having width of flange 2500 mm is required to resist ultimate moment of 1200 KNm, thickness of flange is 150 mm, the width of web is 300 mm and effective depth 900 mm. Find area of steel. Used Fe 250 and M_{15}.

Solution:

To find :- The find area of steel

Given Data:- T beam

Width of flange= b_f = 2500 mm

Depth of flange= D_f = 150 mm

Width of web =b_w =300 mm

Effective depth =d= 900mm

Ultimate Moment= Mu=1200 KNm= 1200 x 10^6 Nmm

M_{15} = Fck = 15 N/mm^2

Fe 250 = Fy = 250 N/mm^2

STEP 1: To find depth of neutral axis:

(From page No. 96 IS CODE)

Assume section is balance section, neutral axis lies in flange

$Mu = 0.36\ Fck\ Xu\ b_f\ \ (d - 0.42\ Xu)$

$1200 \times 10^6 = 0.36 \times 15 \times Xu \times 2500 \times (900 - 0.42\ xu)$

$1200 \times 10^6 = 13500 \times Xu \times (900 - 0.42\ xu)$

$1200 \times 10^6 = 12.15 \times 10^6 \times Xu - 5670X^2u$

$5670X^2u - 12.15 \times 10^6 \times Xu + 1200 \times 10^6 = 0$

$Xu = 103.79\ mm$

$Xu < D_f$

The assumption is correct

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.53 x 900 = 477 mm

STEP 3: To compare Xu and Xu max

$Xu \prec Xu\,max$

$103.79 \prec 477$

then section is under reinforced

STEP 4: To find area of steel

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

$1200 \times 10^6 = 0.87 \times 250 \times Ast\ (900 - 0.42 \times 103.79)$

$Ast = 6442.30\ mm^2$

Assuming diameter of bar ϕ= 40 mm

$$\text{Number of bar} = \frac{A_{st}}{\frac{\pi}{4} x\phi^2} = \frac{6442.30}{\frac{\pi}{4} x40^2} = 5.12 \cong 6$$

5) A 'T' beam having width of flange 1500 mm is required to resist ultimate moment 500 KNm, thickness of flange is 120 mm, the width of web is 300 mm and effective depth 750 mm. Find area of steel. Used Fe 415 and M_{20}.

Solution:

To find :- The find area of steel

Given Data:- T beam

Width of flange= b_f = 1500 mm

Depth of flange= D_f = 120 mm

Width of web =b_w =300 mm

Effective depth =d= 750 mm

Ultimate Moment= Mu=500 KNm= 500 x 10^6 Nmm

M_{20} = Fck = 20 N/mm^2

Fe 415= Fy = 415 N/mm^2

STEP 1: To find depth of neutral axis:

(From page No. 96 IS CODE)

Assume section is balance section, neutral axis lies in flange

$Mu = 0.36\ Fck\ Xu\ b_f\ (d - 0.42\ Xu)$

$500 \times 10^6 = 0.36 \times 20 \times Xu \times 1500 \times (750 - 0.42\ xu)$

$500 \times 10^6 = 8.1x10^6 \times Xu - 4536X^2u$

$4536X^2u - 8.1 \times 10^6 \times Xu + 500 \times 10^6 = 0$

$Xu = 64.02\ mm$

$Xu < D_f$

The assumption is correct

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48d ....For Fe 415

Xu max = 0.48 x 750 = 360 mm

STEP 3: To compare Xu and Xu max

$Xu \prec Xu\max$

$64.02 \prec 360$

then section is under reinforced

STEP 4: To find area of steel

For under reinforced section **(From page No. 96 IS CODE)**

Mu = 0.87 Fy Ast (d- 0.42 Xu)

$500 \text{ x } 10^6 = 0.87 \text{ x } 415 \text{ x Ast } (750 - 0.42 \text{ x } 64.02)$

$Ast = 1915.13 \text{ mm}^2$

Assuming diameter of bar ϕ= 20 mm

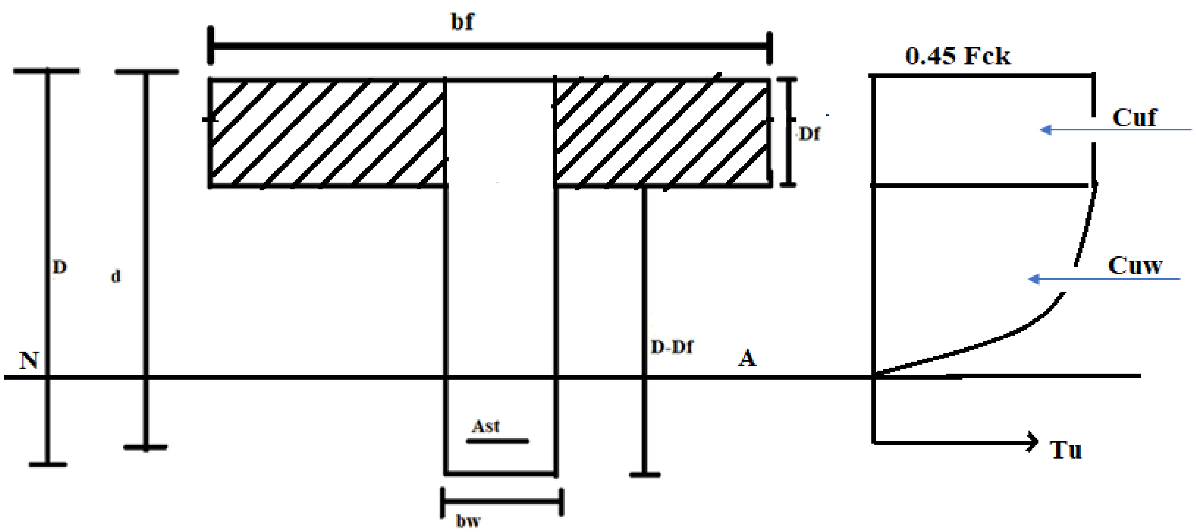
$$\text{Number of bar} = \frac{A_{st}}{\frac{\pi}{4} x\phi^2} = \frac{1915.13}{\frac{\pi}{4} x20^2} = 6.09 \cong 6 \text{ (Provide in two rows)}$$

CASE II: Neutral axis lies in web (Xu> D_f)

But $D_f \leq 0.43$ Xu (IS 456:2000, Page Number: 97)

Or

$$\frac{D_f}{d} < 0.2$$



Area of flange= $(b_f - b_w) \times D_f$
Area of Web= $b_w \times d$
C_{uf} = Compressive Force in Flange
$C_{uf} = (b_f - b_w) \times D_f \times 0.45 \times F_{ck}$
C_{uw} = Compressive Force in Web
$C_{uw} = 0.36 \times F_{ck} \times Xu \times b_w$
Design Procedure
Given Data

STEP 1: To find depth of neutral axis: Xu

(From page No. 96 IS CODE)

Cu=Tu

$C_{uf} + C_{uw} = \mathbf{Tu}$

$(b_f - b_w) \times D_f \times 0.45 \times F_{ck} + 0.36 \times F_{ck} \times Xu \times b_w = 0.87 \times Fy \times Ast$

Xu=?

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.48 d ....For Fe 415

Xu max = 0.46 d ....For Fe 500

STEP 3: To compare Xu and Xu max

a) If $Xu \prec Xu\max$ then section is under reinforced

b) **If** $Xu = Xu\max$ then section is balance section

c) **If** $Xu \succ Xu\max$ then section is over reinforced, if section is over reinforced then consider it as balance section.

STEP 4: To find ultimate moment of resistance

(From page No. 96 , C. No: G.2.2. ,IS 456:2000,)

$$Mu = 0.45 \times F_{ck} \times (b_f - b_w) \times D_f \times \left(d - \frac{D_f}{2} \right) + 0.36 \times F_{ck} \times Xu \times b_w \times (d - 0.42\, Xu)$$

1) Find ultimate moment of resistance of T beam for the following particulars

Width of flange = 1500 mm
Depth of flange = 100 mm
Width of web = 300 mm
Effective depth = 600 mm
Tensile steel = 4500 mm^2
Used Fe 415 and M_{20}.

Solution: Given Data:-

Width of Flange =b_f = 1500 mm

Depth of flange= D_f = 100 mm

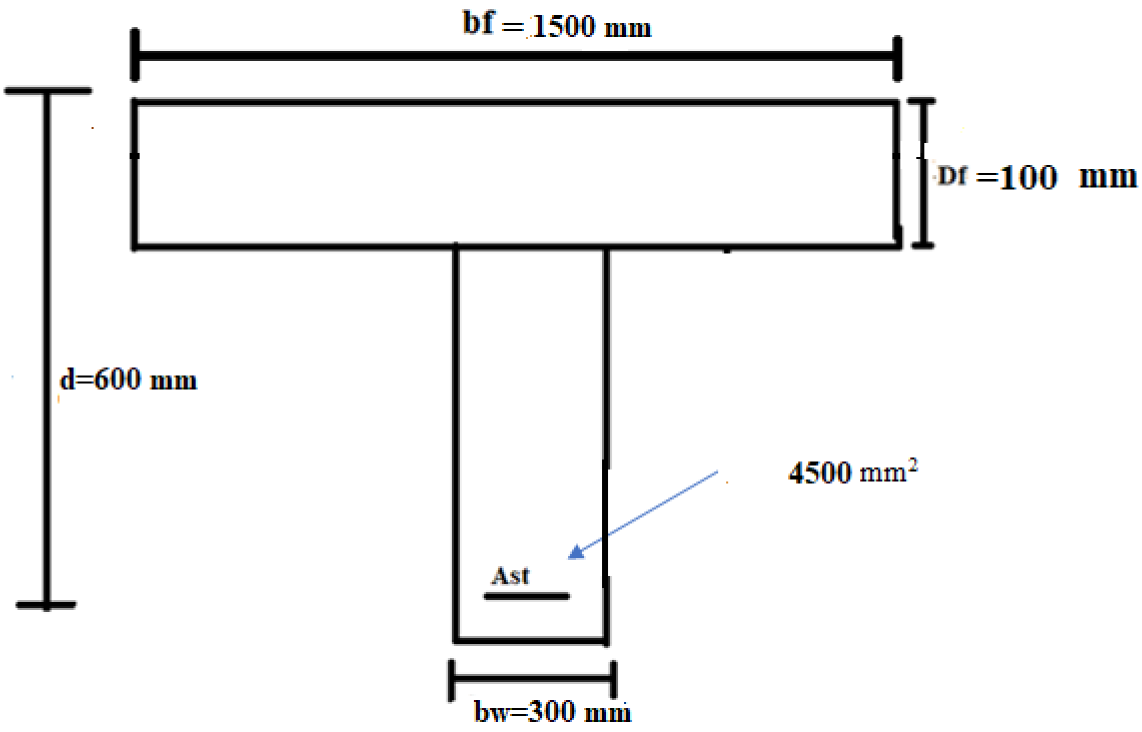
Width of web =b_w =300 mm

Effective depth =d= 600 mm

Ast =4500 mm^2

M_{20} = Fck = 20 N/mm^2

Fe 415 = Fy = 415 N/mm^2



STEP 1: To find depth of neutral axis : Xu < D_f (Neutral axis lies in flange)

(From page No. 96 IS CODE)

Cu = Tu

$$0.36FckXub_f = 0.87\ Fy\ Ast$$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b_f}$$

$$Xu = \frac{0.87 \times 415 \times 4500}{0.36 \times 20 \times 1500} = 150.44\ mm$$

150.44 >100 ,The assumption is wrong

Assuming Neutral axis lies in web (Xu> D_f)

Cu=Tu

$C_{uf} + C_{uw} =$ **Tu**

$(b_f - b_w) \times D_f \times 0.45 \times F_{ck} + 0.36 \times F_{ck} \times Xu \times b_w = 0.87 \times Fy \times Ast$

$(1500- 300) \times 100 \times 0.45 \times 20 + 0.36 \times 20 \times Xu \times 300 = 0.87 \times 415 \times 4500$

Xu=252.19 mm

$xu > D_f$

252.19 >100 ,The assumption is correct

Check: a) $\mathbf{D_f} \leq 0.43\ Xu$ (P. No: 97 , IS 456:2000)

$100 \leq 0.43 \text{ X } 252.19 = 108.44$

$$\frac{D_f}{d} \prec 0.2$$

$$\frac{100}{600} \prec 0.2$$

$0.16 \prec 0.2$ (OK)

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48d ....For Fe 415

Xu max = 0.48 x600 = 288 mm

STEP 3: To compare Xu and Xu max

$$Xu \prec Xu\,max$$

$$252.19 \prec 288$$

then section is under reinforced

STEP 4: To find moment of resistance

(From page No. 96 , C. No: G.2.2 ,IS 456:2000,)

$$Mu = 0.45 \times F_{ck} \times (b_f - b_w) \times D_f \times \left(d - \frac{D_f}{2} \right) + 0.36 \times F_{ck} \times Xu \times b_w \times (d - 0.42\ Xu)$$

$$Mu = 0.45 \times 20 \times (1500-300) \times 100 \times \left(600 - \frac{100}{2} \right) + 0.36 \times 20 \times 252.19 \times 300 \times (600 - 0.42 \times 252.19)$$

$$Mu = 863.14 \times 10^6 \text{ Nmm} = 863.14 \text{ KNm}$$

2) Find ultimate moment of resistance of T beam for the following particulars

Width of flange = 1200 mm
Depth of flange = 100 mm
Width of web = 275 mm
Effective depth = 550 mm
Tensile steel =4 bars of 25 mm diameter & 4 bars of 16 mm diameter
Used Fe 415 and M_{15}.

Solution: Given Data:-

Width of Flange =b_f = 1200 mm

Depth of flange= D_f = 100 mm

Width of web =b_w =275 mm

Effective depth =d= 550 mm

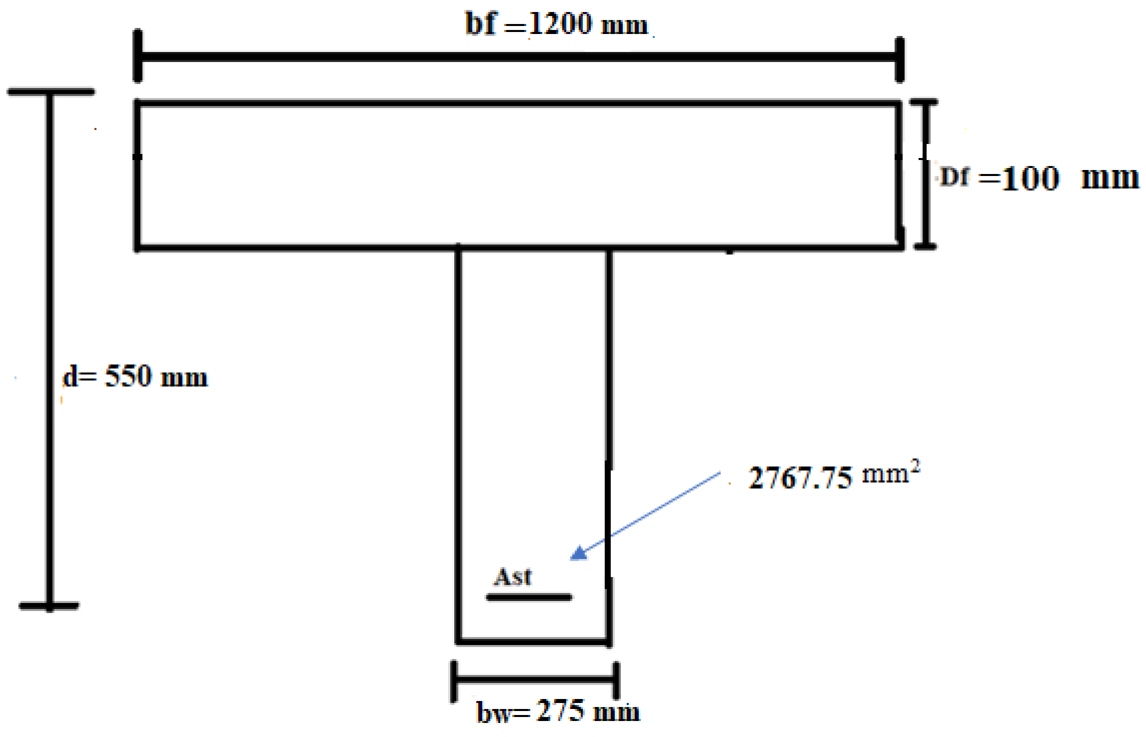
Ast =4x$\frac{\pi}{4}$ X $\phi_1^2 + 4\ \frac{\pi}{4}$ X ϕ_2^2

Ast =4x$\frac{\pi}{4}$ X $25^2 + 4\ \frac{\pi}{4}$ X 16^2

= 2767.75 mm^2

M_{15} = Fck = 15 N/mm^2

Fe 415 = Fy = 415 N/mm^2



STEP 1: To find depth of neutral axis : Xu < D_f (Neutral axis lies in flange)

(From page No. 96 IS CODE)

Cu = Tu

$0.36 Fck Xu b_f = 0.87\ Fy\ Ast$

$$Xu = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck } b_f}$$

$$Xu = \frac{0.87 \times 415 \times 2767.75}{0.36 \times 20 \times 1200} = 152.21 \text{ mm}$$

152.21 >100 ,The assumption is wrong

Assuming Neutral axis lies in web (Xu> D_f)

Cu=Tu

$C_{uf} + C_{uw}$ = **Tu**

$(b_f - b_w) \times D_f \times 0.45 \times F_{ck} + 0.36 \times F_{ck} \times Xu \times b_w = 0.87 \times Fy \times Ast$

(1200- 275) x 100 x 0.45 x 15 + 0.36 x 15 x Xu x 275 = 0.87 x 415 x 2767.75

Xu=252.47 mm

$xu > D_f$

252.47 >100 ,The assumption is correct

Check: a) D_f ≤ 0.43 Xu (P. No: 97 , IS 456:2000)

$100 \leq 0.43 \text{ X } 252.19 = 108.44$

$$\frac{D_f}{d} \prec 0.2$$

$$\frac{100}{600} \prec 0.2$$

$0.16 \prec 0.2$ (OK)

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48d ....For Fe 415

Xu max = 0.48 x550 = 264 mm

STEP 3: To compare Xu and Xu max

$$\text{Xu} \prec \text{Xu max}$$

$$252.47 \prec 264$$

then section is under reinforced

STEP 4: To find moment of resistance

(From page No. 96 , C. No: G.2.2 ,IS 456:2000,)

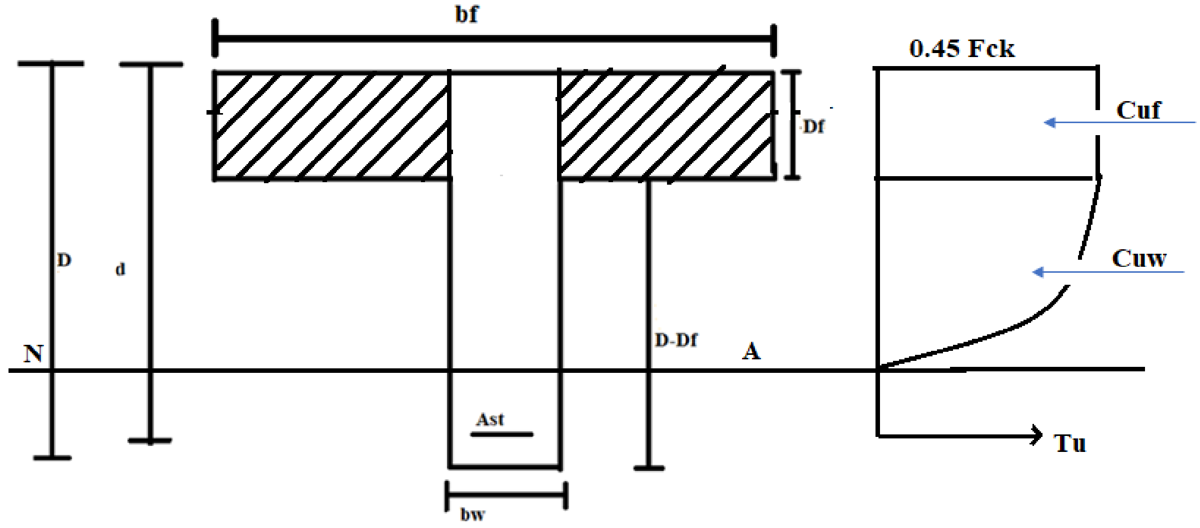
$$Mu = 0.45 \text{ x } F_{ck} \text{ x } (b_f - b_w) \text{ x } D_f \text{ x } \left(d - \frac{D_f}{2} \right) + 0.36 \text{ x } F_{ck} \text{ x } Xu \text{ x } b_w \text{ x } (d - 0.42 \, Xu)$$

$$Mu = 0.45 \text{ x } 15 \text{ x } (1200-275) \text{ x } 100 \text{ x } \left(550 - \frac{100}{2} \right) + 0.36 \text{ x } 15 \text{ x } 252.47 \text{ x } 275 \text{ x } (550 - 0.42 \text{ x } 252.47)$$

$$Mu = 478.64 \text{ x } 10^6 \text{ Nmm} = 478.64 \text{ KNm}$$

CASE III: Neutral axis lies in web ($Xu > D_f$)

$\frac{D_f}{d} > 0.2$ (IS 456:2000, Page Number: 97)



Area of flange= $(b_f - b_w) \text{ x } D_f$
Area of Web= $b_w \text{ x } d$
C_{uf} = Compressive Force in Flange
$C_{uf} = (b_f - b_w) \text{ x } Y_f \text{ x } 0.45 \text{ x } F_{ck}$
C_{uw} = Compressive Force in Web
$C_{uw} = 0.45 \text{ x } F_{ck} \text{ x } Xu \text{ x } b_w$
$Y_f = 0.15 \, Xu + 0.65 \, D_f$ (P. No: 97 , IS 456:2000)
Design Procedure
Given Data

STEP 1: To find depth of neutral axis: Xu

(From page No. 96 IS CODE)

Cu=Tu

$C_{uf} + C_{uw} =$ **Tu**

$(b_f - b_w) \text{ x } Y_f \text{ x } 0.45 \text{ x } F_{ck} + 0.36 \text{ x } F_{ck} \text{ x } Xu \text{ x } b_w = 0.87 \text{ x } Fy \text{ x } Ast$

Xu=?

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.48 d ....For Fe 415

Xu max = 0.46 d ....For Fe 500

STEP 3: To compare Xu and Xu max

a) If $Xu \prec Xu\max$ then section is under reinforced

b) **If** $Xu = Xu\max$ then section is balance section

c) **If** $Xu \succ Xu\max$ then section is over reinforced, if section is over reinforced then consider it as balance section.

STEP 4: To find moment of resistance

(From page No. 97 , C. No: G.2.2.1 ,IS 456:2000,)

$$Mu = 0.45 \text{ x } F_{ck} \text{ x } (b_f - b_w) \text{ x } Y_f \text{ x } \left(d - \frac{Y_f}{2} \right) + 0.36 \text{ x } F_{ck} \text{ x } Xu \text{ x } b_w \text{ x } (d - 0.42\ Xu)$$

1) The simply supported beam space at 2.5 m C/C supported span is 5 m, slab thickness is 100 mm, width of web is 300 mm, overall depth is 450 mm, cover to the steel centre is 65 mm, the reinforcement is 8 bars of 25 mm diameter in two layers. Find ultimate moment of resistance of T beam. Use M_{20} and Fe 415.

Solution: Given Data:-

C/C Spacing = 2.5 m =2500 mm

l= l_0 =5 m= 5000 mm

Depth of flange= D_f = 100 mm

Width of web =b_w =300 mm

Overall depth =D= 450 mm

Effective Cover= d'= 65 mm

Effective depth =d= D-d'=450-65=385 mm

ϕ =25 mm

N=8

For T beam

$b_f = \frac{l_0}{6} + b_w + 6D_f <$ C/C distance

$b_f = \frac{5000}{6} + 300 + 6x100 < 2500mm$

$b_f = 1733.33$ mm $< 2500\ mm$ (ok)

Ast =8x$\frac{\pi}{4}$ X ϕ^2 =**8x**$\frac{\pi}{4}$ X 25^2 =3926.99 mm^2

M_{20} = Fck = 20 N/mm^2

Fe 415 = Fy = 415 N/mm^2

STEP 1: To find depth of neutral axis : $Xu < D_f$ (Neutral axis lies in flange)

(From page No. 96 IS CODE)

Cu = Tu

$0.36FckXub_f = 0.87\ Fy\ Ast$

$$Xu=\frac{0.87\ Fy\ Ast}{0.36\ Fck\ b_f}$$

$$Xu=\frac{0.87 \times 415 \times 3926.99}{0.36 \times 20 \times 1733.33} = 113.61\ mm$$

$xu > D_f$

113.61>100 ,The assumption is wrong

Assuming Neutral axis lies in web ($Xu > D_f$)

$$\frac{D_f}{d} > 0.2$$

$$\frac{100}{385} > 0.2$$

$$0.2597 > 0.2$$

This is CASE III

Y_f =0.15 Xu + 0.65 D_f (P. No: 97 , IS 456:2000)

Cu=Tu

$C_{uf} + C_{uw}$ = **Tu**

$(b_f - b_w) \times Y_f \times 0.45 \times F_{ck} + 0.36 \times F_{ck} \times Xu \times b_w = 0.87 \times Fy \times Ast$

(1733.33- 300) x (0.15 Xu + 0.65 x 100) x 0.45 x 20 + 0.36 x 20 x Xu x 300 = 0.87 x 415 x 3926.99

Xu=141.56 mm

$xu > D_f$

141.56 >100 ,The assumption is correct

Y_f =0.15 Xu + 0.65 D_f (P. No: 97 , IS 456:2000)

Y_f =0.15 Xu + 0.65 D_f =0.15x 141.56 +0.65 x100=86.23 mm

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48d ....For Fe 415

Xu max = 0.48 x385 = 184.8 mm

STEP 3: To compare Xu and Xu max

$Xu \prec Xu\,max$

$141.56 \prec 184.8$

then section is under reinforced

STEP 4: To find moment of resistance

(From page No. 97 , C. No: G.2.2.1 ,IS 456:2000)

$$Mu = 0.45 \text{ x } F_{ck} \text{ x } (b_f - b_w) \text{ x } Y_f \text{ x } \left(d - \frac{Y_f}{2} \right) + 0.36 \text{ x } F_{ck} \text{ x } Xu \text{ x } b_w \text{ x } (d - 0.42\, Xu)$$

$$Mu = 0.45 \text{ x } 20 \text{ x } (1733.33-300) \text{ x } 86.23 \text{ x } \left(385 - \frac{86.23}{2} \right) + 0.36 \text{ x } 20 \text{ x } 141.56 \text{ x } 300 \text{ x } (385 - 0.42 \text{ x } 141.56)$$

$$Mu = 479.84 \text{ x } 10^6 \text{ Nmm} = 479.84 \text{ KNm}$$

2) The simply supported beam having width of flange is 1900 mm, slab thickness is 100 mm, width of web is 350 mm, overall depth is 500 mm, cover to the steel centre is 50 mm, the reinforcement is 8 bars of 25 mm diameter in two layers. Find ultimate moment of resistance of T beam. Use M_{20} and Fe 415.

Solution: Given Data:-

Width of flange =b_f =1900 mm

Depth of flange= D_f = 100 mm

Width of web =b_w =350 mm

Overall depth =D= 500 mm

Effective Cover= d'= 50 mm

Effective depth =d= D-d'=500-50=450 mm

ϕ =25 mm

N=8

Ast =8x$\frac{\pi}{4}$ X ϕ^2 =**8x**$\frac{\pi}{4}$ X 25^2 =3926.99 mm^2

M_{20} = Fck = 20 N/mm^2

Fe 415 = Fy = 415 N/mm^2

STEP 1: To find depth of neutral axis : Xu < D_f (Neutral axis lies in flange)

(From page No. 96 IS CODE)

$Cu = Tu$

$0.36 Fck Xu b_f = 0.87\ Fy\ Ast$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b_f}$$

$$Xu = \frac{0.87 \times 415 \times 3926.99}{0.36 \times 20 \times 1900} = 103.643 \text{ mm}$$

$xu > D_f$

103.643>100 ,The assumption is wrong

Assuming Neutral axis lies in web ($Xu > D_f$)

$$\frac{D_f}{d} > 0.2$$

$$\frac{100}{450} > 0.2$$

$0.2222 > 0.2$

This is CASE III

$Y_f = 0.15\ Xu + 0.65\ D_f$ (P. No: 97 , IS 456:2000)

Cu=Tu

$C_{uf} + C_{uw} = \mathbf{Tu}$

$(b_f - b_w) \times Y_f \times 0.45 \times F_{ck} + 0.36 \times F_{ck} \times Xu \times b_w = 0.87 \times Fy \times Ast$

$(1900 - 350) \times (0.15\ Xu + 0.65 \times 100) \times 0.45 \times 20 + 0.36 \times 20 \times Xu \times 350 = 0.87 \times 415 \times 3926.99$

Xu=110.81 mm

$xu > D_f$

110.81 >100 ,The assumption is correct

$Y_f = 0.15\ Xu + 0.65\ D_f$ (P. No: 97 , IS 456:2000)

$Y_f = 0.15\ Xu + 0.65\ D_f = 0.15 \times 110.81 + 0.65 \times 100 = 81.62$ mm

STEP 2: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48d ....For Fe 415

Xu max = 0.48 x450 = 216 mm

STEP 3: To compare Xu and Xu max

$Xu \prec Xu\,max$

$110.81 \prec 216$

then section is under reinforced

STEP 4: To find moment of resistance

(From page No. 97 , C. No: G.2.2.1 ,IS 456:2000)

$$Mu = 0.45 \text{ x } F_{ck} \text{ x } (b_f - b_w) \text{ x } Y_f \text{ x } \left(d - \frac{Y_f}{2} \right) + 0.36 \text{ x } F_{ck} \text{ x } Xu \text{ x } b_w \text{ x } (d - 0.42 \text{ } Xu)$$

$$Mu = 0.45 \text{ x } 20 \text{ x } (1900-350) \text{ x } 81.62 \text{ x } \left(450 - \frac{81.62}{2} \right) + 0.36 \text{ x } 20 \text{ x } 110.81 \text{ x } 350 \text{ x } (450 - 0.42 \text{ x } 110.81)$$

$$Mu = 578.57 \text{ x } 10^6 \text{ Nmm} = 578.57 \text{ KNm}$$

Type II: To find area of steel

Stepwise Procedure

To find :- The area of steel (Ast)

Given Data:- b_f,b_w,d, Fy, Fck, Design Moment (M_d) or loading

Effective depth =d = D - Effective cover

Effective depth =d = D - d'

Effective cover= d' = Clear cover + $\frac{\phi}{2}$

STEP 1: To find ultimate moment

Assuming Xu =D_f (Balance Section)

Mu_1 = 0.36 x F_{ck} x Xu x b_f x (d-0.42x Xu)

Put Xu= D_f

STEP 2: To compare M_{u1} and M_d

If $M_d \leq Mu_1$ then to find area of steel

The assumption is correct

NA lies in flange

STEP 3: To calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy} \left[1 - \sqrt{1 - \frac{4.6M_d}{Fckb_f d^2}} \right] b_f d$$

Assume diameter of bar = Φ

Number of bars = $\frac{Ast}{(\pi / 4) \text{ x } \phi^2}$

$A_{st\,min} = \dfrac{0.85 \; b_w \; d}{Fy}$ .... IS CODE PAGE NO 47

Maximum Area of steel for beam

$A_{st\,Max} = 0.04 \times b_w \times D$ .... IS CODE PAGE NO 47

$A_{st\,min} < A_{st\,max}$

STEP 4: To find depth of neutral axis (From page No. 96 IS CODE)

$Cu = Tu$

$0.36 Fck Xu b_f = 0.87\ Fy\ Ast$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b_f}$$

$Xu < D_f$ Assumption is correct

STEP 5:: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.53 d ....For Fe 250

Xu max = 0.48 d ....For Fe 415

Xu max = 0.46 d ....For Fe 500

STEP 6: To compare Xu and Xu max

a) If $Xu \prec Xu\,max$ then section is under reinforced

b) **If** $Xu = Xu\,max$ then section is balance section

c) **If** $Xu \succ Xu\,max$ then section is over reinforced, if section is over reinforced then consider it as balance section.

1) Find area of steel required in T beam for the following particulars

Simply supported span = 7 m

Slab thickness = 120 mm

Width of web = 230 mm

Effective depth = 560 mm

Inclusive working load on beam = 50 KN/m

Spacing of beam = 4 m c/c

Used Fe 415 and M_{20}.

Solution: Given Data:- Simply supported beam

C/C Spacing = 4 m =4000 mm

$l = l_0$ =7m= 7000 mm

Depth of flange= D_f = 120 mm

Width of web =b_w =230 mm

Effective depth =d= 560 mm

For T beam

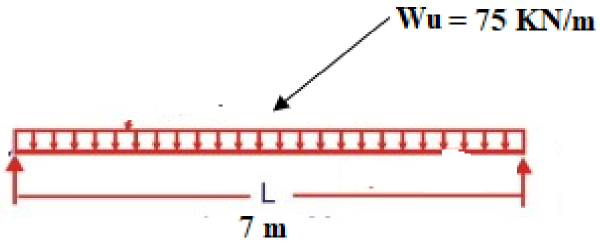
$$b_f = \frac{l_0}{6} + b_w + 6D_f < \text{ C/C distance}$$

$$b_f = \frac{7000}{6} + 230 + 6x120 < 4000mm$$

$$b_f = 2116.67 \text{ mm } < 4000 \; mm \text{ (ok)}$$

Inclusive working load on beam = W=50 KN/m

Factored Load = Wu= 1.5 x W = 1.5 x 50 = 75 KN/m



Maximum bending moment = $\frac{Wul^2}{8} = \frac{75 \text{ x } 7^2}{8}$ =459.38 KNm

M_{20} = Fck = 20 N/mm^2

Fe 415 = Fy = 415 N/mm^2

STEP 1: To find ultimate moment

Assuming Xu =D_f (Balance Section)

Mu_1 = 0.36 x F_{ck} x Xu x b_f x (d-0.42x Xu)

Put Xu= D_f=120 mm

Mu_1 = 0.36 x 20 x 120 x 2116.67 x (560-0.42x 120)

Mu_1 =931.96 X 10^6 Nmm =931.96 KNm

STEP 2: To compare M_{u1} and M_d

$M_d \leq Mu_1$

459.38 $\leq$ 931.96

The assumption is correct

NA lies in flange

STEP 3: To calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckb_f d^2}}\right]b_f d$$

$$A_{st} = \frac{0.5 \text{ x } 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \text{x } 459.38 \text{ x } 10^6}{20 \text{ x } 2116.67 \text{ x } 560^2}}\right] 2116.67 \text{ x } 560$$

A_{st} =2371.64 mm²

Assume diameter of bar = Φ =20 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2} = \frac{2371.64}{(\pi/4) \text{ x } 20^2} = 7.55 \cong 8$

$$A_{st\ min} = \frac{0.85 \text{ b}_w \text{ d}}{Fy}$$ .... IS CODE PAGE NO 47

$$A_{st\ min} = \frac{0.85 \text{ x } 230 \text{ x } 560}{415} = 263.81 \text{ mm}^2$$

Maximum Area of steel for beam

$Ast_{Max} = 0.04 \text{ x } b_w \text{ x } D$ .... IS CODE PAGE NO 47

Assuming Effective cover =d'= 50 mm

Overall depth =D= d+d'= 560+50 =610 mm

$Ast_{Max} = 0.04 \text{ x } 230 \text{ x } 610$

$Ast_{Max} = 5612 \text{ mm}^2$

$A_{st\ min} < A_{st\ max}$

263.81 < 5612 (ok)

STEP 4: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$0.36 Fck Xub = 0.87 \text{ Fy Ast}$

$$Xu = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck } b_f} = \frac{0.87 \text{ x } 415 \text{ x } 2371.64}{0.36 \text{ x } 20 \text{ x } 2116.67} = 54.89 \ mm$$

$Xu < D_f$

54.89<100 , Assumption is correct

STEP 5:: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 560 = 268.8

STEP 6: To compare Xu and Xu max

$Xu \prec Xu \max$

54.89 < 268.8

then section is under reinforced

2) Find area of steel required in T beam for the following particulars

Width of flange = 1100 mm

Slab thickness = 120 mm

Width of web = 275 mm

Overall Depth = 650 mm

Effective cover =50 mm

Effective depth = 600 mm

Ultimate Moment = 380 KNm

Used Fe 415 and M_{15}.

Solution: Given Data:-

Width of flange = b_f =1100 mm

Slab thickness =D_f =120 mm

Width of web =b_w =275 mm

Overall Depth = D= 650 mm

Effective cover =d'=50 mm

Effective depth = d=600 mm

Ultimate Moment = M_d =380 KNm = 380 x 10^6 Nmm

M_{15} = Fck = 15 N/mm^2

Fe 415 = Fy = 415 N/mm^2

STEP 1: To find ultimate moment

Assuming Xu =D_f (Balance Section)

Mu_1 = 0.36 x F_{ck} x Xu x b_f x (d-0.42x Xu)

Put Xu= D_f=120 mm

Mu_1 = 0.36 x 15 x 120 x 1100 x (600-0.42x 120)

Mu_1 =391.75 X 10^6 Nmm =391.75 KNm

STEP 2: To compare M_{u1} and M_d

$M_d \leq Mu_1$

380 $\leq$ 391.75

The assumption is correct

NA lies in flange

STEP 3: To calculate area of steel

$$\mathbf{A_{st}} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckb_f d^2}}\right]b_f d$$

$$\mathbf{A_{st}} = \frac{0.5 \text{ x } 15}{415}\left[1-\sqrt{1-\frac{4.6\text{x } 380 \text{ x } 10^6}{15 \text{ x } 1100 \text{ x } 600^2}}\right]1100 \text{ x}600$$

A_{st} =1907.5 mm²

Assume diameter of bar = Φ =20 mm

Number of bars = $\dfrac{Ast}{(\pi/4) \text{ x } \phi^2} = \dfrac{1907.50}{(\pi/4) \text{ x } 20^2} = 6.07 \cong 7$

$A_{st\ min} = \dfrac{0.85\ b_w\ d}{Fy}$ .... IS CODE PAGE NO 47

$A_{st\ min} = \dfrac{0.85 \text{ x } 275 \text{ x } 600}{415} = 337.91 \text{ mm}^2$

Maximum Area of steel for beam

Ast $_{Max}$ = 0.04 x b_w x D .... IS CODE PAGE NO 47

Ast $_{Max}$ = 0.04 x 275 x 650

Ast $_{Max}$ = 7150 mm²

$A_{st\ min} < A_{st\ max}$

337.95 < 7150 (ok)

STEP 4: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$0.36FckXub = 0.87 \text{ Fy Ast}$

$Xu = \dfrac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck } b_f} = \dfrac{0.87 \text{ x } 415 \text{ x } 1907.50}{0.36 \text{ x } 15 \text{ x } 1100} = 115.94\ mm$

Xu< D_f

115.94<120 , Assumption is correct

STEP 5:: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 600 = 288 mm

STEP 6: To compare Xu and Xu max

$Xu \prec Xu\,max$

115.94 < 288

then section is under reinforced

Type III: Design of 'T' and 'L' section

1) Design a 'T' beam for a hall 6 m X 15 m having beam 3 m centre to centre , the slab thickness is 120 mm cast monolithically with beam having live load is 3 KN/m^2 , floor finish is 1 KN/m^2. Used M_{15} and Fe 415.

Solution: Given Data:- T beam

Hall Size 6 m X 15 m

C/C Spacing = 3m =3000 mm

l=6m= 6000 mm

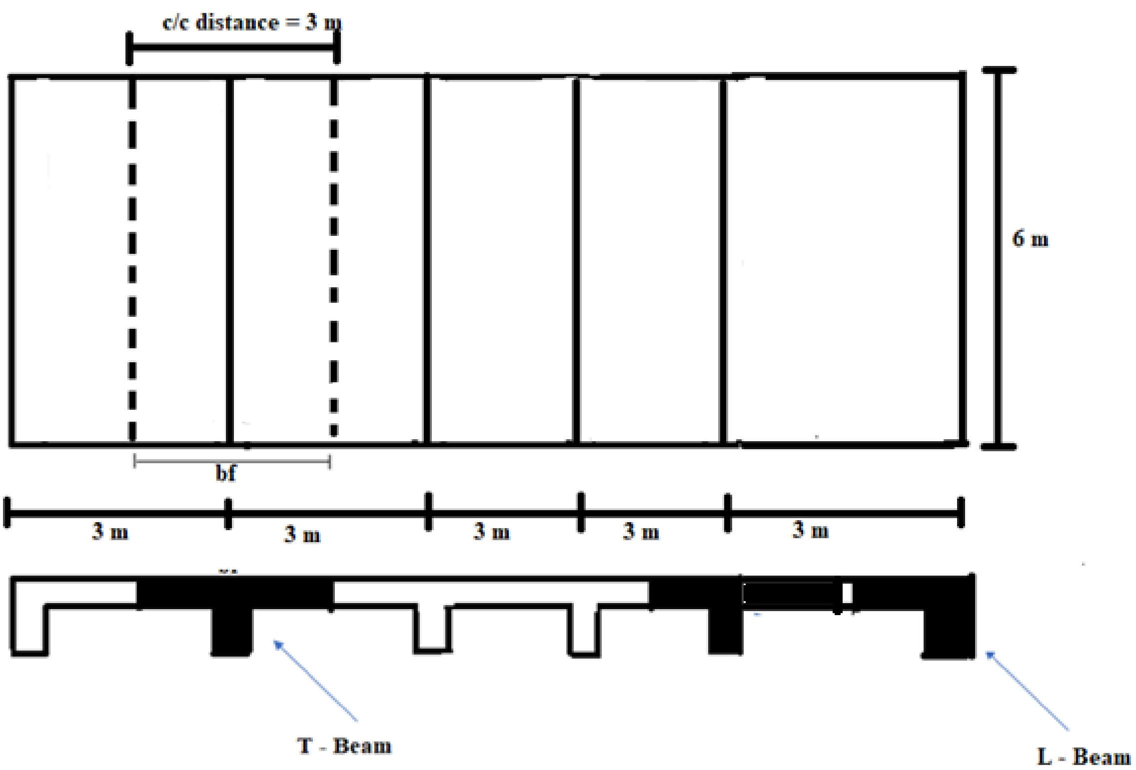
Depth of flange/ slab= D_f = 120 mm

Live Load =3 KN/m^2

Floor Finish = 1 KN/m^2

M_{15} = Fck = 15 N/mm^2

Fe 415 = Fy = 415 N/mm^2



STEP 1: To find dimensions

Overall Depth=D $= \frac{l}{12}$ to $\frac{l}{15}$

Overall Depth=D $= \frac{6000}{12}$ to $\frac{6000}{15}$

Overall Depth=D =500 *mm* to 400 mm

Assuming D= 500 mm

Assuming effective cover = d'= 50 mm

Effective depth = d= D-d'=500-50= 450 mm

Width of beam

$b_w = \frac{D}{3}$ to $\frac{2D}{3}$

$b_w = \frac{500}{3}$ to $\frac{2x500}{3}$

b_w =166.66 mm to 333.33 mm

Assu min *g* b_w =300 mm

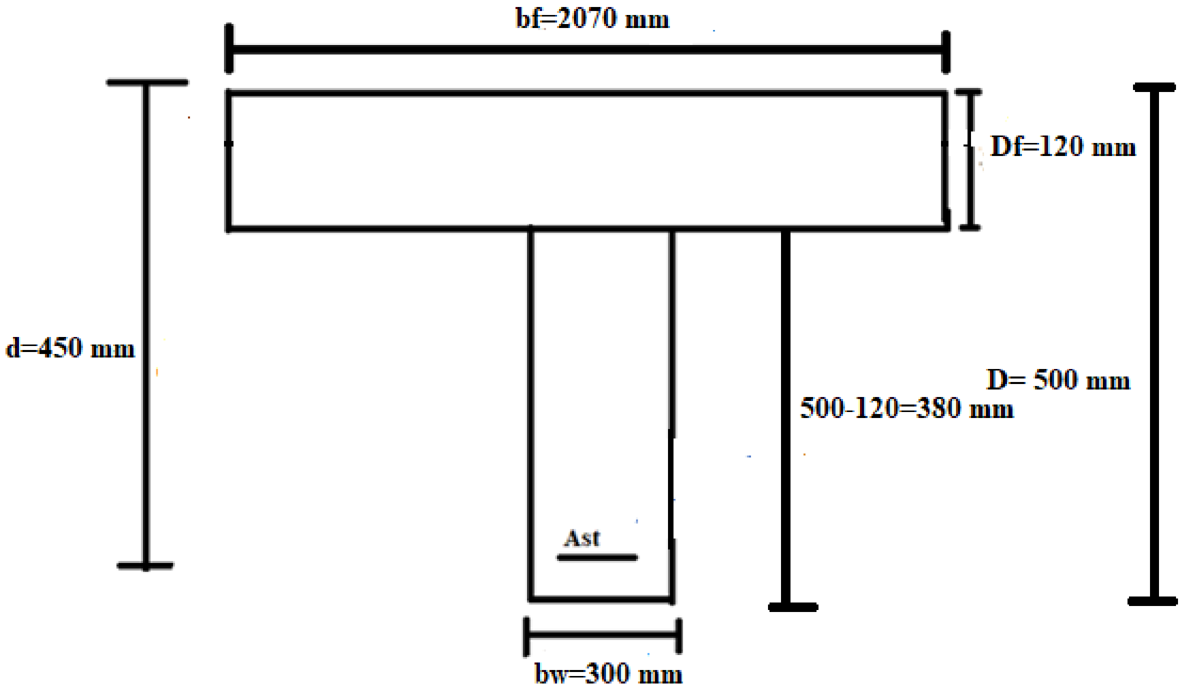
For T beam

$b_f = \frac{l_0}{6} + b_w + 6D_f <$ C/C distance

$l_0 = l + \frac{b_w}{2} + \frac{b_w}{2} = 6000 + \frac{300}{2} + \frac{300}{2} = 6300$

$b_f = \frac{6300}{6} + 300 + 6x120 < 3000mm$

b_f = 2070 mm $< 3000\ mm$ (ok)



STEP 2: To calculate loading

Loading

a) Live Load =3 KN/m^2
b) Floor Finish = 1 KN/m^2
c) Self Weight of slab = Df x Density of concrete
=0.120 x 25 = 3 KN/m^2

Total Load = a+b+ c = 7 KN/m^2

Load per meter = 7 x C/C distance = 7 x3 = 21 KN/m^2

Self Weight of web = Area of web x Density of concrete

Self Weight of web = 0.3 x 0.38 x 25 = 2.850 KN/m^2

Total Load = 21 + 2.850 = 23.850 KN/m^2

Factored load = Wu= 1.5 x 23. 850= 35.775 KN/m^2

Maximum bending moment = $M_d = \frac{Wul_0^2}{8} = \frac{35.775 \text{ x } 6.3^2}{8} = 177.49$ KN.m

STEP 3: To find ultimate moment

Assuming Xu =D_f (Balance Section)

Mu_1 = 0.36 x F_{ck} x Xu x b_f x (d-0.42x Xu)

Put Xu= D_f=120 mm

Mu_1 = 0.36 x 15 x 120 x 2070 x (450-0.42x 120)

Mu_1 =536 X 10^6 Nmm =536 KNm

STEP 4: To compare M_{u1} and M_d

$M_d \leq Mu_1$

177.79 $\leq$ 536

The assumption is correct

NA lies in flange

STEP 5: To calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckb_f d^2}}\right]b_f d$$

$$A_{st} = \frac{0.5 \text{ x } 15}{415}\left[1-\sqrt{1-\frac{4.6\text{x } 177.49 \text{ x } 10^6}{15 \text{ x } 2070 \text{ x } 450^2}}\right]2070 \text{ x}450$$

A_{st} =1132.40 mm^2

Assume diameter of bar = Φ =20 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2} = \frac{1132.40}{(\pi/4) \text{ x } 20^2} = 3.60 \cong 4$

$A_{st\,min} = \frac{0.85 \, b_w \, d}{Fy}$ .... IS CODE PAGE NO 47

$A_{st\,min} = \frac{0.85 \text{ x } 300 \text{ x } 450}{415}$ =276.51 mm^2

Maximum Area of steel for beam

Ast $_{Max}$ = 0.04 x b_W x D .... IS CODE PAGE NO 47

Ast $_{Max}$ = 0.04 x 300 x 500

Ast $_{Max}$ = 6000 mm^2

$A_{st\ min} < A_{st\ max}$

276.51 < 6000 (ok)

STEP6: To find depth of neutral axis (From page No. 96 IS CODE)

$$Cu = Tu$$

$$0.36 Fck Xu b_f = 0.87\ Fy\ Ast$$

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b_f} = \frac{0.87 \times 415 \times 1132.40}{0.36 \times 15 \times 2070} = 36.57\ mm$$

$Xu < D_f$

36.57<120 , Assumption is correct

STEP 7: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 450 = 216 mm

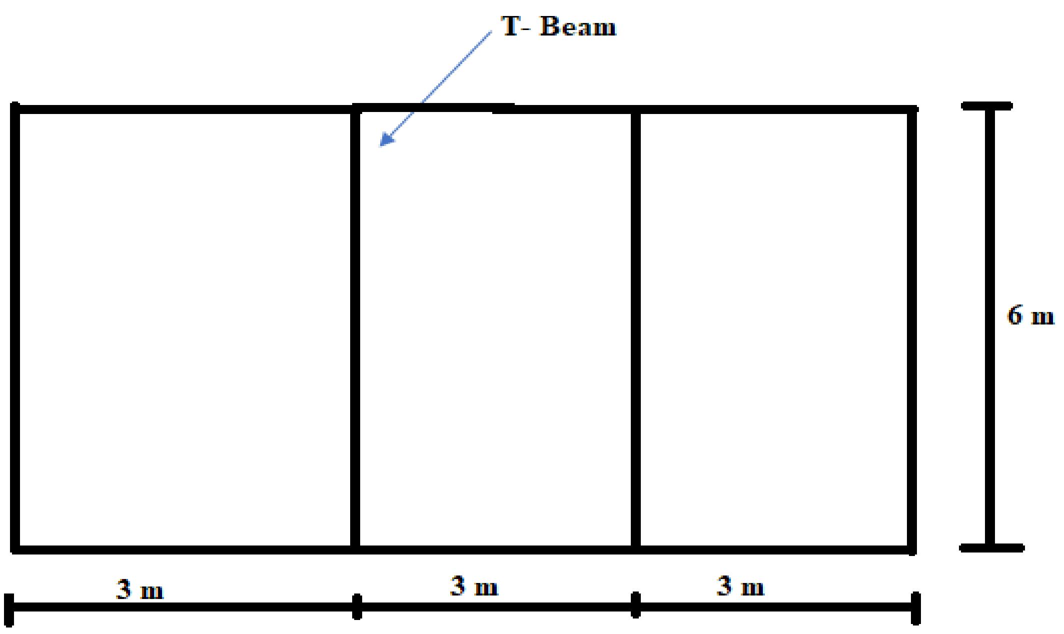
STEP 8: To compare Xu and Xu max

$Xu \prec Xu\ max$

36.57 < 216

then section is under reinforced

2) Design intermediate beam for the slab beam system given below by IS Code method using M_{15} and Fe 415.
   Udl on slab = 5 KN/m^2
   Floor Finish = 1 KN/m^2
   Slab Thickness = 100 mm , Use M_{15} and Fe 415 for following figure



Solution: Given Data:- T beam

Hall Size 6 m X 15

C/C Spacing = 3m =3000 mm

l=6m= 6000 mm

Depth of flange/ slab= D_f = 100 mm

Udl on slab = 5 KN/m^2

Floor Finish = 1 KN/m^2

Fe 415 = Fy = 415 N/mm^2

M_{15} = Fck = 15 N/mm^2

STEP 1: To find dimensions

$$\text{Overall Depth=D} = \frac{l}{12} \text{ to } \frac{l}{15}$$

$$\text{Overall Depth=D} = \frac{6000}{12} \text{ to } \frac{6000}{15}$$

$$\text{Overall Depth=D} = 500\ mm \text{ to } 400 \text{ mm}$$

Assuming D= 500 mm

Assuming effective cover = d'= 50 mm

Effective depth = d= D-d'=500-50= 450 mm

Width of beam

$$b_w = \frac{D}{3} \text{ to } \frac{2D}{3}$$

$$b_w = \frac{500}{3} \text{ to } \frac{2x500}{3}$$

$$b_w = 166.66 \text{ mm to } 333.33 \text{ mm}$$

$$Assu\min g\ b_w = 300 \text{ mm}$$

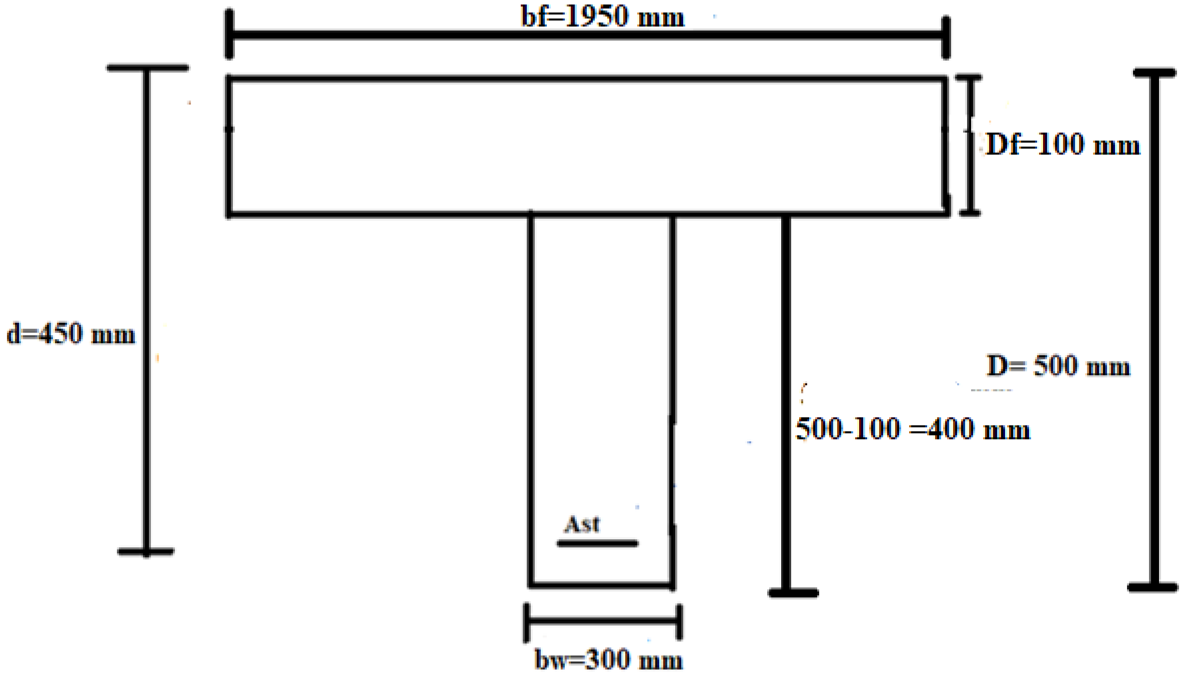
For T beam

$$b_f = \frac{l_0}{6} + b_w + 6D_f < \text{ C/C distance}$$

$$l_0 = l + \frac{b_w}{2} + \frac{b_w}{2} = 6000 + \frac{300}{2} + \frac{300}{2} = 6300$$

$$b_f = \frac{6300}{6} + 300 + 6x100 < 3000mm$$

$$b_f = 1950 \text{ mm } < 3000\ mm \text{ (ok)}$$



STEP 2: To calculate loading

Loading

a) Live Load (Udl) =5 KN/m^2
b) Floor Finish = 1 KN/m^2
c) Self Weight of slab = Df x Density of concrete
=0.1 x 25 = 2.5 KN/m^2

Total Load = a+b+ c = 8.5 KN/m^2

Load per meter = 8.5 x C/C distance = 8.5 x3 = 25.5 KN/m^2

Self Weight of web = Area of web x Density of concrete

Self Weight of web = 0.3 x 0.4 x 25 = 3 KN/m^2

Total Load = 25.5 + 3 = 28.3 KN/m^2

Factored load = Wu= 1.5 x 28.3= 42.75 KN/m^2

Maximum bending moment = $M_d = \frac{Wul_0^2}{8} = \frac{42.75 \text{ x } 6.3^2}{8} = 212.09$ KN.m

STEP 3: To find ultimate moment

Assuming Xu =D_f (Balance Section)

Mu_1 = 0.36 x F_{ck} x Xu x b_f x (d-0.42x Xu)

Put Xu= D_f=100 mm

Mu_1 = 0.36 x 15 x 100 x 1950 x (450-0.42x 100)

Mu_1 =429.64 X 10^6 Nmm =429.64 KNm

STEP 4: To compare M_{u1} and M_d

$M_d \leq Mu_1$

212.09 ≤ 429.64

The assumption is correct

NA lies in flange

STEP 5: To calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckb_f d^2}}\right]b_f d$$

$$A_{st} = \frac{0.5 \text{ x } 15}{415}\left[1-\sqrt{1-\frac{4.6\text{x } 212.09 \text{ x } 10^6}{15 \text{ x } 1950\text{x } 450^2}}\right]1950 \text{ x}450$$

A_{st} =1364.77 mm^2

Assume diameter of bar = Φ =20 mm

Number of bars = $\frac{Ast}{(\pi/4) \text{ x } \phi^2} = \frac{1364.77}{(\pi/4) \text{ x } 20^2} = 4.34 \cong 5$

$$A_{st\,min} = \frac{0.85\ b_w\ d}{Fy}$$ .... IS CODE PAGE NO 47

$$A_{st\,min} = \frac{0.85 \text{ x } 300 \text{ x } 450}{415} = 276.51 \text{ mm}^2$$

Maximum Area of steel for beam

Ast $_{Max}$ = 0.04 x b_w x D .... IS CODE PAGE NO 47

Ast $_{Max}$ = 0.04 x 300 x 500

Ast $_{Max}$ = 6000 mm^2

$A_{st\,min} < A_{st\,max}$

276.51 < 6000 (ok)

STEP 6: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$0.36FckXub_f = 0.87$ Fy Ast

$$Xu = \frac{0.87 \text{ Fy Ast}}{0.36 \text{ Fck } b_f} = \frac{0.87 \text{ x } 415 \text{ x } 1364.77}{0.36 \text{ x } 15 \text{ x } 1950} = 46.79\ mm$$

Xu< D_f

46.79<100 , Assumption is correct

STEP 7: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 450 = 216 mm

STEP 8: To compare Xu and Xu max

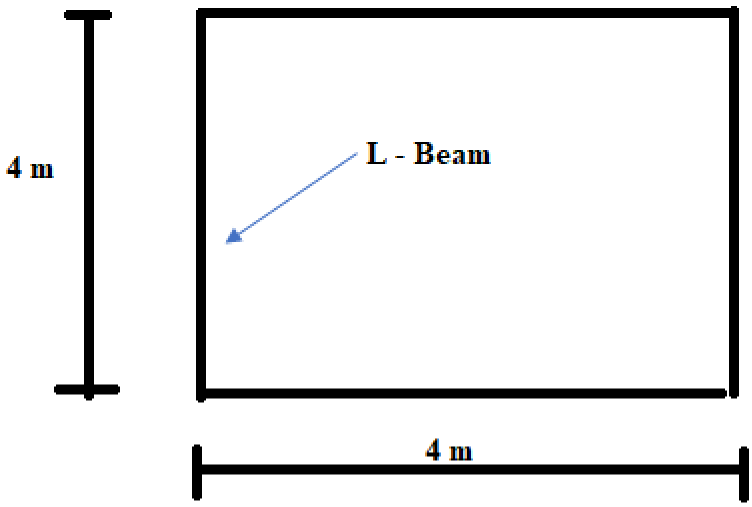
$Xu \prec Xu \max$

46.79 < 216

then section is under reinforced

3) Design ‘L’ beam shown in figure,
Udl on slab= 5 KN/m^2
Floor Finish = 1 KN/m^2
Depth of slab =120 mm

Use M_{20} and Fe 415 for following figure



Solution: Given Data:- L beam

C/C Spacing = 4m =4000 mm

l=4m= 4000 mm

Depth of slab= D_f = 120 mm

Udl on slab = 5 KN/m^2

Floor Finish = 1 KN/m^2

Fe 415 = Fy = 415 N/mm^2

M_{20} = Fck = 20 N/mm^2

STEP 1: To find dimensions

Overall Depth=D $= \frac{l}{12}$ to $\frac{l}{15}$

Overall Depth=D $= \frac{4000}{12}$ to $\frac{4000}{15}$

Overall Depth=D =333.33 *mm* to 266.67 mm

Assuming D= 300 mm

Assuming effective cover = d'= 50 mm

Effective depth = d= D-d'=300-50= 250 mm

Width of beam

$b_w = \frac{D}{3}$ to $\frac{2D}{3}$

$b_w = \frac{300}{3}$ to $\frac{2x300}{3}$

b_w =100 mm to 200 mm

Assu min *g* b_w =200 mm

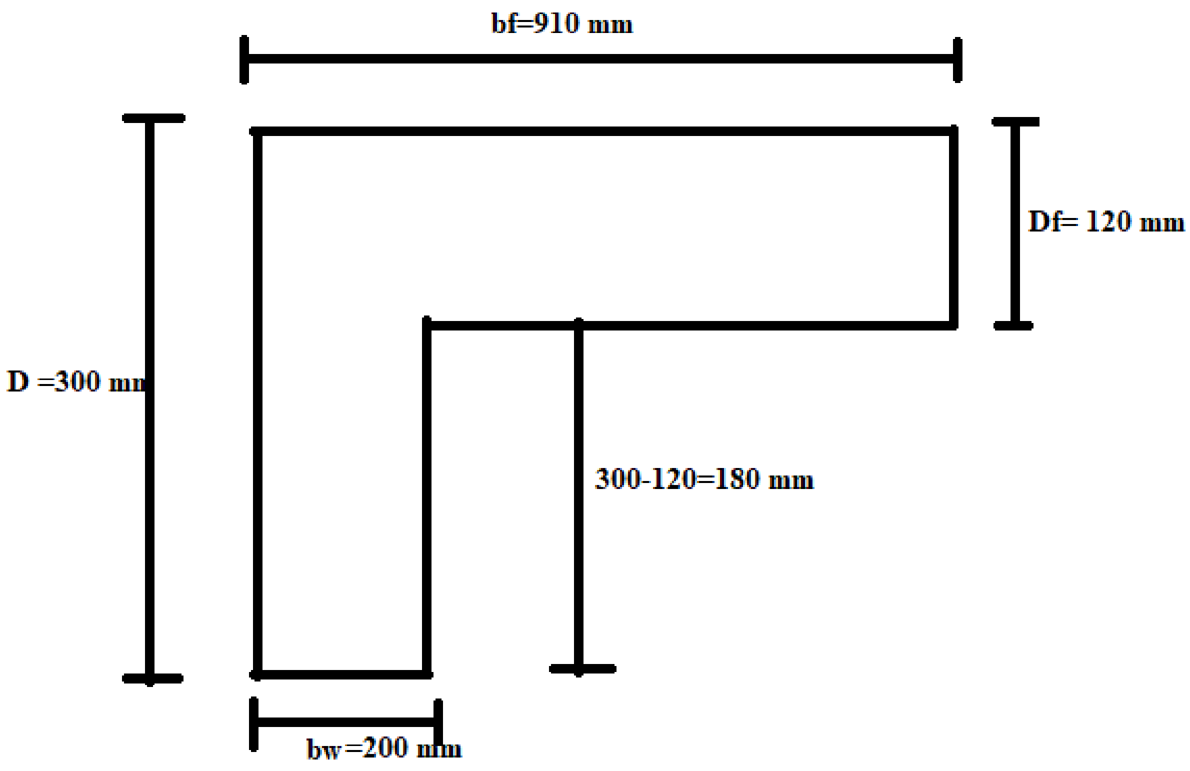
For L beam

$b_f = \frac{l_0}{12} + b_w + 3D_f <$ C/C distance

$l_0 = l + \frac{b_w}{2} + \frac{b_w}{2} = 4000 + \frac{200}{2} + \frac{200}{2} = 4200$

$b_f = \frac{4200}{12} + 200 + 3x120 < 4000mm$

b_f = 910 mm < 4000 *mm* (ok)



STEP 2: To calculate loading

Loading

a) Live Load (Udl) =5 KN/m^2
b) Floor Finish = 1 KN/m^2
c) Self Weight of slab = Df x Density of concrete
=0.12 x 25 = 3 KN/m^2

Total Load = a+b+ c = 9 KN/m^2

Load per meter = 9 x C/C distance = 9 x 4 = 36 KN/m^2

Self Weight of web = Area of web x Density of concrete

Self Weight of web = 0.2 x 0.18 x 25 = 0.9 KN/m^2

Total Load = 36 + 0.9 = 36.9 KN/m^2

Factored load = Wu= 1.5 x 36.9= 55.35 KN/m^2

Maximum bending moment = $M_d = \frac{Wul_0^{\ 2}}{8} = \frac{55.35 \text{ x } 4.2^2}{8} = 122.04$ KN.m

STEP 3: To find ultimate moment

Assuming Xu =D_f (Balance Section)

Mu_1 = 0.36 x F_{ck} x Xu x b_f x (d-0.42x Xu)

Put Xu= D_f=120 mm

Mu_1 = 0.36 x 20 x 120 x 910 x (250-0.42x 120)

Mu_1 =156.93X 10^6 Nmm =156.93 KNm

STEP 4: To compare M_{u1} and M_d

$M_d \le Mu_1$

122.02 ≤ 156.93

The assumption is correct

NA lies in flange

STEP 5: To calculate area of steel

$$A_{st} = \frac{0.5Fck}{Fy}\left[1-\sqrt{1-\frac{4.6M_d}{Fckb_f d^2}}\right]b_f d$$

$$A_{st} = \frac{0.5 \text{ x } 20}{415}\left[1-\sqrt{1-\frac{4.6\text{x } 122.02 \text{ x } 10^6}{20\text{x } 910\text{x } 250^2}}\right]910 \text{ x}250$$

A_{st} =1580.28 mm^2

Assume diameter of bar = Φ =20 mm

Number of bars = $\frac{Ast}{(\pi / 4) \text{ x } \phi^2} = \frac{1580.28}{(\pi / 4) \text{ x } 20^2} = 5.030 \cong 5$

$$A_{st\,min} = \frac{0.85 \text{ b}_w \text{ d}}{Fy}$$ .... IS CODE PAGE NO 47

$$A_{st\,min} = \frac{0.85 \text{ x } 200 \text{ x } 250}{415} = 102.40 \text{ mm}^2$$

Maximum Area of steel for beam

Ast $_{Max}$ = 0.04 x b_w x D .... IS CODE PAGE NO 47

Ast $_{Max}$ = 0.04 x 200 x 300

Ast $_{Max}$ = 2400 mm^2

$A_{st\,min} < A_{st\,max}$

102.40 < 2400 (ok)

STEP 6: To find depth of neutral axis (From page No. 96 IS CODE)

Cu = Tu

$0.36FckXub_f = 0.87$ Fy Ast

$$Xu = \frac{0.87\ Fy\ Ast}{0.36\ Fck\ b_f} = \frac{0.87\ x\ 415\ x\ 1580.28}{0.36\ x\ 20\ x\ 910} = 87.08\ mm$$

$Xu < D_f$

87.08<120 , Assumption is correct

STEP 7: To find depth of critical neutral axis (From page No. 70 IS CODE)

Xu max = 0.48 d ....For Fe 415

Xu max = 0.48 x 250 = 120 mm

STEP 8: To compare Xu and Xu max

$Xu \prec Xu\,max$

87.08 < 120

then section is under reinforced